Interest Rate Smoothing and Inflation-Output Variability in a Small Open Economy*

T. C. Y. Kam† and G. C. Lim‡
Department of Economics
University of Melbourne
Parkville
Victoria 3010, Australia

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Abstract

This paper is concerned with the relationship between the interest rate smoothing behavior of a central bank and the variability of inflation and output. The issue is analyzed through the lens of a small open economy dynamic stochastic general equilibrium model with nontraded goods price rigidities and habit persistence. The benchmark model is calibrated to match certain key business cycle features of a small open economy like Australia. Relative to the benchmark model, experiments on a Taylor rule with interest rate smoothing are conducted. Due to the existence of a short run expectational Phillips curve in the model, monetary policy will imply certain trade-offs between inflation and output variance, under sensible parameter values of the model. More importantly, in a world where there exists such a trade-off, greater interest rate smoothing in the Taylor rule can potentially yield lower sacrifices in terms of output variability in return for lower inflation, thus increasing policy effectiveness.

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‡Email: g.lim@unimelb.edu.au.

1 Introduction

Central banks of several industrialized countries have moved in the direction of an inflation-targeting monetary policy. Examples of such countries, where the inflation target is explicitly announced, are Australia, New Zealand, Canada, United Kingdom, Sweden, Finland, and Spain. In general, the choice of the policy instrument tends to be short-term interest rates. As more central banks implement monetary policies with explicit aims to reduce inflation variability, the question of the impact of such policies on output variability becomes more important. As shown in Cecchetti (1997), the slope of the output-inflation variability frontier is quite steep; hence small reductions in inflation variability can lead to large increases in output variability. Consequently, the impact of monetary policy on the variability of output and inflation should be of concern.

In this regard, the role of interest rate smoothing or interest rate policy gradualism on output and inflation variability is of interest. Sack and Wieland (2000) showed that in a VAR framework, policy gradualism can be the result of an optimal interest rate policy under the central bank's uncertainty about the parameters defining the economy. It is also possible that the optimal policy exhibit interest rate smoothing due to the existence of forward-looking private agents or measurement errors in key macroeconomic variables (e.g. Sack 2000). In a recent conference volume on monetary policy rules, Taylor (1999) made the observation that across the different model specifications considered in the volume, interest rate rules that react to the lagged interest rate tended to perform better (as measured by inflation and output variability) in models that have agents forming rational expectations of future states of the economy. However, these results are generally for closed economies. With an open economy, monetary policy is further complicated in that output and inflation are affected either directly or indirectly by foreign shocks that are transmitted though international trade and asset channels.

This paper is concerned with the impact of interest rate smoothing on the variability of output and inflation for a small open economy like Australia. Since the effect of monetary policy impinge on all aspects of the economy, it is necessary to consider this issue in the context of an economy-wide model. This paper adopts the approach of simulating a calibrated dynamic stochastic general equilibrium (DSGE) model to evaluate the effects of interest rate smoothing.

Recent literature on monetary policy using the DSGE approach have either been studies based on closed-economy (e.g. King and Wolman 1996, Yun 1996) or two-country models (e.g. Betts and Devereux 2000, Chari, Kehoe, and McGrattan 2000, Kollmann 1999), but they share common assumptions about nominal rigidities as propagation mechanisms of shocks. There is a small but rapidly growing approach aimed at analyzing similar issues for small open economies. The typical small open economy model adapts the canonical closed-economy sticky price model with richer Calvo (1983)-style staggered price setting by monopolistic competitive firms. Lane (2000) and Sarno (2001) provide further excellent survey of this new set of literature. The unifying theme across models in this area is the notion of nominal rigidities and market imperfections specified at the level of preferences, technologies and rational optimizing behavior in the face of resource constraints and uncertainty.[1]

The addition of habit formation in consumer behavior in models of monetary economics is a more recent innovation. For example, Fuhrer (2000) shows how habit persistence can generate the observed delays and hump shapes in the impulse responses of real spending to various shocks, including monetary shocks. The open-economy DSGE model developed in this paper includes a role for a Campbell and Cochrane (1999)-type model of habit formation such that household utilities are no longer time-separable.

In this paper, the DSGE model, unlike other small open economy models such as those of Ghironi (2000) and Kollman (1997), also distinguishes between tradable and nontradable goods.[2] With sticky prices and nontradable goods, there are two distinct mechanisms that help to amplify and propagate the effect of shocks on real variables, and also on exchange rates.[3] Firstly, as argued by Sarno (2001), a larger share of nontradables in consumption, compared to a model with purely tradable goods, implies that for international asset market equilibrium, shocks to the system would require large exchange rate adjustments supporting a smaller fraction of tradables. So most of the adjustment will be borne by the nominal exchange rate resulting in greater volatility of this variable. Secondly, coupled with sticky prices in the nontraded goods which feed into the consumer price index (CPI), external shocks to the domestic economy would translate into a more volatile real exchange rate as it bears the brunt of increased nominal exchange rate volatility. As the relative price or the marginal rate of substitution between tradable and nontradable goods varies, this would have allocative effects on production in the two sectors producing these goods and thus on consumption and labor choices as well. In short order, an interest rate rule which reacts, in part, to CPI inflation deviations from some target, would also implicitly take into account large exchange rate fluctuations as that constitutes part of the CPI, as shown by Svensson (2000). It is argued that a model which accounts for these features is more suited for policy analyses in the context of industrialized small open economies like Australia.

The paper is organized as follows. In Section 2, the general equilibrium model in terms of tastes, technology and policy regimes is described. The model is then calibrated in Section 3 to mimic a small open economy such as Australia. Section 4 contains the simulations relating interest rates smoothing and inflation-output variability. Section 5 concludes.

[1] Elsewhere, this approach has been dubbed "neomonetarist" (Kimball 1995) and the "new neoclassical synthesis" (Goodfriend and King 1997).

[2] An exception is Jung (2000), but that paper does not focus on endogenous monetary policy.

[3] Although we do not follow the monetarist convention of introducing money explicitly and thus remain silent on primitive monetary shocks, there is still an influence of monetary policy in the model. That is, our specification of an interest rate rule of the central bank creates endogenous monetary changes given primitive shocks to taste or technology. An independently and identically distributed shock to this rate represents a policy surprise. In the language of Frisch (1965), movements in the monetary policy instrument in this model is a *propagation* mechanism and possibly a primitive *impulse* itself.

2 Model

It is assumed that the small open economy model is populated by identical infinitely-lived households that value consumption and leisure in equilibrium. Households' preferences are assumed to be non-time-separable which arises from habit persistence. That is they prefer more gradual changes in the consumption level compared to permanent income hypothesis consumers. There are two kinds of firms in the economy. The first sort is represented by a typical firm which produces goods that are internationally tradable. That is, it is assumed the law of one price holds with respect to tradable goods. The second class of firms is assumed to be monopolistically competitive firms producing differentiated nontradable goods indexed on a fixed interval of $[0,1]$ as in Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987). Using a discrete-time version of Calvo's (1983) model, these firms are assumed to be able to set new prices at some random time. The government sector involves a central bank that targets inflation using an interest rate rule. It is assumed that the rest of the world is exogenous to the domestic environment as is the case of a small open economy. The stochastic processes governing the rest of the world's economy are thus taken as given by the small open economy.

2.1 Households

The representative household budgets its lifetime wealth according to a sequence of budget constraints. The real budget constraint for each period t is given by

$$\begin{aligned} &\frac{B_{t+1}}{P_t} + \frac{e_t B^*_{t+1}}{P_t} + C_t + \frac{V_{T,t} f_{T,t+1} - V_{T,t-1} f_{T,t}}{P_t} \\ &\leq \frac{(1+i_{t-1}) B_t + e_t \left(1+i^*_{t-1}\right) B^*_t}{P_t} + \frac{d_{T,t} f_{T,t}}{P_t} + \frac{(V_{T,t} - V_{T,t-1}) f_{T,t}}{P_t} \\ &\qquad + \int_0^1 \Pi_{N,t}(i)\, di + \frac{W_t N_t}{P_t} - T_t \quad ; \quad t = 0,1,2...,\infty \qquad (1) \end{aligned}$$

where C_t is a real consumption index, $V_{T,t}$ and $f_{T,t}$, respectively, are the nominal value of and the equity share in the the representative tradable goods firm. Dividends paid by tradable goods firms are denoted by $d_{T,t}$. labor income is $W_t N_t$, $P_t T_t$ is nominal lump-sum tax, e_t is the nominal exchange rate and i_{t-1} is the one-period nominal interest rate paid at time t on the risk-free bond, B_t, held between time $t-1$ and t.[4] Lump sum profits from nontraded goods firms and taxes are respectively $\Pi_{N,t}(i)$ and T_t. Equation (1) constrains the household's expenditure each period according to it's wealth. On the left-hand-side of (1), holdings of domestic and foreign bonds, consumption expenditure on goods and services and changes to share holdings in traded goods firms are being financed by wealth derived from interest income accruing from bonds, dividend income, capital gains from a rise in equity value, lump sum profit distributions and labor income, all net of lump-sum tax.

[4] The relevant foreign variables are denoted with an asterisk.

The household has a consumption choice over traded goods, C_T, and differentiated nontraded goods, $C_N(i)$, measurable on firm type, $i \in [0,1]$. Let $H_t = H\left(C^a_{t-1}, ..., C^a_{t-s}\right)$ denote the *external* habit level associated with a list of *aggregate* consumption history, as in Abel's (1990) model of "catching up with the Joneses" or the relative income model of Duesenberry (1949). The household chooses sequences of the consumption index, C_t, labor hours, N_t, and a share in tradable goods firms, $f_{T,t}$, to solve the following problem:

$$\max_{\{C_t, N_t, f_{T,t}\}_{t=0}^{\infty}} \mathbb{E}\left[\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t}\beta_s\right) u\left(C_t, H_t, N_t\right)\middle| \mathcal{F}_0\right] \tag{2}$$

subject to the sequence of budget constraints in (1) and it is assumed that $\beta_0 = 1$. The Bernoulli or period utility takes the isoelastic form

$$u\left(C_t, H_t, N_t\right) = \lim_{\widetilde{\sigma}\to\sigma} \frac{\left[\left(C_t - H_t\right)^{\nu}\left(1 - N_t\right)^{1-\nu}\right]^{1-\widetilde{\sigma}} - 1}{1-\widetilde{\sigma}};\ \sigma \geq 1, \nu \in (0,1) \tag{3}$$

and $E\left(\mathbf{x}_{t+1}|\mathcal{F}_t\right) := E_t\left(\mathbf{x}_{t+1}\right)$ is the expectations operator on the collection of random variables $\mathbf{x}_{t+1}$ conditional on the information set $\mathcal{F}_t$ at time t, when the expectation or forecast is made.

It is convenient to introduce a service process which captures the relationship between the consumption index and habit. Define the *surplus consumption ratio* as $S^a_t := \left(C^a_t - H_t\right)/C^a_t$. In an extremely bad state of nature, $S_t \downarrow 0$, as consumption falls to the subsistence or habit level. In an "upswing" when consumption rises relative to habit, $S_t \uparrow 1$. With this specification of habit formation, the coefficient of relative risk aversion is state-dependent, $-C_t u_{cc}(.)/u_c(.) = \sigma/S_t$. That is, in each period, if S_t is low (bad state), the local curvature of the Bernoulli utility function rises, implying greater aversion toward risk. If S_t is high, the converse is true. Since identical households choose the same level of consumption in equilibrium, one obtains $C_t = C^a_t$ and $S_t = S^a_t$. Following Campbell and Cochrane (1999), the assumptions are that habit is predetermined at the steady state $S_t = S_{ss}$, $\forall t$; and that habit is predetermined near the steady state or, equivalently, it moves nonnegatively with consumption everywhere.

Thus, unlike standard habit formation models, consumption in this model does not fall below habit. The requirement of Campbell and Cochrane (1999) that the risk-free interest rate is constant is relaxed here as the riskless rate in the model is now endogenously determined. The log surplus consumption ratio, $s_t := \ln S_t$, evolves according to a Markov process,

$$s_{t+1} = (1-\zeta)\, s_{ss} + \zeta s_t + \lambda\left[\widehat{c}_{t+1} - \widehat{c}_t\right] \tag{4}$$

where innovations to the consumption process is transferred to s_t via the factor loading λ. This persistent service process will then give rise, indirectly, to a habit stock which is a function of current and past consumption levels.

The household's subjective discount factor is modeled as a first-order exponential Markov process:

$$\ln\beta_{t+1} = (1-b)\ln\beta + b\ln\beta_t + u_{\beta,t+1}; \quad b \in (0,1), u_\beta \sim \text{i.i.d.}\left(0, \sigma^2_\beta\right) \tag{5}$$

where $\beta_0 = 1$. The rationale for including an exogenous time-dependent process for β is twofold. Firstly, it allows for more variability in the consumption process of households which is rationalized here as random effects impinging on households' psychology and the way they perceive utility of consumption and leisure in an uncertain future. However, the model is silent on the issue of what those random factors may be. The same model does appear in Canton (2001) in a model of stochastic endogenous growth. Secondly, this shock at the level of preferences is used to motivate a shock to a forward-looking IS equation.

The demand for differentiated nontradables $C_{N,t}(i)$ is aggregated by some sub-utility function defined by the CES aggregator:

$$C_{N,t} = \left(\int_0^1 C_{N,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{6}$$

where the elasticity of substitution among goods within the index is given by the scalar $\varepsilon > 1$. The indices of consumption demand for tradables and nontradables, $C_{T,t}$ and $C_{N,t}$, in turn are aggregated as C_t for the purposes of the household's intertemporal choice:

$$C_t = \left[\alpha^{\frac{1}{\eta}} C_{T,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{N,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{7}$$

where the elasticity of substitution between the indices of home and foreign goods is given by $\eta > 0$.

Thus the household first solves dynamically for the optimal consumption index C_t, given the budget allocated for consuming C_t, and the household then allocates this among the demand for the two goods, $C_{T,t}$ and $C_{N,t}$, which is a static optimization problem. In turn, $C_{N,t}$ is optimally subdivided amongst differentiated goods, $C_{N,t}(i)$. Define the stochastic discount factor as $Q_{t,t+1} := \beta_{t+1} \left(\frac{C_{t+1}}{C_t} \frac{S_{t+1}}{S_t} \right)^{\nu(1-\sigma)-1} \left(\frac{1-N_{t+1}}{1-N_t} \right)^{(1-\nu)(1-\sigma)}$ which is the ratio of marginal utilities of consumption over two periods. The resulting first order conditions for the household's maximum problem for all time periods indexed by $t = 0, 1, ..., \infty$:

$$N_t: \quad \left(\frac{1-\nu}{\nu} \right) \frac{C_t S_t}{1-N_t} = \frac{W_t}{P_t} \tag{8}$$

$$B_{t+1}: \quad \mathbb{E}_t \left\{ Q_{t,t+1} \left(\frac{P_t}{P_{t+1}} \right) \right\} = \frac{1}{1+i_t} \tag{9}$$

$$B^*_{t+1}: \quad \mathbb{E}_t \left\{ Q_{t,t+1} \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{e_{t+1}}{e_t} \right) \right\} = \frac{1}{1+i^*_t} \tag{10}$$

$$f_{t+1}(i): \quad \mathbb{E}_t \left\{ Q_{t,t+1} \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{V_{T,t+1} + d_{T,t+1}}{V_{T,t}} \right) \right\} = 1 \tag{11}$$

$$C_{T,t}: \quad C_{T,t} = \alpha \left(\frac{P_{T,t}}{P_t} \right)^{-\eta} C_t \tag{12}$$

$$C_{N,t}: \quad C_{N,t} = (1-\alpha)\left(\frac{P_{N,t}}{P_t}\right)^{-\eta} C_t \tag{13}$$

$$C_{N,t}(i): \quad C_{N,t}(i) = \left(\frac{P_{N,t}(i)}{P_{N,t}}\right)^{-\varepsilon} C_{N,t}; \qquad \forall i \in [0,1] \tag{14}$$

where

$$P_{N,t} = \left(\int_0^1 P_{N,t}(i)^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}} \tag{15}$$

and the consumer price index (CPI) is given by

$$P_t = \left[\alpha P_{T,t}^{1-\eta} + (1-\alpha) P_{N,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{16}$$

Equation (8) defines the household's labor supply by virtue of the household equating the marginal rate of substitution between consumption and leisure with the real wage rate. Condition (9) has the household equating the marginal utility from consumption at time t with the discounted marginal utility of consumption at time $t+1$. Like the consumption Euler equation in (9), equation (10) characterizes optimal holdings of an index of foreign risk-free bonds. Equation (11) is the Euler equation for holding an asset of type i or share in firm of type i. The last three equations given in (12)-(14) are, respectively, the demand functions for the indices of traded and nontraded goods, and the individual differentiated nontraded goods demand, under intratemporal optimality conditions.

It is useful to note the asset pricing implications of the optimality conditions for the household. Intuitively, the Euler equations for holding bonds and the share in each firm i, impose restrictions on the conditional moments of the financial assets. That is, functional equations (9), (10) and (11) are the pricing kernels for the respective assets in that the expected one-period payoffs from holding those assets are equal to the price of each basic security. Under the law of one price or no-arbitrage conditions, the value of a portfolio equals its costs and any two portfolios with the same payoffs must have the same price. Therefore, from (9) and (10), the *ex-ante* uncovered interest rate (UIP) condition can be obtained:

$$(1+i_t) = \left(\frac{e_{t+1}}{e_t}\right)(1+i_t^*) \tag{17}$$

which implies that, in the absence of shocks, the consumer is indifferent between domestic and foreign bonds as long as the gross real return on the two assets are equal, adjusting for exchange rate movements. Let r_t and r_t^* denote the domestic and world real interest rates. In moving from the notation using nominal interest rates to real rates the simple form of the Fisher parity condition is used,

$$(1+r_t) = \frac{P_t}{P_{t+1}}(1+i_t), \quad (1+r_t^*) = \frac{P_t^*}{P_{t+1}^*}(1+i_t^*) \tag{18}$$

which yields the result that *ex-ante*, the domestic real interest rate has to be tied to the world real interest rate, $r_t = r_t^*$. However, it is possible that this condition is violated at the end of period t, since unexpected shocks to the system at the beginning of period t will cause the central bank in the model to react according to an interest rate rule, resulting in the domestic interest rate deviating from this parity *ex-post*. But after the period of the shock, the domestic real interest rate will again be tied to the world real interest rate.

2.2 Technology

Labor-augmenting technological shifts in the small open economy, Z_t, is assumed to follow an exponential Markov process,

$$z_t = (1-\gamma)\,\overline{z} + \gamma z_{t-1} + u_{z,t};\ u_{z,t} \sim \text{i.i.d.}\left(0, \sigma_z^2\right) \tag{19}$$

where $z_t = \ln Z_t$ and $\overline{z}$ is the limit point of the sequence $\{z_t\}_{t=0}^{\infty}$. For simplicity assume that this technology is economy-wide such that a shock to (19) affects supply in both traded and nontraded goods sectors equally.

2.2.1 Traded Goods Firms

The representative firm exists in the perfectly competitive traded goods sector. The firm can be thought of as maximizing its real present value, taking the price of its output and factor rentals as given, by solving the following problem:

$$\max V_{T,t} := \mathbb{E}\left[\sum_{s=t}^{\infty} Q_{t,s}\left(Y_{T,s} - X_{T,s} - \frac{W_{T,s}}{P_{T,s}} N_{T,s}\right)\middle| \mathcal{F}_t\right] \tag{20}$$

subject to

$$Y_{T,s} = F\left(K_{T,s}, Z_s N_{T,s}\right) := K_{T,s}^{\rho_T}\left(Z_s N_{T,s}\right)^{1-\rho_T} \tag{21}$$

$$K_{T,s+1} = \Phi\left(\frac{X_{T,s}}{K_{T,s}}\right) K_{T,s} + (1-\delta)\, K_{T,s} \tag{22}$$

where the capital installation function $\Phi(\cdot)$ is assumed to be increasing, concave and twice-continuously differentiable with $\Phi(0) = \delta$, and $\Phi'(\delta) = 1$, as in Uzawa (1969).[5] It is convenient to partition the firm's problem into those of a production unit and an investment unit of the same firm.

The Production Unit The production unit takes the product price and factor rentals as given in solving profit maximization problem such that for each period t the classical efficiency conditions for production which ensure that the firm equates the real factor rentals to their marginal products are

$$\frac{R_{T,t}}{P_{T,t}} = \rho_T \frac{Y_{T,t}}{K_{T,t}} \tag{23}$$

[5] An alternative is to model the cost explicitly as a negative item in the firms profit function as in Lucas (1967), Gould (1968) and Treadway (1969). See Hayashi (1982) for a comparison. Either method would yield similar investment dynamics.

where capital rental R_T is implicitly the transfer price of capital from the firm's investment unit and

$$\frac{W_{T,t}}{P_{T,t}} = (1 - \rho_T) \frac{Y_{T,t}}{N_{T,t}}. \tag{24}$$

yields the labor demand function.

The Investment Unit Taking the real rental of capital and product price as given, the investment unit chooses investment and capital to maximize the firm's real present value. Efficient investment choice yields an investment function which determines the rate of investment as a function of Tobin's q, which is defined as the ratio of the shadow price of installed capital to the price of replacement capital, which is normalized to one:

$$q_{T,t} = \left[\Phi' \left(\frac{X_{T,t}}{K_{T,t}} \right) \right]^{-1} \tag{25}$$

Efficient choice of capital yields a difference equation which describes the evolution of Tobin's q:

$$q_{T,t} = \mathbb{E}_t Q_{t,t+1} \left\{ \frac{R_{T,t+1}}{P_{T,t+1}} + \left[(1-\delta) + \Phi \left(\frac{X_{T,t+1}}{K_{T,t+1}} \right) - \Phi' \left(\frac{X_{T,t+1}}{K_{T,t+1}} \right) \frac{X_{T,t+1}}{K_{T,t+1}} \right] q_{T,t+1} \right\} \tag{26}$$

That is, the shadow price of capital at time t takes into account the discounted value of capital rentals accruing in the next period as well as the effect of capital accumulation on next period's capital stock and investment costs, which is given by the second term on the right-hand side of (26). Because the capital installation function $\Phi(\cdot)$ is concave, a unit of investment good purchased translates into a less-than-one unit of capital stock used in the following period. The functional form for $\Phi(\cdot)$ can be left unspecified. However, a parameter which reflects the size of the adjustment cost for capital, χ, has to be defined. This parameter has the interpretation of the elasticity of the investment-capital ratio of the firm with respect to the shadow price of installed capital or Tobin's q at steady state.[6]

2.2.2 Firms in the Differentiated Nontraded Goods Sector

Let the commodity produced by each differentiated nontradable goods firm indexed by $i \in [0,1]$ be

$$Y_{N,t}(i) = F\left(\overline{T}_{N,t}(i), Z_t N_{N,t}(i)\right) := \overline{T}_N^{\rho_N}(i) \left[Z_s N_N(i)\right]^{1-\rho_N} \tag{27}$$

where $\overline{T}_{N,t}(i)$ and $N_{N,t}(i)$ are, respectively, some input fixed in the short run and labor used by firm i in its production. Because each nontradable goods firm can differentiate its product, it faces a downward sloping market demand curve. That is, each firm has the power to set its own price. Again, it is useful to think of each firm i as consisting of a production unit and a pricing unit.

[6] To be specific, define the implicit function from (25) as $G(X_{T,t}/K_{T,t}, q_{T,t}) := \Phi'(X_{T,t}/K_{T,t}) - q_{T,t}^{-1} = 0$. The elasticity of $X_{T,t}/K_{T,t}$ with respect to $q_{T,t}$, evaluated at the steady state is $\chi \equiv -\frac{\partial G_1}{\partial G_2}\left(\frac{q}{X/K}\right)\Big|_{q=1, X/K=\delta} = -\left[\Phi''(\delta)\,\delta\right]^{-1}$.

The Production Unit: Marginal Cost and labor Demand Assume that rentals and wages are perfectly flexible in perfectly competitive factor markets. Therefore, R_N and W_N are independent of the firm's output. The short run cost function of firm i can be written as

$$TC_N(i) = \min_{N_N(i)} \left\{ R_N \overline{T}_N(i) + W_N N_N(i) : D_N(i) \leq F\left(\overline{T}_N(i), ZN_N(i)\right)\right\}$$

That is, $TC_N(i)$ gives the least-cost combination of factor inputs from which the demand determined output, $Y_N(i)$, can be produced. The short-run cost minimizing condition at time t yields a labor demand function for firm i :

$$W_N = MC_{N,t}(1-\rho_N)\frac{Y_{N,t}(i)}{N_{N,t}(i)}$$

Given production technology which is homogeneous of degree one across all firms, the cost minimization conditions above also hold for aggregate quantities:

$$\frac{W_N}{P_{N,t}} = mc_{N,t}(1-\rho_N)\frac{Y_{N,t}}{N_{N,t}}. \tag{28}$$

where $mc_{N,t}$ is the real marginal cost.

The Pricing Unit: Limited Price Adjustment It is assumed that firms in the nontraded goods sector set prices according to a discrete-time version of Calvo's (1983) model, where firms face a constant probability of price adjustment each period and the duration of price stickiness is random. That is, the signal for a price change is a stochastic time-dependent process governed by a geometric distribution. The expected lifetime of price stickiness is $(1-\theta)^{-1}$. Recall that the nontraded goods price index was given in (15). In a symmetric equilibrium all firms that get to set their price in the same period choose the same price. Thus prices evolve according to

$$P_{N,t} = \left[(1-\theta)\left(P_{N,t}^{new}\right)^{1-\varepsilon} + \theta\left(P_{N,t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{29}$$

That is, each period a fraction $1-\theta$ of all the firms gets to charge a new price and the remaining fraction θ must charge the previous period's price.

The price set at time t, $P_{N,t}^{new}$ will be the solution to the following problem where firms face a probability θ that a new price commitment, $P_{N,t}^{new}$, in period t will still be charged in period $t+k$. Thus, when setting $P_{N,t}^{new}$, each firm will seek to maximize the value of expected discounted profits:

$$\max_{P_{N,t}^{new}} \mathbb{E}\left\{ \sum_{k=0}^{K-1} Q_{t,t+k}\theta^k \left[P_{N,t}^{new} D_{N,t+k}\left(P_{N,t}^{new}, i\right) - TC_{t,t+k}\right] \middle| \mathcal{F}_t \right\} \tag{30}$$

where $Q_{t,t+k} = \prod_{k=0}^{K}(1+r_{t+k})^{-1}$ and $Q_{t,t} = 1$, is the stochastic discount factor on nominal profits and

$$D_{N,t+k}\left(P_{N,t}^{new}, i\right) = \left(\frac{P_{N,t}^{new}}{P_{N,t+k}}\right)^{-\varepsilon} D_{N,t+k} \tag{31}$$

The optimal pricing strategy is thus one of choosing an optimal path of price markups as a function of rational expectations forecast of future demand and marginal cost conditions,

$$P_{N,t}^{new} = P_{N,t}\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\mathbb{E}_t\sum_{k=0}^{\infty}Q_{t,t+k}\theta^k\left(\frac{P_{N,t}}{P_{N,t+k}}\right)^{-1-\varepsilon}\left(\frac{MC_{N,t+k}}{P_{N,t+k}}\right)D_{N,t+k}}{\mathbb{E}_t\sum_{k=0}^{\infty}Q_{t,t+k}\theta^k\left(\frac{P_{N,t}}{P_{N,t+k}}\right)^{-\varepsilon}D_{N,t+k}}. \tag{32}$$

Notice that if the chance for stickiness in price setting is nil, $\theta = 0$ for all $k = 0, 1, 2, ..., \infty$, the first order condition in (32) reduces to $mc_{N,t} = (\varepsilon - 1)/\varepsilon, \forall t$, which says that the optimal price is a constant markup over marginal cost, or that the real marginal cost is constant over time. This is the same result as that for a static model of a firm with monopoly power. Thus with price-setting behavior, the markup is positive. Straightforward algebra and manipulation of the pricing decision determines the inflation dynamics of nontraded goods as:

$$\pi_{N,t} = \beta\mathbb{E}_t\{\pi_{N,t+1}\} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\widehat{mc}_{N,t}. \tag{33}$$

This is a forward-looking or New Keynesian Phillips curve derived from the assumption of Calvo (1983) price-stickiness, where inflation is positively related to deviations of real marginal cost from steady state.

2.3 Market Clearing in the Small Economy

2.3.1 Product and Asset Market Clearing

Denote aggregate supply of the nontraded good by $Y_{N,t} = \left[\int_0^1 Y_{N,t}(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}$. It would be useful to relate absorption to aggregate supply expressed as the function of factors of production. To do so, follow Yun (1996) by defining an alternative price index $P_N^{'}$ which satisfies the aggregation $\left(P_N^{'}\right)^{-\varepsilon} = \int_0^1 P_N(i)^{-\varepsilon}di$ and evolves according to

$$P_{N,t}^{'} = \left[(1-\theta)\left(P_{N,t}^{'new}\right)^{-\varepsilon} + \theta\left(P_{N,t-1}^{'}\right)^{-\varepsilon}\right]^{-\frac{1}{\varepsilon}} \tag{34}$$

and the aggregate

$$Y_{N,t}^{'} = \int_0^1 Y_{N,t}(i)\,di = \overline{T}_{N,t}^{\rho_N}(Z_t N_{N,t})^{1-\rho_N} \tag{35}$$

where $\overline{T} = \int_0^1 \overline{T}_N(i)\,di$ and $N_N = \int_0^1 N_N(i)\,di$ by construction. The resulting relationship between the alternative index, $Y_N^{'}$ and Y_N is $Y_N^{'} = \left(P_N/P_N^{'}\right)^{\varepsilon}Y_N$. So product market equilibrium for nontraded goods is given by the condition $Y_{N,t} = C_{N,t}$ or,

$$Y_{N,t}^{'} = \overline{T}_{N,t}^{\rho_N}\left(Z_t\overline{N}_{N,t}\right)^{1-\rho_N} = (1-\alpha)\left(\frac{P_{N,t}^{'}}{P_{N,t}}\right)^{-\varepsilon}\left(\frac{P_{N,t}}{P_t}\right)^{-\eta}C_t. \tag{36}$$

Also, market clearing in traded goods implies that aggregate supply and demand in the traded goods sector must balance

$$Y_{T,t} = K_{T,t-1}^{\rho_T} \left(Z_t N_{T,t}\right)^{1-\rho_T} = \alpha \left(\frac{P_{T,t}}{P_t}\right)^{-\eta} C_t + X_{T,t} + \frac{B_{t+1}^d - B_t^d \left(1 + r_{t-1}\right)}{P_t} \tag{37}$$

where B_t^d is net foreign assets held. Equilibrium in the asset markets requires that demand for net foreign assets equal its supply:

$$B_t^d = B_t^s. \tag{38}$$

2.3.2 Labor Market Equilibrium

Assume that there is perfect labor mobility across the traded and nontraded goods sectors such that wages are equalized, $W_{T,t} = W_{N,t} = W_t$. Labor market equilibrium is characterized by the optimality conditions for per capita labor supply (8) and labor demand in the traded and nontraded goods sectors, (24) and (28). Note that there will be an exchange rate channel (via a relative price effect) to labor demand, where the relative price is expressed as the ratio of the prices of nontraded and traded goods, $RP_{NT,t} := P_{N,t}/P_{T,t}$. This is because of the exchange rate pass through effect to CPI which is used to deflate nominal wages. Finally, total labor supply must equal total labor demand in the small economy such that

$$N_t = N_{N,t} + N_{T,t}. \tag{39}$$

2.3.3 Exchange Rates and CPI inflation

The law of one price holds only for traded goods. That is, $P_{T,t} = e_t P_{T,t}^*$. Normalizing the foreign price to one, the nominal exchange rate in the model is thus the domestic traded goods price, $e_t = P_{T,t}$, and e_t will be determined by the no-arbitrage condition given in the uncovered interest parity condition (17). The real exchange rate is defined as $\epsilon_t := e_t P_t^* / P_t$ which is the ratio of national consumer price indices in domestic terms.

2.4 Monetary Policy and Interest Rate Rules

To obtain closure of the general equilibrium model, the nominal interest rate is assumed to be controlled by a central bank. In general the loss function minimization problem for the central bank can be written as

$$\max_{i_t} L\left(\widehat{i}_{t-1}, \widehat{y}_t, \pi_t\right) = -\frac{1}{2}\mathbb{E}\left\{ \sum_{t=0}^{\infty} \beta^t \left[\kappa_0 \left(\widehat{i}_t\right)^2 + (1-\kappa_1)\,\widehat{y}_t^2 + \kappa_1 \pi_t^2\right] \middle| \mathcal{I}_t \right\}. \tag{40}$$

subject to the equilibrium evolution of $\widehat{i}_t$, $\widehat{y}_t$ and π_t and $\mathcal{I}_t$ may or may not be equal to $\mathcal{F}_t$, the information set available to all other agents in the economy.

The optimal solution is an interest rate rule which solves the policy-maker's optimal linear regulator problem.[7] It is well know that the solution to the first-order condition

[7] See Clarida, Gali, and Gertler (1999).

of a maximization of a quadratic objective function like (40) subject to linear constraints for the dynamics of the arguments in the loss function, is a linear policy rule. Instead of solving for the optimal rule in this model, a simple rule is postulated:

$$\widehat{i}_t = \psi \widehat{i}_{t-1} + \mu_1 \widehat{y}_t + \mu_2 \pi_t + u_{m,t}; \; u_m \sim i.i.d.(0, \sigma_m^2) \tag{41}$$

That is, a Taylor interest rate rule, with a preference for interest rate smoothing, is postulated to capture the role of the central bank in setting monetary policy. This decision rule can be thought of as summarizing the first-order condition to the central bank's loss function minimization problem. There is considerable argument in favor of simple monetary policy rules. First, they have been shown to be robust across various models or that they are robust when there is considerable uncertainty about the true structure of the economy. Second, simple rules are easy to implement and more credible. Recall that the interest rate i_t is the rate which is determined at time t but which applies in time $t+1$. A positive realization of the innovation u_m represents an unanticipated contractionary monetary policy.

2.5 Recursive Competitive Equilibrium

Formally, a recursive competitive equilibrium in the model is given by sequences of allocations and price functions that satisfy the agents' optimal decisions and markets clear in international assets, in domestic goods and labor. This is defined below. Let Ξ^t denote the history of realized shocks up to and including time t.

Definition 2.1 *A recursive competitive equilibrium is given by sequences of allocation functions* $\left\{\mathbf{A}_t\left(\Xi^t\right)\right\}_{t=0}^{\infty}$ *where* $\mathbf{A}_t = (C_{T,t}, C_{N,t}(i), B_{t+1}, V_{T,t}, d_{T,t}, K_{T,t+1}, X_{T,t}, N_{T,t}, N_{N,t})$ *and sequences of price and shadow price functions,* $\left\{\mathbf{P}_t\left(\Xi^t\right)\right\}_{t=0}^{\infty}$ *where* $\mathbf{P}_t = (P_{T,t},\, P_{N,t}(i),\, P_{N,t}^{new},\, R_{T,t},\, W_{T,t},\, W_{N,t},\, i_t,\, e_t,\, MC_{N,t},\, q_{T,t},\, Q_{t,t+1})$ *for* $i \in [0,1]$ *satisfying: (i) Households' optimal choices in (1), (8)-(16); (ii) firm's optimal decisions in (23)-(26) and (29) and (32); and (iii) markets clear according to (17), (36), (37), (38) and (39), given initial values of* $K_{T,0}, P_{T,-1}, P_{N,-1}$*, the policy rule (41), and the exogenous stochastic processes* $\{i_t^*, y_t^*, \pi_t^*, z_t, \beta_t\}_{t=0}^{\infty}$.

3 Quantitative Features

The model is calibrated to mimic the behavior of a small open economy like Australia. Most of the parameter values used are by now standard in this literature. This is reported in Table 1. Some explanation of a few of the calibrations will be required. The sum of imports and exports for the small economy, as a ratio of output, is about 38 per cent for Australia, between 1993:1 to 2000:4. Calibration of the share of traded goods price in the CES price index for domestic CPI, $\alpha = 0.3$, approximates this fact. Following existing literature, it is assumed that a probability $\theta = 0.75$ that prices are unchanged in each period. Given the geometric distribution assumed for pricing signals, this corresponds to an average duration of price stickiness of 4 quarters. The autocorrelation for the log surplus consumption ratio process is retained from Campbell and Cochrane (1999), and this is set as $\zeta = 0.87$. Whereas in the endowment economy of Campbell and Cochrane (1999), the steady-state surplus consumption ratio, S_{ss}, is a function of taste parameters, it is now determined by labor market equilibrium in this model, reflecting both the preference and production sets which exist in this model. To be consistent with a steady-state labor hours proportion of time, $N_{ss} \simeq 0.2$, this is set as $S_{ss} = 0.25$. This is used to restrict the parameter λ in the habit process under the Campbell and Cochrane (1999) assumptions, $\lambda = 1/S_{ss} - 1$. Steady-state real marginal cost is equivalent to the inverse of the steady-state markup. The markup is set to 1.47, as estimated for Australia in Ubide (1999) and close to the value used by Schmitt-Grohe (1998). The standard Taylor rule with interest rate smoothing, $\psi = 0.8$, and with the weights of $\mu_1 = 0.5$ and $\mu_2 = 1.5$, implies that the central bank is more concerned about targeting inflation than output. Since the rule is also written in log-linear terms the parameters have elasticity interpretations.

Table 1: Parameter calibrations in the benchmark model

Parameter description	**Mnemonics**	**Value**
Share of tradable goods in CPI	α	0.3
Steady state world interest rate (quarterly)	i^*_{ss}	0.01
Elasticity of substitution between home and foreign goods	η	1.5
Curvature of utility function	σ	2
Consumption share in utility	ν	0.1
Capital share in tradables output	ρ_T	0.4
Labor share in nontradables output	$1-\rho_T$	0.6
Probability of price-stickiness per period	θ	0.75
Autocorrelation of log surplus consumption ratio	ς	0.87
Steady-state surplus consumption ratio	S_{ss}	0.25
Steady-state real marginal cost	mc_{ss}	1/1.47
Elasticity of investment-capital ratio w.r.t. Tobin's q	χ	2
Depreciation of capital stock	δ	0.025
Autocorrelation of nominal interest rate	ψ	0.8
elasticity of interest rate w.r.t output	μ_1	0.5
elasticity of interest rate w.r.t inflation	μ_2	1.5
Autocorrelation of technology shock	γ	0.6
Standard deviation of technology shock	σ_z	0.007
Autocorrelation of discount factor shock	b	0.6
Standard deviation of discount factor shock	σ_β	0.002

The parameters describing the processes which govern the foreign variables are estimated using a $VAR(1)$ model with Wold causal ordering $\mathbf{z}^*_t = (\pi^{*\prime}_t, y^{*\prime}_t, i^{*\prime}_t)'$. The reduced form is estimated as $\mathbf{z}^*_t = \mathbf{\Pi}\mathbf{z}^*_t + \varepsilon^*_t$ where $\mathbb{E}\left(\varepsilon^*_t \varepsilon^{*\prime}_t\right) = \mathbf{\Theta}^{-1}\mathbf{D}^*\left(\mathbf{\Theta}^{-1}\right)'$. The estimated $\mathbf{\Pi}$, $\mathbf{\Theta}$ and $\mathbf{D}^*$ are defined below (standard errors are reported in parentheses):

$$\mathbf{\Pi} = \begin{bmatrix} -0.17 & 0.11 & -0.08 \\ 0.14 & 0.94 & -0.04 \\ 0.07 & -0.02 & 0.87 \end{bmatrix}, \mathbf{\Theta} = \begin{bmatrix} 1 & 0 & 0 \\ \underset{(0.11)}{-0.24} & 1 & 0 \\ \underset{(0.47)}{-0.51} & \underset{(0.48)}{0.05} & 1 \end{bmatrix}, \tag{42}$$

$$\mathbf{D}^{*} = \begin{bmatrix} \underset{(0.0002)}{0.002} & 0 & 0 \\ 0 & \underset{(0.0002)}{0.002} & 0 \\ 0 & 0 & \underset{(0.0001)}{0.001} \end{bmatrix}.$$

This allows for contemporaneous correlations of a recursive nature in the innovations to foreign variables.

As is the case with most DSGE models there is no analytical solution to the system of nonlinear expectational dynamic equations. The model is approximated by log-linearizing the various first-order conditions and identities as in King, Plosser, and Rebelo (1987) or Campbell (1994) and this linear counterpart is then solved. In other words, the dynamics of the model are constrained to evolve within an ε-ball of the model's steady state, where ε is some arbitrarily small positive scalar. The solution method employed is the method of undetermined coefficients described in Uhlig (1999). The validity of the calibrated model is examined via second moments and a number of impulse response simulations. The impulse response functions are available on request.

3.1 Volatility

An informal assessment of the quantitative performance of the model's assumed data generating processes and dynamic propagational mechanisms can be conducted by comparing the second moments of the simulated series of certain key macroeconomic variables implied by the benchmark model with their observable counterparts. Data relating to the Australian business cycle from a sample period between 1993:1 and 1997:4 is used. This relatively short sample period is chosen so that one can focus on data relevant to a monetary policy regime which explicitly targets inflation using an interest rate rule. The sample moments are calculated for Hodrick-Prescott filtered data.

Table 2 documents the facts about volatility and correlation between the time series for the vector of interest (e, ϵ, y, c, π) which contains the nominal and real exchange rates, output, consumption, CPI inflation.[8] In the data for the chosen sample period, the exchange rates are countercyclical and more volatile than the Australian GDP by about 7 to 8 times. Consumption is also procyclical, where consumption is less volatile than GDP. Contemporaneous inflation and GDP are weakly and positively correlated.

Using Monte Carlo simulation of the model to produce an average of 100 simulated sets of time series with length of 100, some reasonable matches between the model's artificial

[8] Data is obtained from the Reserve Bank of Australia Bulletin and the ABS Treasury Model (for G7 output and inflation) databases. The nominal exchange rate is taken to be the Australian-U.S. dollar nominal exchange rate and the constructed series of this exchange rate multiplied into the ratio of Australian CPI to U.S. CPI yields the real exchange rate.

						j				
variable	s.d.	-4	-3	-2	-1	0	1	2	3	4
e	0.054	-0.21	-0.24	-0.19	-0.22	-0.19	0.09	0.19	0.30	0.26
ϵ	0.056	-0.26	-0.28	-0.23	-0.23	-0.15	0.13	0.20	0.33	0.29
y	0.007	-0.14	0.03	0.21	0.50	1.00	0.50	0.21	0.03	-0.14
c	0.006	0.10	0.24	0.34	0.37	0.54	0.26	-0.01	-0.07	-0.19
π	0.005	0.13	0.09	0.11	0.08	0.02	-0.44	0.05	0.02	-0.05

Table 2: Data standard deviations and correlations with y for j leads and lags

						j				
variable	s.d.	-4	-3	-2	-1	0	1	2	3	4
e	0.010	-0.24	-0.31	-0.38	-0.43	-0.44	0.02	0.24	0.32	0.33
ϵ	0.020	0.05	0.02	-0.03	-0.11	-0.22	-0.14	-0.09	-0.06	-0.02
y	0.009	-0.11	-0.02	0.15	0.47	1.00	0.47	0.15	-0.02	-0.11
c	0.001	0.20	0.29	0.39	0.49	0.45	0.10	-0.10	-0.20	-0.24
π	0.004	0.07	-0.05	-0.28	-0.67	-0.93	-0.42	-0.11	0.05	0.13

Table 3: Benchmark model standard deviations and correlations with y for j leads and lags

data and the measured reality is obtained.[9] In Table 3, designated as the set of standard deviations and correlations of variables with output generated by the benchmark model is presented. First, the model has been calibrated such that the volatility of output implied in the model is close to that in the data. The lag and lead autocorrelations for output in the model are also quite close to those of the data in sign and magnitude. The model performs reasonably well with respect to the exchange rates. Correlations of the nominal and real exchange rates with output are generally countercyclical as in the data. The benchmark model predicts that the exchange rates are much more volatile than output as in the data, although the relative volatility is not as high as 7 to 8 in the data. Thus the model ranks the volatilities correctly but does not explain all of the excess volatility in exchange rates.[10] Consumption in the model is smoother than output compared as in the data. There is some lagged phase shift in the correlation between consumption and output in the model, relative to actual data. Nevertheless consumption is still moderately procyclical in the model. The model generally does well, with respect to matching the output and inflation standard deviation in the data. This is of importance when experiments on the interest rate rule is conducted relative to this benchmark in the following section.

[9] Standard errors have not been reported but can be reproduced upon request.

[10] Perhaps with the addition of government spending shocks, the model would yield even higher relative volatilities of the exchange rates.

4 Interest Rate Smoothing in a Taylor Rule

Levin, Wieland, and Williams (1999) showed that simple rules such as the Taylor rule tend to be more robust across alternative large scale models than complicated rules. Further, allowing for lags of inflation and output, and other variables in place of contemporaneous does not matter much for the model outcomes. More relevantly, they also show in the context of their four models, that there are gains from not changing interest rates too much over time. In this section, the issue of interest rate smoothing is examined in the context of the small open economy model. Firstly, it is assumed that all the shocks underlying the model are present. These, as enumerated above, are the foreign inflation, output, and interest rate and domestic productivity and demand shocks. Under the benchmark parameterization of the model for this case, the generalized Taylor rule was given by equation (41) where $\psi = 0.8$, $\mu_1 = 0.5$ and $\mu_2 = 1.5$. For comparative purposes, the benchmark variances in the previous section are standardized to unity.

4.1 Inflation-Output Trade-offs

4.1.1 Changing Weights on Output, μ_1

The elasticity of the interest rate instrument with respect to output, μ_1, is varied over the interval $[0.5, 0.6]$ and simulated pairs of inflation and output variances are generated.[11] The variances of inflation and output are plotted as functions of the varying parameter μ_1 in Figure 1. It can be seen from Figure 1 that increasing μ_1 lowers the variance of output measured as deviations from steady state but increases the variance of inflation. The trade-off frontier is also plotted in Figure 1. It was seen that inflation variance is monotonically increasing with μ_1 in the bounded domain of $[0.5, 0.6]$, while the opposite is true for output variance and this results in inflation to output variance ratio rising with μ_1 as expected. That is the sacrifice ratio in terms of variances, describing the trade-off of having less inflation uncertainty in return for more output variability falls as the central bank places more concern on output deviations relative to inflation.

4.1.2 Changing Weights on Inflation, μ_2

While most studies focus on the output-inflation variability trade-off use the approach discussed in Section 4.1.1, a similar approach of increasing the concern for inflation relative to a fixed weight on output is considered here. By effecting a small change in the elasticity of the policy instrument with respect to inflation, μ_2, while holding μ_1 constant, it is also possible to obtain a trade-off between having lower inflation volatility and output volatility in the short run, although the numerical simulation as shown in Figure 2 do not necessarily yield points on a smooth trade-off curve.

[11] It should be noted that authors using simple structural models to analyze Taylor rules (e.g. Debelle 1999) vary the weight on output in the loss function rather than the policy rule itself as there is a direct analytical mapping between the parameters in the loss function and the decision rule. In this case, the model is no longer analytically computable, so that the only way to analyse the trade-off is to vary the weights indirectly via the policy rule.

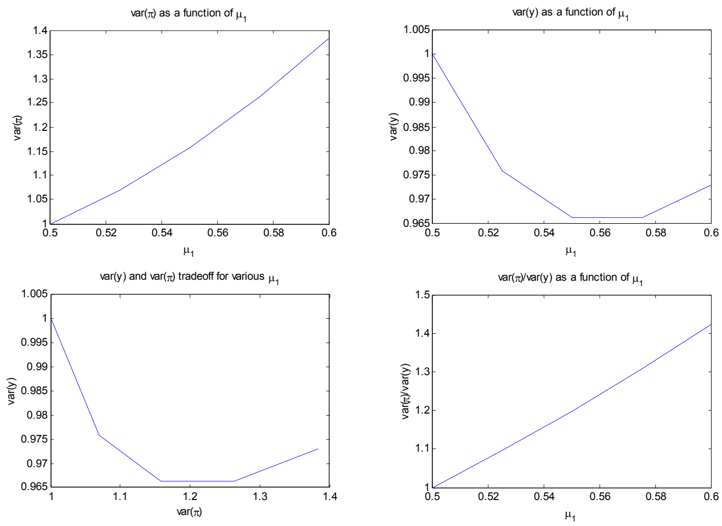

Figure 1: The effect of increasing μ_1 on inflation variance, output variance, inflation-output varaince trade off and the ratio of inflation variance to output variance.

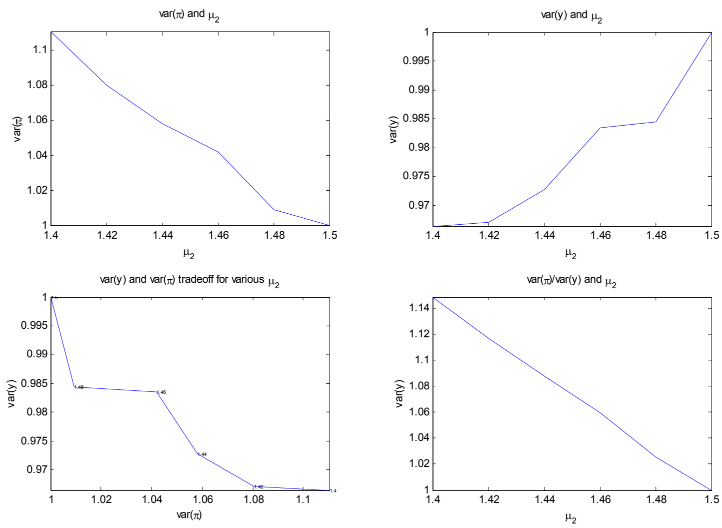

Figure 2: Inflation variance, output variance, inflation-output variance trade off, and inflation-output variance ratio as μ_2 varies.

4.1.3 Combinations of μ_1 and μ_2

In the following Figure 3, the surface maps of the variances of output and inflation as functions of different pairs of elasticities of the policy instrument with respect to output, μ_1, and inflation, μ_2, is displayed. The benchmark model parameterization places output and inflation variances on the highest and lowest points of the variance surfaces in the left panels of Figure 3, respectively. Moving along the line where $\mu_2 = 1.5$ in a northeasterly direction in the left panels or equivalently, moving along the top line of the contour boxes to the right, yields the same graphs as in Figures 1 earlier. There is a valley-like surface for output variance while inflation variance is a plane which slopes up in the direction of lower inflation weight but higher output weight. As the central bank places more weight on output while decreasing concern for inflation, output variance falls towards the valley but inflation variance is rising in the given parameter subspace. However, too little inflation concern relative to output also causes output variance to rise. This suggests that if central bankers are not very strict on inflation, and thus allowing greater uncertainty in inflation, output can also be more volatile, yielding a positive relationship between output and inflation variance.

In Figure 4 the ratio of inflation variance to output variance is shown as a function of the two target variable weights. Overall, this experiment is consistent with the output-inflation variability story. From this surface, it can be seen that stricter targeting of inflation relative to output lowers the ratio, implying a sacrifice of output variability for lower inflation variability. That is, the central bank policy-makers cannot have their cake and eat it as well. In the short run, due to the existence of a Phillips curve in the model economy, there has to be a trade-off in the emphasis placed on either targets.

4.2 Interest rate smoothing

In this section the idea that central banks do not alter the interest rate from time to time by large jumps is considered within the model for interest rate smoothing. One argument (e.g. Sack and Wieland 2000) for interest rate smoothing is that if agents are forward looking, then an expectation that a small initial policy change will be followed by subsequent moves in the same direction will only increase the effectiveness of the policy. Figures 5 shows the impact of varying the degree of interest rate smoothing $\psi \in [0.75 < ... < 0.9]$ under the assumption of varying μ_1. There is a tilting or swivel of the output-inflation variance frontier in an anti-clockwise manner as ψ is increased.

The curves under the various values of ψ form a lower envelope which yields the lowest variability frontier so that one can find some value of μ_1 while moving from a low value of ψ to a higher one to obtain large reductions in inflation variance at the low expense of some output variability compared to a move along the individual curves. This suggests that with a fixed weight on inflation, the central bank can choose combinations of the output weight and lagged interest rate smoothing parameter, (μ_1, ψ), such that it can afford to lower inflation by more with decreased sacrifices of rising output volatility.

Further, interest rate smoothing alone, may be beneficial in terms of lower output and inflation variance. By fixing (μ_1, μ_2) at the benchmark values, it is found that for certain

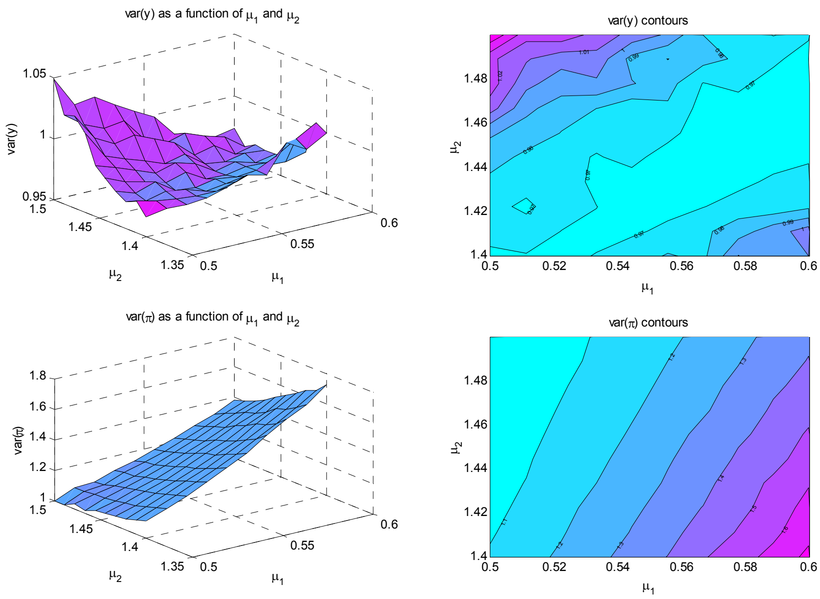

Figure 3: Output and inflation variance surfaces under varying μ_1 and μ_2.

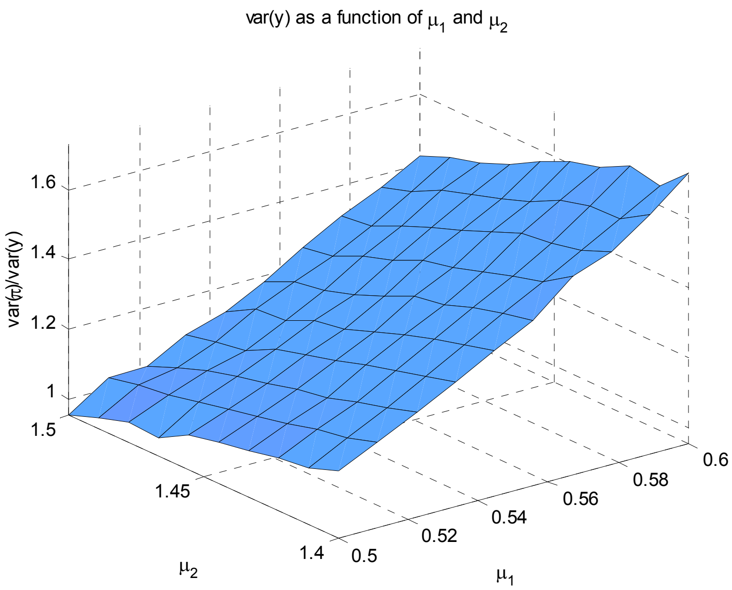

Figure 4: Ratio of inflation variance to output variance under varying weights for output and inflation, μ_1 and μ_2, respectively.

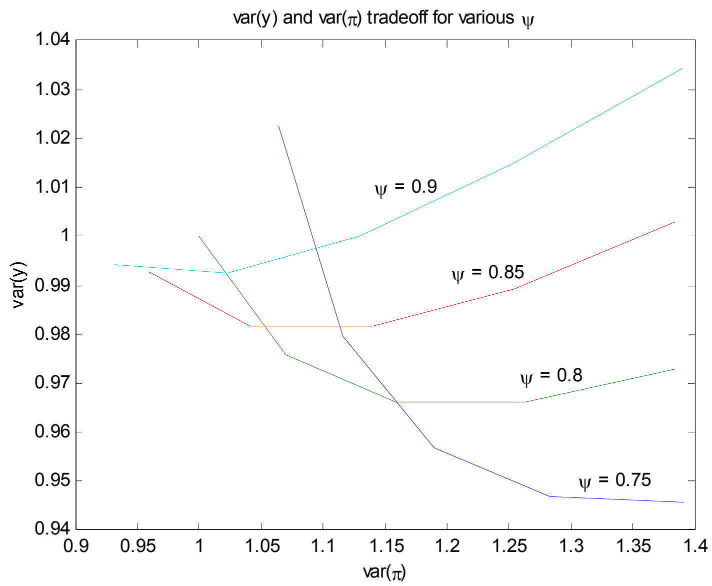

Figure 5: Gains from interest rate smoothing under varying μ_3. Varying μ_3 creates the trade-off frontiers, while varying ψ tilts the curves.

degrees of inflation smoothing, ψ, one can lower variances of both output and inflation by increasing the smoothing factor for interest rate. This is displayed as the figures in 8. In the simulation and benchmark parameterization, these were found to be values of ψgreater than 0.6, placing the standard assumption in the literature (e.g. Monacelli 2000) of $\psi = 0.8$ to be quite valid.[12] For $\psi \geq 0.7$, the inflation-output variance ratio falls at a decreasing rate with ψ. That is, with greater smoothing of i_t, the central bank can achieve a larger fall in inflation relative to output variance at an increasingly lower cost. In fact, values of ψ in excess of 1 were also admissible and yield unique equilibrium solutions in the model. Again this is consistent with the result for a closed economy model in Rotemberg and Woodford (1999). Intuitively, a value of $\psi \geq 1$ (but not too excessive), implies, for instance, that higher inflation at time t would cause the long-run real interest rate to rise by more because future short rates will be expected (as agents know the policy rule) to be even higher while agents expect future inflation to be low. Thus $\psi \geq 1$ can be stabilizing.

In short, policy is more effective and less costly in terms of inflation-output variability if central bankers desire to smooth out their policy variable over time, so that private agents' expectations about prices can be made in a less uncertain environment.

[12] Note that too low a value of ψ can yield multiple equilibria in the model. This is labeled as "indeterminacy" in the figures. The model in that case is solved by selecting the equilibrium path consistent with the smallest stable eigenvalues.

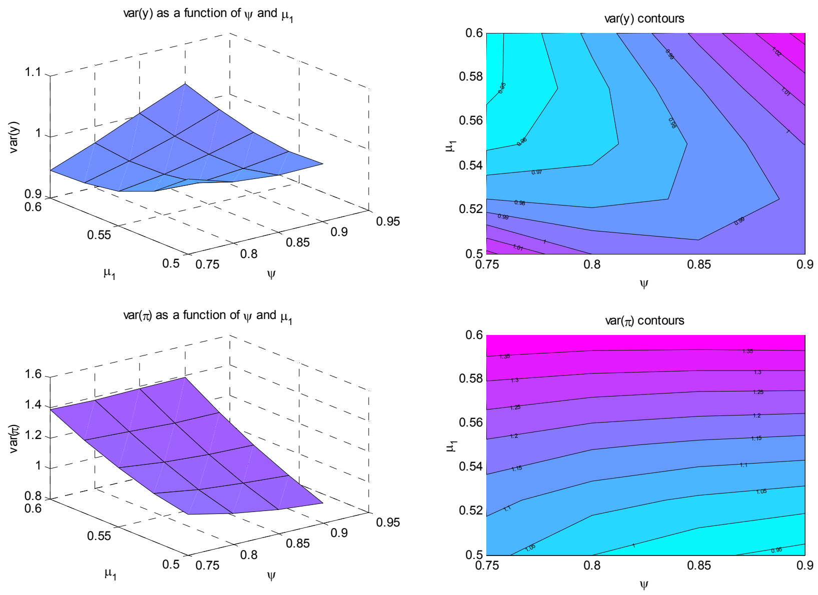

Figure 6: Output and inflation variances as functions of μ_1 and ψ.

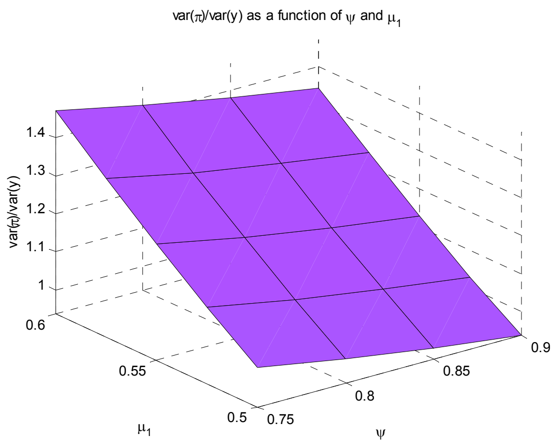

Figure 7: Inflation-output variances ratio.

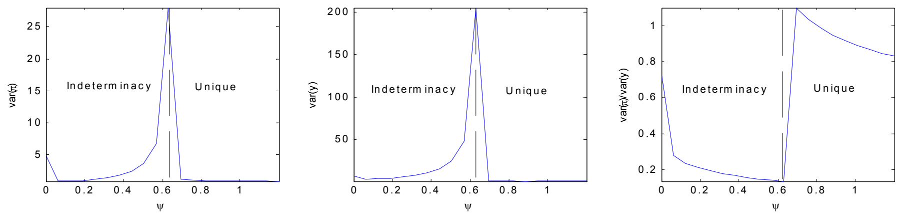

Figure 8: Variance of inflation, output and their ratio under increasing ψ. The regions to the left of the dashed lines signify rational expectations solutions that have multiple equilibria. A single equilibrium path in those cases is chosen using the smallest stable eigenvalues.

5 Conclusion

This paper is concerned with the implications of interest rate smoothing on inflation and output variability. The issue is analyzed through the lens of a small open economy dynamic stochastic general equilibrium model with nontraded goods price rigidities and habit persistence. The model is first calibrated to match certain key business cycle features of a small open economy like Australia. This was labeled the benchmark model. Relative to the benchmark model, experiments on a Taylor rule with interest rate smoothing was conducted. Due to the existence of a short run expectational Phillips curve in the model, monetary policy will imply certain trade-offs between inflation and output variance, under sensible parameter values of the model. More importantly, in a world where there exists such a trade-off, greater interest rate smoothing in the Taylor rule, combined with an adequate balance between a concern for inflation and output deviations from the long run, can potentially yield lower sacrifices in terms of output variability in return for lower inflation, thus increasing policy effectiveness. Furthermore, with given weights placed on output and inflation in the interest rate rule, increased interest rate smoothing within a reasonable range, can be beneficial for both output and inflation volatility.

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