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The Optimal Composition of Government Expenditure

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Abstract

This paper examines the optimal ratio of transfer payments to expenditure on public goods, for a given income tax rate. The transfer payment is then determined by the government's budget constraint. The optimal ratio of transfers to public good expenditure per person is expressed as a function of the ratio of the median to the mean wage, and of the tax rate. Reductions in the skewness of the wage rate distribution are associated with reductions in transfer payments relative to public goods expenditure, at a decreasing rate. Furthermore, increases in the tax rate, from relatively low levels, are associated with increases in the relative importance of transfer payments. But beyond a certain level, further tax rate increases are associated with a lower ratio of transfers to public goods.

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1 Introduction

The aim of this paper is to examine the optimal composition of government expenditure, that is, the composition that maximises a social welfare function which is considered to be a function of individuals' (indirect) utilities. In particular the division between expenditure on public goods and on transfer payments is examined. A simple static model is constructed in which individuals have similar preferences, but differing abilities and thus wages, and expenditure is financed by a proportional income tax.[1] The analysis, while being in the tradition of optimal tax models, therefore differs from the standard problem in the optimal tax literature, which is concerned with the determination of the tax level and its associated transfer payment.[2] The present paper considers the optimal allocation between the two categories for a given tax rate, and thus the way in which the allocation varies as the tax rate varies. The tax rate may be regarded as being determined by other considerations, for example relating to taxable capacity or 'conventional' or politically acceptable levels. Unlike the standard optimal tax framework where closed-form solutions are seldom available, explicit expressions for transfer and public good expenditure are derived. Comparisons are made with the case where the composition of expenditure is determined by majority voting.

The basic model and framework of analysis are described in Section 2, which derives the indirect utility function of each individual, expressed in terms of expenditure on the public good, the transfer payment and the given tax rate. Section 3 derives the optimal allocation of expenditure for an exogenous tax rate. The solutions are shown to depend on the ratio of

[1] This focus contrasts with the large literature concerned with the optimal allocation between consumption and investment expenditure in a growth framework. In such models, a social planner, or representative agent, chooses the optimal composition of government expenditure to maximise a multi-period welfare function: for recent examples see, for example, Chen (2006), Ghosh and Gregoriou (2006), Piras (2001) and Lee (1992).

[2] With three policy instruments and a government budget constraint, there are two degrees of freedom. It would be possible to use numerical methods to carry out a two-dimensional search – over the public good expenditure and the tax rate – to obtain the values which maximise a specified social welfare function. But the focus here is on the expenditure composition where explicit solutions are derived, giving insights into the determinants.

a measure of location of the wage rate distribution to the arithmetic mean wage. The location measure is a weighted average of wage rates, with weights depending on the properties of the social welfare function being maximised. In this section a comparison with the median voter outcome is also made. Since each model involves the use of a wage ratio, comparisons between the relevant ratios are made in Section 4. Section 5 reports numerical examples to investigate the relationship between the two approaches, and between the composition of expenditure and the tax rate, as well as examining the effect of changes in the wage rate distribution. In particular the relationship between the composition of expenditure and the ratio of the median to the average wage rate, and the income tax rate, are investigated. Brief conclusions are in Section 7.

2 Individual Preferences

This section derives individuals' indirect utility functions. The direct utility function and optimal consumption and labour supply, for an individual who faces a given wage rate and tax rate and receives a non means-tested transfer payment, or basic income, are examined in subsection 2.1. Subsection 2.2 gives the indirect utility function in terms of public good expenditure, the transfer payment and the tax rate. Earnings are the only source of income and tax revenue is devoted only to the provision of the pure public good and the transfer payment.

2.1 Individual Consumption and Labour Supply

Each individual is assumed to derive utility from consumption, c, leisure, h, and the public good, G. By definition all individuals consume the same amount of the pure public good which must be tax-financed. Individuals have similar preferences but different productivities and therefore wage rates, w. The direct utility function is assumed to be Cobb-Douglas, so that, omitting individual subscripts:

$$U = c^{\alpha} h^{\beta} G^{1-\alpha-\beta} \tag{1}$$

Although all individuals consume the same amount of the public good, they do not receive the same benefits: higher wage individuals experience higher marginal utility.

The choice of G is not determined at the individual level, since individuals cannot be excluded. The price of the consumption good is normalised to unity, so that consumption and net earnings are equal. Suppose there is an unconditional and untaxed transfer payment of b per individual. There is a simple proportional income tax, with the rate, t, so that the price of leisure is $w(1-t)$. Therefore the form of individual's budget constraint is:

$$c = w(1-h)(1-t) + b \tag{2}$$

The transfer payment per person is restricted to be positive, so that, for example, public goods expenditure cannot be financed from a poll tax.

Defind full income, M, as the net income obtained if all the individual's endowment of one unit of time is devoted to work, so that $M = w(1-t) + b$ and the budget constraint can is:

$$c + hw(1-t) = M \tag{3}$$

Using the standard properties of the Cobb-Douglas utility function, the demand for private goods and leisure can be written as:

$$c = \left(\frac{\alpha}{\alpha+\beta}\right) M = \alpha' M \tag{4}$$

$$h = \left(\frac{\beta}{\alpha+\beta}\right) \frac{M}{w(1-t)} = \beta' \frac{M}{w(1-t)} \tag{5}$$

Where $h < 1$, that is the individual works, if the wage rate exceeds a threshold, $w_{\min}$, such that $w_{\min} = \frac{\beta'}{1-\beta'} \frac{b}{1-t}$.

2.2 Indirect Utility

The indirect utility function, V, is obtained by substituting the solutions for c and h given above into the direct utility function, so that:

$$V = \left(\alpha'^{\alpha} \beta'^{\beta}\right) \left(w(1-t)\right)^{\alpha} \left(\frac{M}{w\,(1-t)}\right)^{\alpha+\beta} G^{1-\alpha-\beta} \tag{6}$$

Furthermore, writing:

$$k = \alpha'^{\alpha}\beta'^{\beta}\left(w(1-t)\right)^{\alpha} \tag{7}$$

indirect utility becomes:

$$V = k\left(\frac{M}{w(1-t)}\right)^{\alpha+\beta} G^{1-\alpha-\beta} \tag{8}$$

The indirect utilities provide the arguments of the social welfare function, examined in the following section.

3 The Optimal Composition of Expenditure

This section examines the optimal composition of expenditure where social welfare, W, is defined in terms of individuals' indirect utilities. The associated social indifference curves, showing combinations of G and b which leave W unchanged, are derived in subsection 3.1. An optimal allocation of expenditure is obtained as a point of tangency of the highest social indifference curve which can be reached subject to the government's budget constraint relating G and b. This constraint is derived in subsection 3.2. The optimal policy is thus given from the condition:

$$\left.\frac{db}{dG}\right|_{t} = \left.\frac{db}{dG}\right|_{W} \tag{9}$$

Closed-form solutions are obtained in subsection 3.3. Finally, in subsection (3.4) a comparison with the majority voting outcome is made.

3.1 Social Indifference Curves

The social evaluation, or social welfare, function of the planner is considered to be a general function of indirect utilities, so that:

$$W = W\left(V_1, ...V_n\right) \tag{10}$$

Differentiating W totally with respect to public good expenditure and the transfer payment gives:

$$dW = \left(\sum_{i=1}^{n}\frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b}\right)db + \left(\sum_{i=1}^{n}\frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial G}\right)dG \tag{11}$$

Consider social indifference curves relating combinations of G and b for which W is constant. Setting $dW = 0$ gives the slope of indifference curves, $\left(\frac{db}{dG}\Big|_W\right)$, as:

$$\frac{db}{dG}\bigg|_W = -\frac{\sum_{i=1}^{n} \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial G}}{\sum_{i=1}^{n} \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b}} \tag{12}$$

This slope can be expressed more conveniently by defining v_i as the welfare weight attached to an increase in i's income. Hence:

$$v_i = \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b} \tag{13}$$

and:

$$\begin{aligned}\sum_{i=1}^{n} \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial G} &= \sum_{i=1}^{n} \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b}\left(\frac{\partial V_i/\partial G}{\partial V_i/\partial b}\right) \\ &= \sum_{i=1}^{n} v_i \left(\frac{\partial V_i/\partial G}{\partial V_i/\partial b}\right) \end{aligned} \tag{14}$$

Substituting (13) and (14) into (12) gives:

$$\frac{db}{dG}\bigg|_W = -\frac{\sum_{i=1}^{n} v_i \left(\frac{\partial V_i/\partial G}{\partial V_i/\partial b}\right)}{\sum_{i=1}^{n} v_i} \tag{15}$$

$$= -\sum_{i=1}^{n} v_i' \left(\frac{\partial V_i/\partial G}{\partial V_i/\partial b}\right) \tag{16}$$

with $v_i' = v_i / \sum_{i=1}^{n} v_i$. The slope of social indifference curves is therefore a weighted sum of the ratio of $\partial V_i/\partial G$ to $\partial V_i/\partial b$. Differentiating (6) with respect to G and b gives :

$$\frac{\partial V_i}{\partial G} = (1-\alpha-\beta)k_i\left(1+\frac{b}{w_i(1-t)}\right)^{\alpha+\beta} G^{-(\alpha+\beta)} \tag{17}$$

$$\frac{\partial V_i}{\partial b} = \frac{(\alpha+\beta)k_i}{w_i(1-t)}\left(1+\frac{b}{w_i(1-t)}\right)^{\alpha+\beta-1} G^{1-\alpha-\beta} \tag{18}$$

Consequently, $\frac{\partial V_i/\partial G}{\partial V_i/\partial b}$ is obtained from (17) and (18), so that:

$$\frac{\partial V_i/\partial G}{\partial V_i/\partial b} = \frac{(1-\alpha-\beta)}{(\alpha+\beta)}\left\{\frac{w_i(1-t)+b}{G}\right\} \tag{19}$$

The slope of social indifference curves is therefore:

$$\begin{aligned}\left.\frac{db}{dG}\right|_W &= -\sum_{i=1}^{n} v_i' \frac{(1-\alpha-\beta)}{(\alpha+\beta)} \left\{ \frac{w_i(1-t)+b}{G} \right\} \\ &= -\frac{(1-\alpha-\beta)}{(\alpha+\beta)} \frac{1}{G} \left((1-t) \left(\sum_{i=1}^{n} v_i' w_i \right) + b \right) \end{aligned} \qquad (20)$$

This is the right hand side of the tangency condition in (9). The left hand side is derived in the following subsection.

3.2 The Government Budget Constraint

The government budget constraint requires that total revenue from the proportional income tax, equal to $t \sum_{i=1}^{n} y_i$ for a population of n individuals, is sufficient to finance the transfer payment and the public good, $nb + G$. Hence:

$$b + \frac{G}{n} = t\overline{y} \qquad (21)$$

where $\overline{y}$ denotes arithmetic mean earnings. The analysis is simplified by the assumption that $w_i > w_{\min}$ for all individuals, implying that everyone works.[3] Gross earnings, y, of workers are $y = w(1-h) = w(1-\beta') - \frac{b\beta'}{(1-t)}$ and average income, since individuals have similar preferences, is:

$$\overline{y} = \overline{w}(1-\beta') - \frac{b\beta'}{(1-t)} \qquad (22)$$

where $\overline{w}$ denotes the arithmetic mean wage rate. By substituting (22) in (21), it is possible to express b in terms of average earnings, the tax rate and G, as:

$$b = \frac{t\overline{w}(1-\beta') - \frac{G}{n}}{1 + \beta' \frac{t}{(1-t)}} \qquad (23)$$

[3] If not all individuals have a wage level above the value $w_{\min} = \frac{\beta'}{1-\beta'} \frac{b}{1-t}$, average earnings become $\overline{y} = \overline{w} \left(1-\beta'\right) H\left(w_{\min}\right)$ where $H\left(w_{\min}\right) = \left\{1 - F_1\left(w_{\min}\right)\right\} - \frac{\theta}{(1-\beta')\overline{w}} \left\{1 - F\left(w_{\min}\right)\right\}$ and $\theta = b\beta'/(1-t)$, and $F_1\left(w_{\min}\right)$ and $F\left(w_{\min}\right)$ denote respectively the proportion of total wage (rates) and the proportion of people with $w < w_{\min}$. On functions of the form $H(.)$, see Creedy (1996).

Therefore, the slope of the government's budget constraint for a given tax rate is:

$$\left.\frac{db}{dG}\right|_t = -\frac{1}{n\left(1+\beta'\frac{t}{(1-t)}\right)} \tag{24}$$

This provides the left hand side of the condition in (9).

3.3 The Tangency Solution

Substituting (20) and (24) into (9), and defining $\tilde{w} = \sum_i^n v_i' w_i$ as a weighted average of wage rates, gives:

$$\frac{1}{n\left(1+\beta'\frac{t}{(1-t)}\right)} = \frac{(1-\alpha-\beta)}{(\alpha+\beta)G}\left((1-t)\tilde{w}+b\right) \tag{25}$$

Finally, substituting for b using (23), it is found that the optimal public expenditure per person, G_W/n, can be expressed as:

$$\frac{G_W}{n} = (1-\alpha-\beta)\left\{\tilde{w} + t(1-\beta')(\overline{w}-\tilde{w})\right\} \tag{26}$$

The resulting value of b_W is given by appropriate substitution into (23).

These values apply for positive values of the social transfer, so the given tax rate must exceed the value, $t_{\min}$, where:[4]

$$t_{\min} = \left(\frac{1}{1-\beta'}\right)\left[1+\left(\frac{\overline{w}}{\tilde{w}}\right)\left(\frac{\alpha+\beta}{1-(\alpha+\beta)}\right)\right]^{-1} \tag{27}$$

The focus here is on the ratio of the transfer payment to the expenditure on the public good per person, rather than absolute values. It can be shown that this ratio, $R_W = b_W/\left(G_W/n\right)$, is given by:

$$R_W = \frac{1-t}{1-t\left(1-\beta'\right)}\left[\left(\frac{1}{1-\alpha-\beta}\right)\left\{\frac{\tilde{w}}{\overline{w}}\left(\frac{1}{t\left(1-\beta'\right)}-1\right)+1\right\}^{-1}-1\right] \tag{28}$$

This result shows that the ratio of the transfer payment to public goods expenditure per person depends, among other things, on the ratio of the welfare-weighted wage rate to the average wage rate.

[4] In addition, the tax rate is also subject to an upper limit, given the assumption that all individuals work, so that for sensible values the social transfer must remain sufficiently below the minimum wage.

The expression in (28) is highly nonlinear in $\tilde{w}/\bar{w}$ and t. However, the relationship between increasing $\frac{\tilde{w}}{\bar{w}}$ and the ratio of the transfer payment to public goods is:

$$\frac{\partial R_W}{\partial\left(\frac{\tilde{w}}{\bar{w}}\right)}=-\frac{t(1-t)(1-\beta')(1-t(1-\beta'))}{(1-\alpha-\beta)\left\{\frac{\tilde{w}}{\bar{w}}(1-t(1-\beta'))+t(1-\beta')\right\}^2}<0 \tag{29}$$

Higher inequality aversion reduces $\tilde{w}$ relative to $\bar{w}$, and (29) demonstrates that this leads to a higher ratio of expenditure on transfer payment to public goods. Hence, both higher wage inequality and inequality aversion lead to a more redistributive expenditure policy.

Furthermore, it can be shown that

$$\frac{\partial^2 R_W}{\partial t^2}<0 \tag{30}$$

so that there is a concave relationship between R_W and t. The first derivative $\partial R_W/\partial t$ is positive for low values of t and negative for relatively higher values. This is dominated by the concave relationship between b_W and t, since $\partial G_W/\partial t$ is positive, while $\partial^2 G_W/\partial t^2=0$ for all relevant values of t. Hence G_W increases linearly with t.

The concavity of R_W with respect to t is therefore strongly affected by the labour supply effects of taxes and transfers. Hence initial increases in the tax rate from a relatively low level are used to increase income redistribution by increasing the proportion of expenditure devoted to transfer payments. But beyond a certain level, further increase in t have the effect of reducing the optimal proportion spent on transfers.

The partial derivatives $\partial\left(G_W/n\right)/\partial\left(\bar{w}\right)$ and $\partial\left(b_W\right)/\partial\left(\bar{w}\right)$ are both positive so that an upward shift in the distribution of wage rates unambiguously increases the optimum expenditure on the public goods and transfer payment. The partial derivatives $\partial\left(G_W/n\right)/\partial\left(\tilde{w}\right)$ and $\partial\left(b_W\right)/\partial\left(\tilde{w}\right)$ are positive and negative respectively. Hence a increase in $\tilde{w}$ (with an unchanged arithmetic mean wage) has a positive effect on public goods and total expenditure, but reduces the absolute social transfer and the ratio of the transfer to public good expenditure.

3.4 Comparison with Majority Voting

The previous section obtained an expression for the optimal expenditure on public goods, G_W, in terms of a welfare-weighted average wage rate, $\tilde{w}$. In an earlier paper, Creedy and Moslehi (2007), considered the majority choice of the composition of expenditure within the same basic framework.[5] The conditions required for the median voter theory to hold were found to be satisfied and the median voter is unambiguously identified as the individual with median wage, w_m. The resulting expenditure on the public good, G_m, was found to be:

$$\frac{G_m}{n} = (1 - \alpha - \beta) \{w_m + t(1 - \beta')(\bar{w} - w_m)\} \tag{31}$$

and of course the transfer payment, b_m is obtained from the government budget constraint in (23). Hence the only difference between the two approaches concerns the wage ratio used. In the median voter model, the relevant variable is $w_m/\bar{w}$ whereas social welfare maximisation involves $\tilde{w}/\bar{w}$, the ratio of the welfare-weighted average wage to the arithmetic mean wage. The following subsection therefore examines these two ratios in further detail.

4 Alternative Wage Ratios

To examine how the two relevant wage ratios, $w_m/\bar{w}$ and $\tilde{w}/\bar{w}$, differ, it is useful to assume a specific functional form for the wage rate distribution. Suppose that wages follow a lognormal distribution with mean and variance of logarithms of μ and σ^2 respectively. Hence the median and mean wage rates are e^{μ} and $e^{\mu+\sigma^2/2}$ respectively and the ratio of median to mean wage rate is:

$$\frac{w_m}{\bar{w}} = \frac{\exp(\mu)}{\exp\left(\mu + \frac{\sigma^2}{2}\right)} = \exp(-\frac{\sigma^2}{2}) \tag{32}$$

[5] This framework again differs from the majority voting models concerned with the choice of government size (the tax rate used to finance a transfer payment). Those models, like optimal tax models, rarely provide explicit or closed-form solutions. On such models see, for example, Roberts (1977), Meltzer and Richard (1981), Tabellini and Alesina (1990), Tridimas and Winer (2005) and Borck (2007).

This expression shows that w_m/w_m depends only on the variance of logarithms of wages. Decreasing the ratio of median to mean wage rates implies an increase in the skewness of the distribution and in this positively skewed case it also reflects an increasing in inequality, as measured by σ^2.

In examining the welfare-weighted average, $\tilde{w}$, it is clear that the term $v_i = \frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b}$, and hence the weight v_i', is highly complex even for simple forms of W. For example, suppose the social planner maximizes an additive social welfare function with iso-elastic weights applied to the V_is such that:

$$\begin{aligned} W &= \frac{1}{1-\varepsilon}\sum_{i=1}^{n} V_i^{1-\varepsilon} \qquad \varepsilon \neq 1, \varepsilon > 0 \\ &= \log y \qquad\qquad\qquad \varepsilon = 1 \end{aligned} \tag{33}$$

where ε is the degree of concavity of the weighting function and represents the degree of constant relative inequality aversion of the planner. Hence $\partial W/\partial V_i = V_i^{-\varepsilon}$ and substituing for V from (8), with (7), along with $\partial V_i/\partial b$ from (18) gives a very awkward expression for $\frac{\partial W}{\partial V_i}\frac{\partial V_i}{\partial b}$, making comparisons with the median voter model difficult.

Suppose instead that the welfare weights can be treated as depending directly on wage rates.[6] The weighted average $\tilde{w}$ can be regarded as approximated by an 'equally distributed equivalent' value, w_e: this is the wage which, if obtained by everyone, gives the same welfare, defined in terms of the w_is, as the actual distribution. This concept is associated with the Atkinson measure of inequality, A, which is expressed as the proportional difference between the arithmetic mean wage and the equally distributed equivalent wage level, so that:

$$A = \frac{\bar{w} - w_e}{\bar{w}} = 1 - \frac{w_e}{\bar{w}} \tag{34}$$

Again using the iso-elastic welfare function expressed in terms of wages, w_e is defined by:

$$\frac{1}{1-\varepsilon} w_e^{1-\varepsilon} = \left(\frac{1}{1-\varepsilon}\right)\frac{1}{n}\sum_{i=1}^{n} w_i^{1-\varepsilon} \tag{35}$$

[6] This kind of assumption is commonly made. For example, in the marginal indirect tax reform literature it is assumed that the welfare weights are a function of total expenditure, rather than utility.

so that:

$$w_e = \left(\frac{1}{n} \sum_{i=1}^{n} w_i^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \tag{36}$$

The expression in (36) can be further simplified using the properties of the lognormal distribution where, as above, w is distributed as $\Lambda(\mu, \sigma^2)$. Since $\log w^{1-\varepsilon} = (1-\varepsilon)\log w$, the term $w^{1-\varepsilon}$ is lognormally distributed as $\Lambda((1-\varepsilon)\,\mu, (1-\varepsilon)^2\,\sigma^2)$. Using the result for the arithmetic mean of a lognormal variable, (36) becomes:

$$w_e = \left\{ \exp\left((1-\varepsilon)\,\mu + (1-\varepsilon)^2 \frac{\sigma^2}{2} \right) \right\}^{\frac{1}{1-\varepsilon}} \tag{37}$$

so that:

$$w_e = \exp\left(\mu + (1-\varepsilon) \frac{\sigma^2}{2} \right) \tag{38}$$

Consequently, the ratio of the equally distributed equivalent wage to the arithmetic mean wage is:

$$\begin{aligned} \frac{w_e}{\bar{w}} &= \frac{\exp\left(\mu + (1-\varepsilon)\frac{\sigma^2}{2}\right)}{\exp\left(\mu + \frac{\sigma^2}{2}\right)} \\ &= \exp\left(-\varepsilon \frac{\sigma^2}{2}\right) = \left(\exp\left(-\frac{\sigma^2}{2}\right)\right)^{\varepsilon} \end{aligned} \tag{39}$$

Comparing equations (32) and (39) shows that the relationship between the two relevant ratios is:

$$\frac{w_e}{\bar{w}} = \left(\frac{w_m}{\bar{w}} \right)^{\varepsilon} \tag{40}$$

Thus, if $\tilde{w}$ is approximated by w_e, (40) gives the required relationship between the two wage ratios. This result shows that, when $\varepsilon = 1$ the optimal choice is identical to that of the median voter. However, it is not appropriate to suggest that the median voter has inequality aversion of 1. Indeed, the majority voting outcome arises from entirely selfish behaviour where the utility of other individuals is not taken into account at all.[7]

[7] However, Creedy and Moslehi (2007) show how the majority voting outcome is modified in cases where voters do have an aversion to inequality.

Substitution of $\tilde{w} = (w_m/\bar{w})^\varepsilon$ into (28), gives the ratio of expenditure on the transfer payment to public goods as a function of the ratio of median to mean wage rates:

$$R_W = \frac{1-t}{1-t(1-\beta')}\left[\left(\frac{1}{1-\alpha-\beta}\right)\left\{\left(\frac{w_m}{\bar{w}}\right)^\varepsilon\left(\frac{1}{t(1-\beta')}-1\right)+1\right\}^{-1}-1\right] \tag{41}$$

The first derivative of equation (41) with respect to ε gives the relationship between inequality aversion of the social planner and the ratio of the transfer payment to public goods expenditure.

$$\frac{\partial R_W}{\partial \varepsilon} = -\frac{\left(\frac{w_m}{\bar{w}}\right)^\varepsilon \log\left(\frac{w_m}{\bar{w}}\right)}{(1-\alpha-\beta)(1-\beta')}\frac{1-t}{t} > 0 \tag{42}$$

Since w_m is less than $\bar{w}$, $\log\left(\frac{w_m}{\bar{w}}\right)$ is negative and the relationship between R_W and inequality aversion is positive. This result confirms that higher inequality aversion leads to a larger proportion of expenditure devoted to the transfer payment.

5 Some Numerical Examples

This section provides numerical examples of the sensitivity of optimal outcomes to variations in selected parameters of the model, in particular the degree of relative inequality aversion, ε, the tax rate, t, and the ratio of the welfare-weighted average wage to the arithmetic mean wage, $\tilde{w}/\bar{w}$.

The examples are obtained using preference parameters of $\alpha = 0.58$ and $\beta = 0.4$. These produce, with a tax rate of $t = 0.25$, a sensible proportion of time devoted to labour supply. When reporting absolute values of b and G/n, the arithmetic mean and median wage rates used, expressed in annual terms, are \$70000 and \$60000 respectively.[8] Using (40), $\tilde{w}/\bar{w}$ is obtained as $(w_m/\bar{w})^\varepsilon$ for different ε.

[8] These are consistent with a lognormal distribution with mean and standard deviation of logarithms of hourly wage rates of 2.87 and 0.56: these are similar to those for Australia. Using the properties of the lognormal distribution the arithmetic mean and the median hourly wage rate are $20.64 = \exp(2.87 + 0.56/2)$ and $17.64 = \exp(2.87)$: see Aitchison and Brown (1957). Furthermore, the maximum hours per day are set at 13 to obtain annual equivalents.

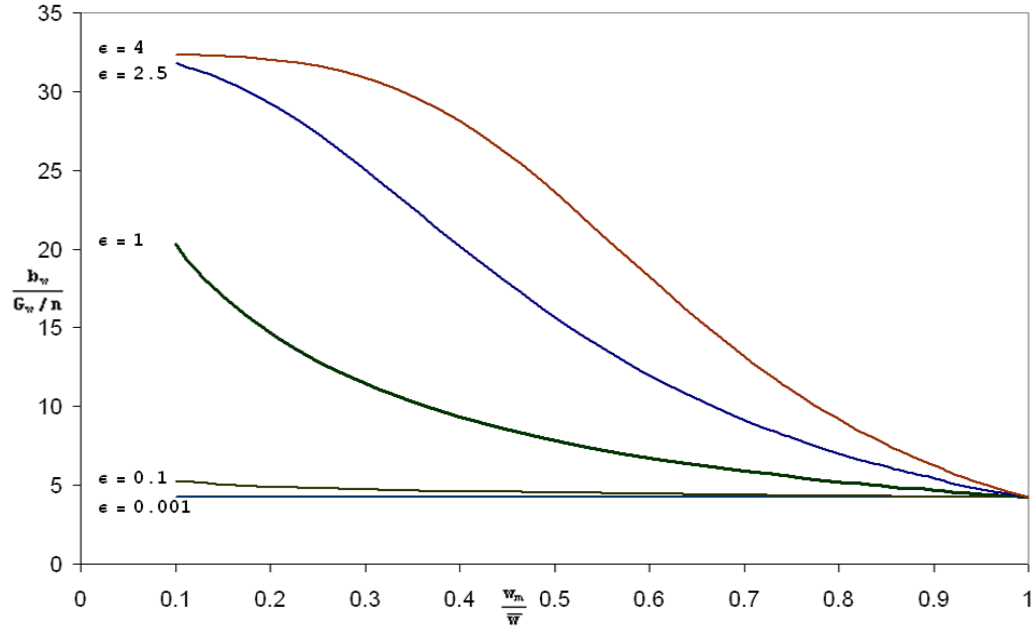

Figure 1: Variation in $b_W/(G_W/n)$ with $w_m/\bar{w}$ for Diffrent ε

Figure1 shows the relationship between $b_W/(G_W/n)$ and $w_m/\bar{w}$ for the different value of ε and benchmark preference parameters, with $t = 0.25$. It illustrates the property mentione above that the social planner's choice of $b_W/(G_W/n)$ falls as inequality falls, that is as $w_m/\bar{w}$ increases towards unity.

When ε is less than one, the optimal proportion of expenditure devoted to redistributive transfer payments is substantially less than the majority voting outcome, and is less sensitive to variations in the ratio $w_m/\bar{w}$. Where the social welfare function has a high degree of inequality aversion, Figure 1 shows that the response of the expenditure pattern to changes in wage rate inequality is very different from that of the median voter. Increasing values of $w_m/\bar{w}$, when there is substantial inequality, has little effect on the ratio $b_W/(G_W/n)$. But around the range where the median is half the arithmetic mean, further increases in $w_m/\bar{w}$ (that is, reductions in inequality) have a considerably effect on the optimal choice of redistributive transfers relative to public good expenditure. The relationship between $b_W/(G_W/n)$ and $w_m/\bar{w}$ is sigmoid for high ε, whereas that for the median voter is closer to being

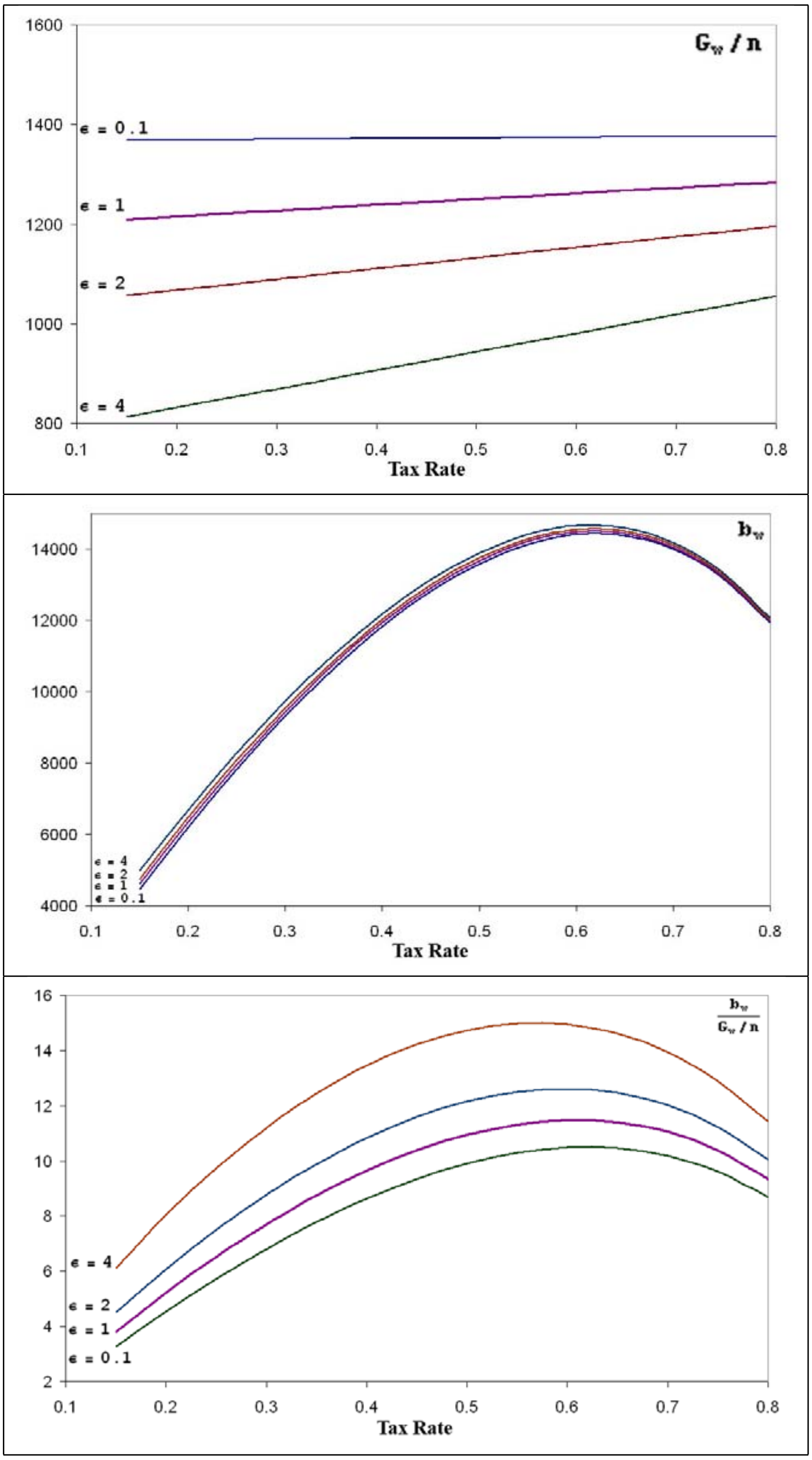

Figure 2: The Optimal Composition and Variations in the Tax Rate

quadratic.

Figure 2 shows the variation in different types of government expenditure as the tax rate (considered here to be exogenous) increases. As shown in the individual figures there is an inverted U-shaped relationship between the absolute level of the transfer payment and the tax rate, and between the ratio $b_W/(G_W/n)$ and the tax rate. Nevertheless, expenditure on public goods is linearly related to the tax rate: this was established analytically above, where it was shown that $\partial G_W/\partial t$ is positive and $\partial^2 G_W/\partial t^2 = 0$. The shapes of the various profiles are very similar for different ε values. As expected higher inequality aversion is consistently associated with higher optimal expenditure on the transfer payment and lower expenditure on the public good. Care is needed in comparing the first two parts of the figure, in view of the fact that the scales on the vertical axes are substantially different.

Figure 3 shows the relationship between $b_W/(G_W/n)$ and ε for different ratios of median to mean wage rate. As expected from the analytical result, raising inequality aversion reduces the optimal ratio of expenditure on the transfer payment to public goods. From the analytical result given above, R does not necessarily increase at an increasing rate as inequality aversion increases, although it does so for the range of values shown in Figure 3. For the lower values of $w_m/\bar{w}$, the profiles were found to be concave over higher ranges of ε, but such high values of inequality aversion are not relevant.[9]

6 Empirical Specifications

Since the relationship between R_m and $w_m/\bar{w}$ is exactly the same as the relationship between R_W and $\tilde{w}/\bar{w}$, the question arises of whether it is possible to distinguish empirically between the two models. Hence, if the composition of expenditure were actually determined by a process that corresponds to the maximisation of a welfare function, is it likely that estimates of the relevant relationship would mistakenly support the median voter model?

Despite the complexity of the relationship in (28) it can be shown that

[9] Beyond the maximum value of ε shown in the figure, aversion is close to becoming 'extreme', thereby corresponding to the 'maxi-min' case.

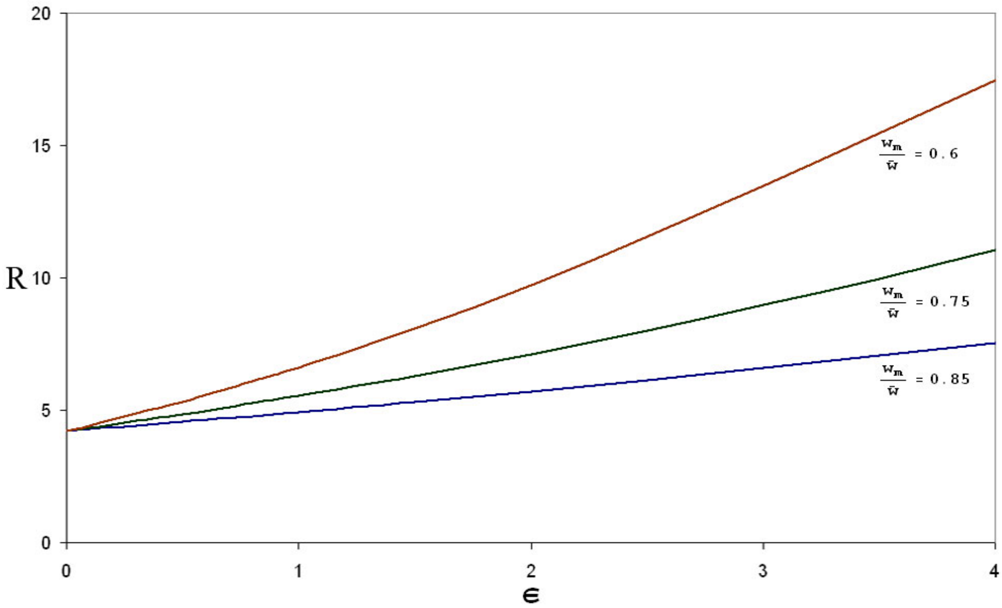

Figure 3: Variation in $b_W/(G_W/n)$ with ε for Diffrent $w_m/\bar{w}$

R_m can be approximated by a linear relationship involving $w_m/\bar{w}$ and its square, and t and its square. Hence any attempt to test the median voter model involves estimation of the regression equation:

$$R = \alpha_0 + \alpha_1 \left(\frac{w_m}{\bar{w}}\right) + \alpha_2 \left(\frac{w_m}{\bar{w}}\right)^2 + \alpha_3 t + \alpha_4 t^2 \tag{43}$$

If the resulting values of α_1 and α_2 are such that R continues to decline as $w_m/\bar{w}$ increases towards 1, and if the values of α_3 and α_4 are such that R reaches a maximum with respect to t at a sensible value of t, the estimates would seem to provide support for the median voter model.

However, if policy is more appropriately modelled 'as if' it is determined by the maximisation of a social welfare function reflecting inequality aversion, the specification, also involving $w_m/\bar{w}$, takes the form:

$$R = \alpha_0 + \alpha_1 \left(\frac{w_m}{\bar{w}}\right)^{\varepsilon} + \alpha_2 \left(\frac{w_m}{\bar{w}}\right)^{2\varepsilon} + \alpha_3 t + \alpha_4 t^2 \tag{44}$$

As mentioned above, the median voter model and the social welfare maximising models are observationally equivalent in the case where $\varepsilon = 1$. If a time series of observations is available for a single country, and if it can be assumed that inequality aversion underlying policy remains constant over time, then a simple first approach would be to estimate (44) for alternative assumed values of ε: the value giving the maximum R^2 may then be taken as the implicit inequality aversion used in policy making. Of course, it could not be taken as an 'estimate' of the actual inequality aversion of policy-makers, which can be substantially different from that of the value implicit in actual policy decisions (which are obviously not made following consideration of a fully specified optimisation problem). Alternatively, it would be useful if a relationship between α_1 and α_2 could be established, for example by imposing an assumed rate of change in R when, say, $w_m/\bar{w} = 1$. But from the above model, this change depends on the tax rate and is thus not constant. Nevertheless, a check on the minimum implied slope of the relationship between R and $w_m/\bar{w}$ provides a valuable check, because it would be expected to be a small negative number.[10] However, in practice is is very difficult to construct such a time series dataset, and it is likely that the variation in $w_m/\bar{w}$ would not be sufficiently wide for estimation purposes.

Alternatively, from (28) it can be seen that:

$$\begin{aligned}\left(\frac{w_m}{\bar{w}}\right)^{\varepsilon} &= \frac{t\left(1-\beta'\right)}{1-t\left(1-\beta'\right)}\left[\left\{\left(1-\alpha-\beta\right)\left(1+\frac{R\left(1-t\left(1-\beta'\right)\right)}{1-t}\right)\right\}^{-1}-1\right] \\ &= K \end{aligned} \tag{45}$$

so that, given extraneous estimates of α and β, and values of the relevant variables for a single period (or averages over a short period) it would be possible to solve for ε as:

$$\varepsilon = \frac{\log K}{\log\left(w_m/\bar{w}\right)} \tag{46}$$

Such values would clearly need to be treated with great care.

[10] In practice an examination of (43) using cross-sectional data providing sufficient variation in wage rate inequality and tax rates needs to rely on the application of this type of criterion to judge the credibility of the results.

7 Conclusions

This paper has examined the optimal composition of government expenditure, in terms of the ratio of transfer payments to expenditure on public goods, for a given income tax rate. A social welfare function, in terms of indivduals' utilities, was maximised subject to the government's budget constraint, involving a loss of one degree of freedom in policy choices. An explicit solution to this optimal tax problem was obtained in which the optimal ratio of transfers to public good expenditure per person is expressed as a function of the ratio of the welfare-weighted mean wage rate to the arithmetic mean wage rate, and of the tax rate.

Reductions in the skewness of the wage rate distribution are associated with reductions in transfer payments relative to public goods expenditure, at a decreasing rate. Furthermore, increases in the tax rate, from relatively low levels, are associated with increases in the relative importance of transfer payments. But beyond a certain level, further tax rate increases are associated with a lower ratio of transfers to public goods. A comparison of the welfare maximising solution with the majority voting outcome was made. It was shown that the difference involves the use of the ratio of median to mean wages in the voting model. Using an assumption that wage rates are lognormally distributed, a simple relationship between an approximation to the welfare-weighted mean and the median wage was obtained, involving the degree of constant relative inequality aversion. Numerical examples were provided, showing the sensitivity of policy choices to a range of parameter values.

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