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**Discounting and the Time
Preference Rate: An Introduction**

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Discounting and the Time Preference Rate: An Introduction*

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Abstract

This paper provides an introduction to the evaluation of alternative time streams of consumption and the closely related concept of time preference. The potential sensitivity of comparisons, especially to the choice of time preference rate and elasticity of marginal valuation, is demonstrated. The nature of time preference, based on an axiomatic approach, is then discussed. The analysis of optimisation over time leads to the concept of the social time preference rate, and a difficulty with using this rate is highlighted. Finally, complications introduced by non-income differences between individuals are examined. Emphasis is placed on the central role of value judgements.

*We are grateful to Tony Scott and Denis O'Brien for comments on an earlier version of this paper.

1 Introduction

This paper provides an introduction to the evaluation of alternative time streams and the closely related concept of time preference. Many investment projects involve a present cost incurred in order to achieve future benefits. These might be in the context of investments in health technology, civil engineering projects, or environmental protection. It is therefore necessary to evaluate alternative outcomes, involving different time streams of net benefits. In any exercise of this kind there are obviously huge problems associated with measurement issues and uncertainty about the future. But the aim of the present paper is to discuss a central issue in the evaluation of alternative time streams – that of discounting. Despite the long-standing nature of this problem, it remains controversial and even the basic issues are far from being settled.¹ One of the problems concerns a lack of clarity over the concepts. Another problem arises from the fact that there is no escape from fundamental value judgements, while protagonists on different sides of debates often conceal their value judgements. The role of the professional economist in these situations is to examine the implications of adopting alternative value judgements. Hence it is important to be clear about precisely how they enter the calculations and how they may be specified.

Section 2 begins by introducing the concept of the social welfare function that is dominant in the literature concerned with evaluating alternative consumption or income streams. This form of welfare function involves, as well as attaching different weights to different levels of consumption irrespective of their timing, the discounting of future flows using what is called a ‘pure time preference rate’. There are alternative views about the way to proceed. One approach is simply to say that the social welfare function is meant to represent alternative value judgements and therefore results should be reported for alternative time preference rates. Some economists attempt to impose their own value judgements, using rhetorical arguments suggesting for example that pure time preference is in some sense ‘ethically indefensible’.

¹For example, the controversial nature of discounting is demonstrated by the debate over the so-called Stern Report (2006) on climate change.

However, it is desirable to have a clear understanding not only of what is implied by pure time preference – or its absence – but what value judgements may lie behind it. That is, it is useful to appreciate how time preference can arise from more basic axioms stating specific value judgements in a clear way. Section 3 provides an explanation of an axiomatic approach to time preference, following the argument of Koopmans (1960).

Section 4 turns from social evaluations of exogenous time profiles to decisions regarding the socially optimal allocation over time. It therefore concerns the planning, again by an independent judge, of optimal saving and consumption patterns but uses the same kind of social welfare function. Section 5 returns to the evaluation of alternative streams in the context of cost-benefit analyses. It discusses the concept of the social time preference rate and highlights a problem with its application. Section 6 introduces some modifications to the basic form of social welfare function discussed in earlier sections.

2 Social Evaluations

Suppose it is required to evaluate a time stream $C = [c_1, c_2, \dots]$ of consumption. For simplicity, it is assumed that the population consists only of individuals (rather than families), that the size of the population remains unchanged over time, and that consumption is the only economic variable considered to be relevant by the judge.² The term c_t refers to aggregate consumption in period t . Hence there is, by assumption, no concern for within-period inequality among individuals. An evaluation cannot avoid the use of value judgements. Hence, the usual approach is to examine the implications of adopting a range of value judgements.

²These assumptions are relaxed in section 6 below.

2.1 A Social Welfare Function

Consider social evaluations based on an additive Paretian social welfare function – representing the views of an independent judge – which takes the form:

$$W(C) = \sum_{t=1}^T U(c_t) \left(\frac{1}{1+\rho} \right)^{t-1} \quad (1)$$

where $U(c_t)$ is the weight attached by the judge to period t 's consumption. It is a cardinal measure of the contribution to W , before discounting, of period t 's consumption. The term ρ is the rate of pure time preference – the focus of attention in much of the discussion below.

The weighting function U is often called a utility function, although this is somewhat misleading unless it refers to a single-person framework. Hence the time preference rate is sometimes also called a ‘utility discount rate’. The social welfare function W does not in general represent the wellbeing of society. Above all, it does not represent ‘society’s views’, although it is remarkable how often writers slip into the use of such expressions.

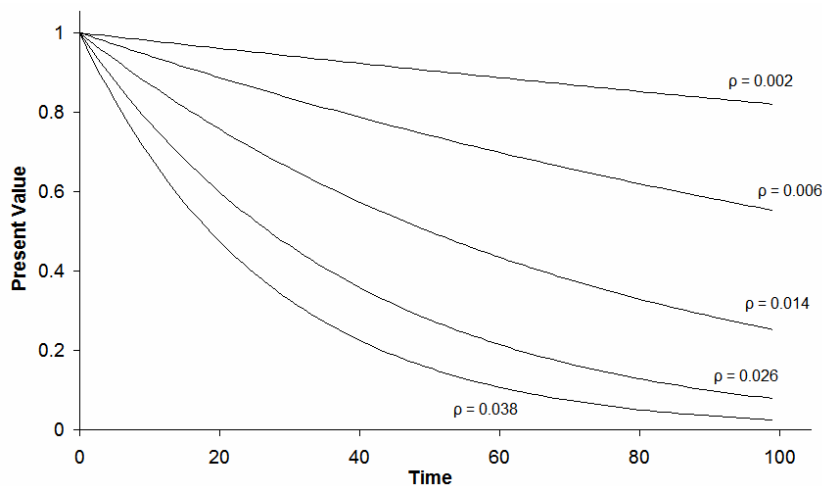


Figure 1: Present Values and Discounting

The effects of discounting alone can be seen in Figure 1, which shows how the present value of \$1 falls as the time period increases: that is it plots $\left(\frac{1}{1+\rho} \right)^{t-1}$ against t , for several alternative values of ρ .

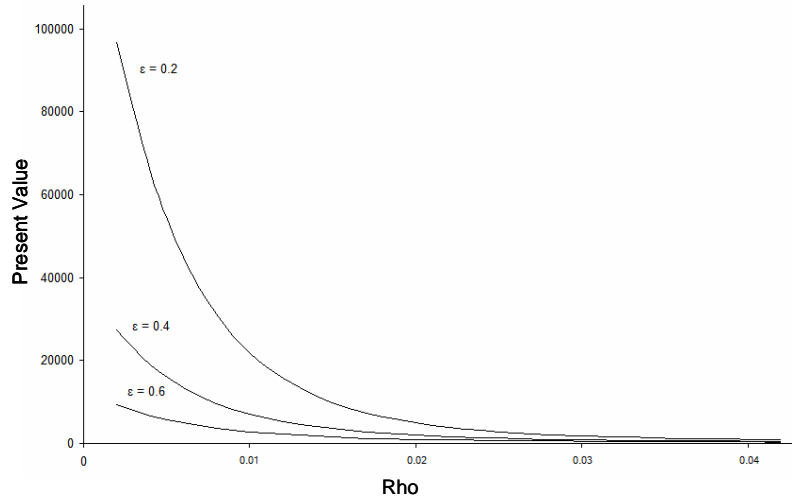


Figure 2: Sensitivity to Choice of Epsilon (Less Than 1):

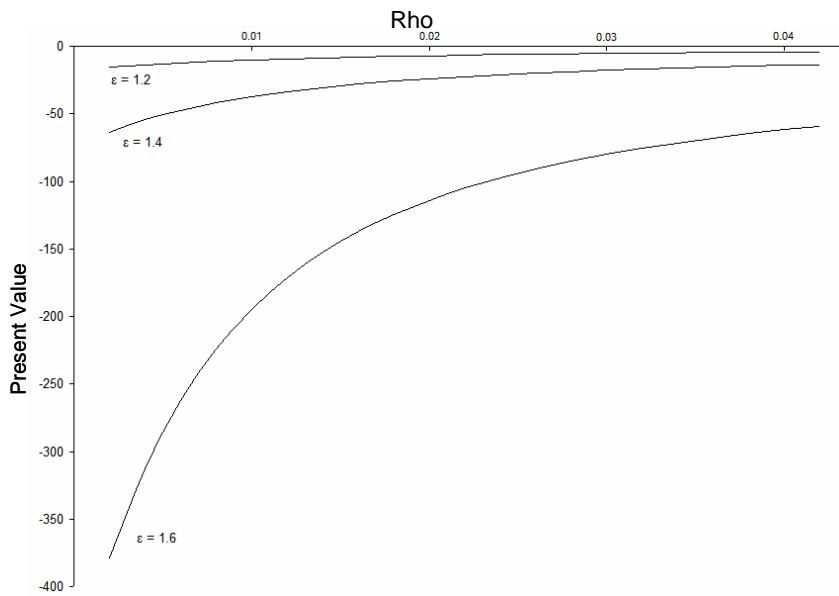


Figure 3: Sensitivity to Choice of Epsilon (Greater Than 1):

Consideration of alternative value judgements regarding U is facilitated by the use of the iso-elastic form:

$$U(c_t) = \frac{c_t^{1-\varepsilon}}{1-\varepsilon} \quad (2)$$

The term ε measures the degree of constant relative aversion to variability on the part of the judge. Those who refer to U as a utility function typically refer to ε as the constant (absolute value of the) elasticity of marginal utility. However, the term ‘elasticity of marginal valuation’ would be clearer.

Alternative value judgements – within the context of this class of welfare functions – can be examined by investigating W for a range of values of ε and ρ . The values of $W(C)$ are highly sensitive to the choice of ε , as shown in Figures 2 and 3, where each profile shows the variation in the present value of a time stream as ρ is increased, for a given value of ε . These show the value of $W(C)$ for a consumption stream over 250 periods, where the initial value is 30 units and there is smooth growth at the constant rate of 2.3 per cent per period. Figure 2 shows the reduction in the present value as ε is increased from 0.2 to 0.6, while Figure 3 shows variations for values of $\varepsilon > 1$, for which $W(C)$ is negative.

Illustrative examples of social indifference curves for consumption in periods 1 and 2, based on (1) combined with (2), are shown in Figure 4. In this case the judge’s marginal rate of substitution of period 1’s for period 2’s consumption is:

$$MRS_{c_1, c_2} = \left(\frac{c_2}{c_1} \right)^\varepsilon (1 + \rho) \quad (3)$$

At the point of intersection with the 45 degree line from the origin, along which consumption is equal in both periods, the solid indifference curve shown is steeper than the downward sloping 45 degree line, indicating pure time preference. The elasticity, reflecting the concavity of U , is also a measure of the convexity of indifference curves, so that the solid curve reflects a lower value than the broken curve.

The introduction of the terms ρ and ε in the social welfare function makes it clear that these reflect the value judgements of a hypothetical judge or decision maker. However, there is a literature attempting to ‘estimate’ values,

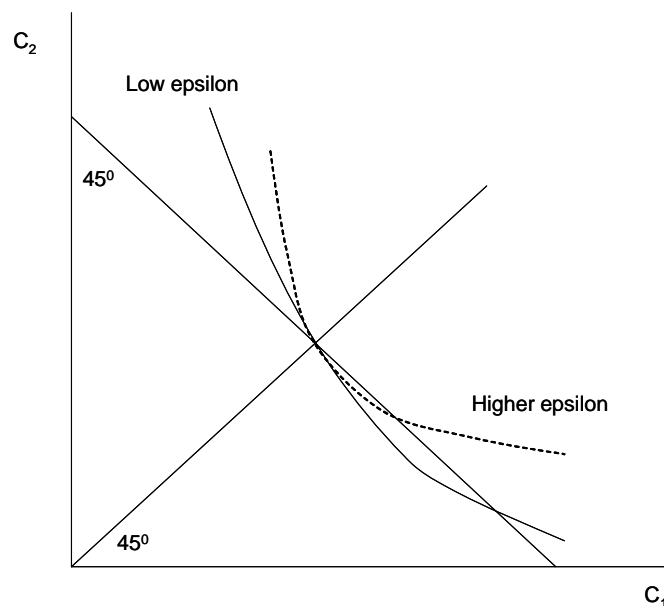


Figure 4: Time Preference

using a variety of methods, which (their authors argue) should then be used in evaluations. Estimation methods include questionnaires, the analysis of cross-sectional consumption patterns and saving behaviour for various population groups, and the recovery of implicit value judgements involved in previous tax and transfer policies. However, there is no escape from the fact that value judgements are involved: the economist can only compare the results of imposing alternative values.³

2.2 Comparing Alternative Time Streams

In view of the sensitivity of present values to the choice of ε and ρ it cannot be expected that alternative time profiles, or projects, have the same ranking, independent of the choice of elasticity of marginal valuation and time preference rate. Consider the two profiles A and B in Figure 5, where B has the fastest constant growth rate of 1.6 per cent, compared with A of 0.9 per cent, but the starting value of B is 5 while that of A is 15. Time profile

³For discussion of various 'estimation' methods and criticism of their use in social evaluations, see Creedy (2006).

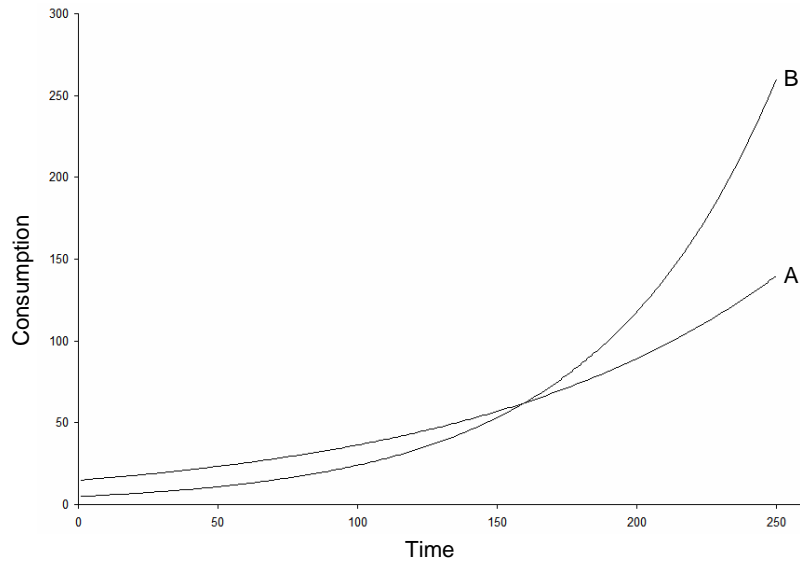


Figure 5: Alternative Time Profiles of Consumption

B is expected to dominate A only for relatively low values of ρ , though the particular value of ρ for which the ranking changes depends crucially on the choice of ε . Present values are shown in Figure 6 for $\varepsilon = 0.6$. For values of $\varepsilon > 0.88$, there is no change in the ranking of the two profiles as ρ varies.

More complex comparisons may result from more variable time profiles, making the choice of alternative streams more sensitive to the choices of ε and ρ . Consider Figure 7, where time stream A results from a constant growth rate of 2.3 per cent (starting from 10 units), but profile B results from a fixed trend rate of growth of 1.8 per cent (starting from 4 units) combined with a cyclical growth component having an amplitude of 5 per cent and a wavelength of 165 periods. From the multiple intersections, it is likely that stream A has the highest value of $W(C)$ for both low and high values of ρ , while stream B is likely to dominate for intermediate values, though the precise values are again likely to be sensitive to the choice of ε . Examples are given in Figures 8 and 9, for two different values of the elasticity of marginal valuation, ε .

It is important to recognise that (1) represents a particular set of value judgements, as well as those giving rise to pure time preference: the evalua-

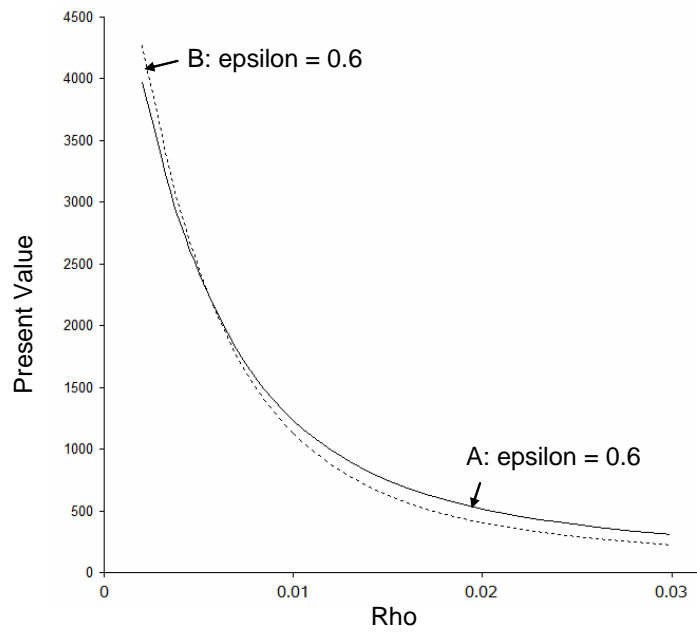


Figure 6: Alternative Rankings of Time Profiles

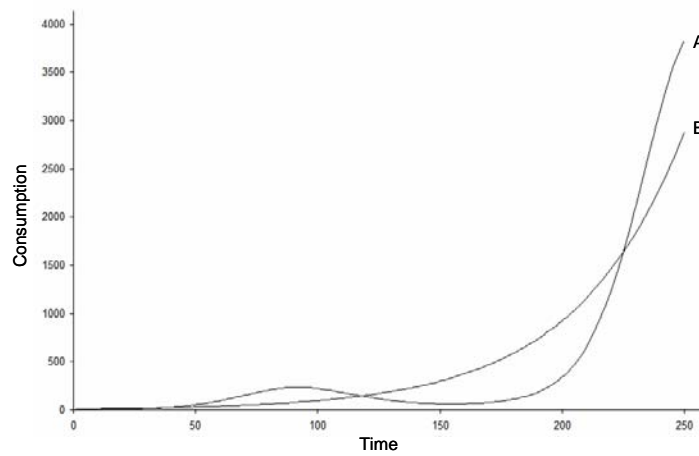


Figure 7: Alternative Time Profiles

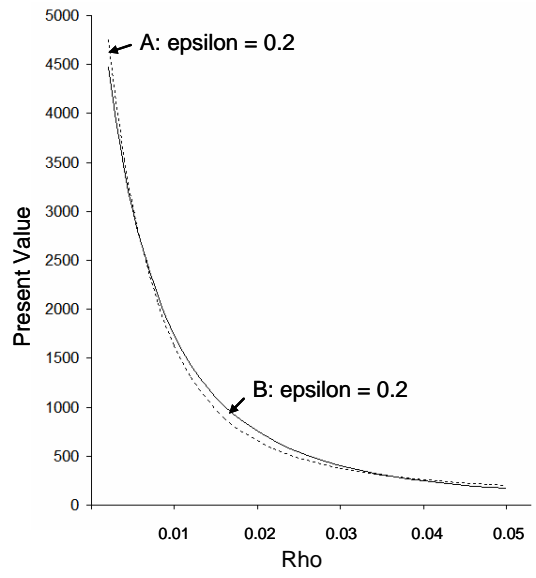


Figure 8: Rankings for Epsilon = 0.2

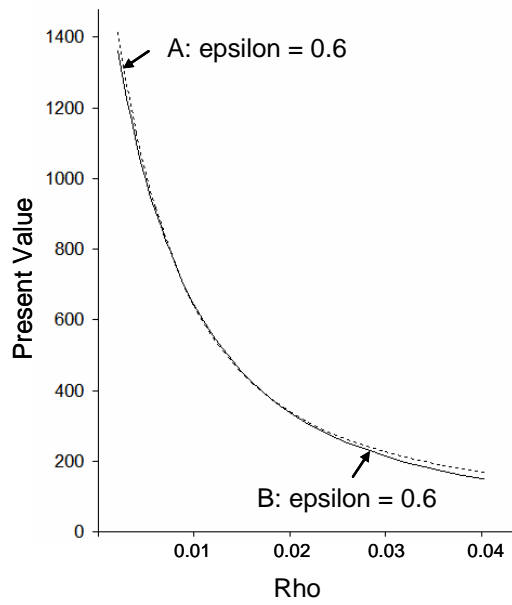


Figure 9: Rankings for Epsilon = 0.6

tion function is additive and Paretian. Alternative views about the desirable evaluation of a time stream of consumption are obviously possible, and professional economists cannot make prescriptions about the form to be used, but can only investigate the implications of adopting alternative forms. The following section considers the implications of the absence of time preference and examines a set of fundamental value judgements, in the form of axioms, giving rise to positive time preference.

3 Existence of Time Preference

The question considered here is whether time preference arises from a clear set of axioms describing an independent judge's or social planner's views (value judgements) about time profiles of consumption. This makes it easier to identify precisely why individuals may differ in their attitudes towards time preference. The following discussion is a highly simplified version of the argument put forward by Koopmans (1960).⁴

3.1 An Axiomatic Approach

Consider an independent judge with an ordinal evaluation function, given by $P(C) = P(c_1, c_2, c_3, \dots)$ and defined over a time stream of consumption represented by the vector, $C = [c_1, c_2, c_3, \dots]$. It is simply assumed that this function has the usual properties of evaluation functions, such as monotonicity and transitivity. For simplicity, it is assumed that the population consists only of individuals (rather than families), that the size of the population remains unchanged over time, and that consumption is the only economic variable considered to be relevant by the judge. The term c_t refers to aggregate consumption in period t . Hence there is, by assumption, no concern for

⁴Other demonstrations are available. Marina and Scaramozzino (1999, p.6) provided an interesting analysis of growth in an overlapping generations framework. They stated that, 'a social rate of pure time preference is justifiable on purely ethical grounds'. A clearer statement of what the authors showed is that *if* the objective of maximising average steady-state consumption per capita is adopted, then *an implication* of this *ethical value judgement*, combined with a model containing productivity and population growth, is that positive time preference exists that does not reflect myopia.

within-period inequality among individuals.

Stated informally, the *continuity* axiom states that any slight variation in C does not lead to big changes in the valuation of C , while a *boundedness* axiom states that paths C_A and C_B exist such that $P(C_A) \leq P(C) \leq P(C_B)$. If alternative paths were to produce unbounded values of P , they could not be ranked.⁵

The *sensitivity* axiom says that if paths C_0 and C_1 differ in only the first period, then $P(C_0) \neq P(C_1)$. Essentially this is stating that the first period matters, in that it cannot be swamped by all other periods. Without the sensitivity axiom, a small gain to each of an infinitely large number of future periods, achieved at the expense of reducing consumption in the present period to zero, would be regarded as acceptable.

A *non complementarity* (or *independence*) axiom states that if two time streams differ only by the first period, their ranking does not depend on the form of the remaining stream. Here, it is convenient to introduce the notation $C^{[2]} = (c_2, c_3, c_4, \dots)$, so that $C = (c_1, C^{[2]})$. Hence, for two time profiles $C_0 = [c_{0,1}, c_{0,2}, c_{0,3}, \dots]$ and $C_1 = [c_{1,1}, c_{1,2}, c_{1,3}, \dots]$, where $c_{k,t}$ refers to consumption in the t th time period and the k th time stream, independence implies that if:

$$P(c_{0,1}, C_0^{[2]}) \geq P(c_{1,1}, C_0^{[2]}) \quad (4)$$

then:

$$P(c_{0,1}, C_1^{[2]}) \geq P(c_{1,1}, C_1^{[2]}) \quad (5)$$

and vice versa.

Finally, a *stationarity* axiom states that if paths C_0 and C_1 have the same consumption in the first period, so that $c_{0,1} = c_{1,1} = c_1$, then the ranking:

$$P(c_1, C_0^{[2]}) \geq P(c_1, C_1^{[2]}) \quad (6)$$

implies also that:

$$P(C_0^{[2]}) \geq P(C_1^{[2]}) \quad (7)$$

⁵Alternative (positive) time streams of consumption over an infinite period could not be compared in the absence of time preference, because they would be unbounded.

Hence the rankings of the alternative streams (with a common first element) remain unchanged if they are simply moved earlier one period in time.

Having stated the axioms, consider two time paths C_1 and C_2 such that $c_{1,t} > c_{2,t}$ for all t , and all $c_{k,t}$ are positive consumption levels ('all goods are good'). It must therefore be the case that $P(C_1) \geq P(C_2)$. Suppose there are two other time streams, $C_3 = (c_{3,1}, C_1)$ and $C_4 = (c_{4,1}, C_2)$ where $c_{3,1} = c_{4,1}$. Hence streams C_3 and C_4 have a common first period's consumption level, and thereafter have precisely the same streams, respectively, as C_1 and C_2 . The stationarity axiom therefore implies that $P(C_3) \geq P(C_4)$.

By definition, the paths C_3 and C_4 , by having a common first element, are less different than C_1 and C_2 . Since, from above, each period matters, this implies that:

$$P(C_1 - C_2) > P(C_3 - C_4) \quad (8)$$

This property implies that the difference is smaller, the more distant in time it is: this is referred to as 'time perspective'; see Koopmans, Diamond and Williamson (1964).

Next, consider alternative streams such that C_1 and C_2 differ only in the first time period, such that $c_{1,1} - c_{2,1} = 1$. Hence the streams C_3 and C_4 differ only in their second period, by the same amount. Using (8) it can be seen that:

$$P(1, 0, 0, 0, 0, \dots) > P(0, 1, 0, 0, 0, \dots) \quad (9)$$

Hence, with only one unit of consumption available, there is a preference for having this in the first period, rather than having nothing in the first period and waiting to consume the unit in the second period. There is therefore a preference for bringing the consumption forward from the second to the first period. This result clearly implies pure time preference.

3.2 A Measure of Pure Time Preference

It is necessary to have a measure of the extent of this pure time preference. Consider for simplicity the two-period case. Time preference can be interpreted in a diagram with period 2's consumption on the vertical axis and period 1's consumption on the horizontal axis, using the concept of social

indifference curves, along which P is constant. In general, the absolute slope of the social indifference curve, the marginal rate of substitution of period 1's consumption for period 2's consumption, MRS_{c_1, c_2} , is given by:

$$MRS_{c_1, c_2} = - \left. \frac{dc_2}{dc_1} \right|_P = \frac{\partial P / \partial c_1}{\partial P / \partial c_2} \quad (10)$$

Where a social indifference curve passes through the point where consumption is the same in each period, the curve must be steeper than a downward sloping 45 degree line, which has an absolute slope of 1. This is because time preference implies that the social planner is prepared to give up one unit in the second period in order to get less than one extra unit in the first period. Hence:

$$MRS_{c_1, c_2} \Big|_{c_1=c_2} = \frac{\partial P / \partial c_1}{\partial P / \partial c_2} > 1 \quad (11)$$

A precise measure of pure time preference can be based on the extent to which the absolute slope of the social indifference curve at $c_1 = c_2$ exceeds 1, as follows. Suppose the evaluation function P is additively separable, so that $P(c_1, c_2) = P_1(c_1) + P_2(c_2)$. In the case where $c_1 = c_2 = c$ and consumption is the same in both periods, time preference implies that $P_1(c) > P_2(c)$. Writing $P_1(c) = U(c)$, it must be possible to write $P(c, c) = U(c) + \gamma U(c)$, where $\gamma < 1$, and hence when $c_1 = c_2 = c$:

$$MRS_{c_1, c_2} \Big|_{c_1=c_2} = \frac{1}{\gamma} \quad (12)$$

To express the fact that $\gamma < 1$, write $\frac{1}{\gamma} = 1 + \rho$. Clearly ρ reflects the extent to which the social indifference curve at $c_1 = c_2 = c$ is steeper than 45 degrees. Hence ρ measures the rate of pure time preference of the social planner, and $\gamma = 1 / (1 + \rho)$.

In general it can be shown that if $P(C_0) > P(C_1)$, for two streams C_0 and C_1 , then it is possible to write:

$$\sum_{t=1}^{\infty} \left(\frac{1}{1 + \rho} \right)^{t-1} U(c_{0,t}) \geq \sum_{t=1}^{\infty} \left(\frac{1}{1 + \rho} \right)^{t-1} U(c_{1,t}) \quad (13)$$

where, as above, $U(c_t)$ represents an evaluation function defined over a single period, t , in contrast with the multi-period P . Hence, the ranking according to $P(C)$ is the same as the ranking according to:

$$W(C) = \sum_{t=1}^T \left(\frac{1}{1+\rho} \right)^{t-1} U(c_t) \quad (14)$$

The evaluation function $W(C)$ has the same form as the welfare function in (1) above. The difference is that in the latter case, pure time preference is simply assumed to be a feature of the social planner, who uses the cardinal weighting function $U(c)$ in each period: it is necessarily cardinal because the values are added in (14). However, following Koopman's axiomatic approach, time preference is seen to be implied by a set of basic axioms, where evaluation of a time stream is based on an ordinal evaluation function, P .

In general, the absolute slope of a social indifference curve associated with the social welfare function in (14) is:

$$MRS_{c_1, c_2} = - \left. \frac{dc_2}{dc_1} \right|_W = \left(\frac{1}{\gamma} \right) \frac{\partial U / \partial c_1}{\partial U / \partial c_2} = (1 + \rho) \frac{\partial U / \partial c_1}{\partial U / \partial c_2} \quad (15)$$

4 Choice of Optimal Time Stream

Consider a planner, with value judgements represented by the social welfare function in (1), who must decide on the optimal consumption and saving path of the economy. The welfare function is maximised subject to an intertemporal budget constraint which can be written in the form:

$$\sum_{t=1}^T \left(\frac{1}{1+r} \right)^{t-1} c_t = Y \quad (16)$$

where Y represents a measure of the present value of resources available for consumption over the period, and r is the rate of interest in a perfect capital market. The Lagrangean for this problem is:

$$L = W + \lambda \left\{ Y - \sum_{t=1}^T \left(\frac{1}{1+r} \right)^{t-1} c_t \right\} \quad (17)$$

Hence first-order conditions, for $t = 1, \dots, T$, are:

$$\frac{\partial L}{\partial c_t} = \left(\frac{1}{1 + \rho} \right)^{t-1} \frac{dU}{dc_t} - \lambda \left(\frac{1}{1 + r} \right)^{t-1} \quad (18)$$

so that for two periods t and $t + 1$:

$$(1 + \rho) \frac{dU/dc_t}{dU/dc_{t+1}} = 1 + r \quad (19)$$

Convenient analytical results can be obtained where U takes the isoelastic form $U(c) = \frac{c^{1-\varepsilon}}{1-\varepsilon}$, discussed above, so that the absolute value of the elasticity of marginal valuation, $\left(\frac{c}{dU/dc} \right) \frac{d}{dc} \left(\frac{dU}{dc} \right)$, is constant and equal to ε . Hence (19) becomes:

$$(1 + \rho) \left(\frac{c_{t+1}}{c_t} \right)^\varepsilon = 1 + r \quad (20)$$

Defining $g_t = \frac{c_{t+1}}{c_t} - 1$, taking logarithms and using the approximation $\log(1 + x) = x$, gives:

$$g_t = \frac{1}{\varepsilon} (r - \rho) \quad (21)$$

This expression is known as the Euler equation for optimal consumption: it describes the optimal time path of consumption. In this simple problem, if the various rates are constant, consumption either grows or declines at a constant rate, depending on the value of $r - \rho$. If the pure time preference rate is equal to the market rate of interest, consumption smoothing is implied, with $g_t = 0$.

Rearrangement of (21) gives:

$$r = \rho + \varepsilon g_t \quad (22)$$

so that at the optimal position the equates the market rate of interest with $\rho + \varepsilon g_t$. It may therefore be said that along the optimal path, the planner equates the marginal return from saving, represented by the market rate of interest, r , with the marginal cost of saving, represented by $\rho + \varepsilon g_t$.

The above analysis of optimal consumption is often used in macroeconomic models of optimal saving; see, for example, Blanchard and Fischer (1989) and Barro and Sala-i-Martin (1995). In such models macroeconomic

behaviour is assumed to be captured by the behaviour of a single individual described as a ‘representative agent’, rather than a social planner as discussed here. There is therefore no consideration of aggregation requirements. In some growth models, the representative individual is assumed to be infinitely lived. The introduction of population growth and other complications can produce a different Euler equation from that given in (21), as discussed in section 6 below.

5 The Social Time Preference Rate

Previous sections have shown that the pure time preference rate, ρ , of a hypothetical judge is used in the context of a social welfare function to discount the weighted values $U(c_t)$ for each period. However, in cost-benefit analyses it is common to compare present values of time streams of money values of consumption, using a ‘consumption discount rate’, rather than the ‘utility discount rate’, ρ . Following (22), the consumption discount rate, δ , is defined as:

$$\delta = \rho + \varepsilon g_t \tag{23}$$

This rate, δ , is more commonly referred to as the ‘social time preference rate’.⁶ In the context of cost-benefit analyses where money values of an exogenous consumption stream are evaluated, the social time preference rate, δ , does not need to be set equal to the market rate of interest.⁷ The terms ρ and ε reflect the value judgements of the independent judge, and in carrying out cost-benefit analyses these values have not surprisingly been the focus of much attention. Equation (23) is the fundamental equation that takes a

⁶Pearce and Ulph (1998) actually refer to the pure time preference rate simply as the ‘rate of time preference (the rate at which *utility* is discounted’, and decompose it into a ‘pure rate’ and a term reflecting the rate of growth of life chances. They refer to $\delta = \rho + \varepsilon g$ as the ‘consumption rate of interest’.

⁷This contrasts with determination of the optimal growth path, as in the previous section, where δ must be equal to the market rate of interest, r . The latter is determined by, for example, the marginal product of capital – depending on the precise nature of the growth model considered.

central role when discussing social time preference rates to be used in cost-benefit analysis. However, there is a serious problem with such an approach, which does not seem to be well-understood in the literature. This problem can be seen from the following comparisons: for further details, see Creedy (2007).

The approach, focusing on the primary role of the social time preference rate, as in (23), is thus to produce a ‘social evaluation’ of the time path, c_t for $t = 1, \dots, T$, using W^* , where:

$$W^* = \sum_{t=1}^T c_t \left(\frac{1}{1 + \rho + \varepsilon g} \right)^{t-1} \quad (24)$$

It is usually taken for granted that this welfare function gives the same ranking of projects as does the function in (1).

In comparing the two forms of evaluation, it is convenient to begin with the most favourable case, that is where consumption does in fact grow at the constant proportional rate, g . Hence $c_t = c_1 (1 + g)^{t-1}$, for $t = 1, \dots, T$, and substitution into (1) gives:

$$W = \sum_{t=1}^T \frac{\{c_1 (1 + g)^{t-1}\}^{1-\varepsilon}}{1 - \varepsilon} \left(\frac{1}{1 + \rho} \right)^{t-1} \quad (25)$$

Rearrangement of this expression gives:

$$W = \sum_{t=1}^T \frac{c_1^{1-\varepsilon} (1 + g)^{(t-1)-\varepsilon(t-1)}}{1 - \varepsilon} \left(\frac{1}{1 + \rho} \right)^{t-1} \quad (26)$$

and:

$$W = \frac{c_1^{-\varepsilon}}{1 - \varepsilon} \sum_{t=1}^T c_1 (1 + g)^{(t-1)} \left(\frac{(1 + g)^{-\varepsilon}}{1 + \rho} \right)^{t-1} \quad (27)$$

Furthermore, using the approximation $(1 + \rho)(1 + g)^\varepsilon = 1 + \rho + \varepsilon g$, this becomes:

$$W = \frac{c_1^{-\varepsilon}}{1 - \varepsilon} \sum_{t=1}^T c_t \left(\frac{1}{1 + \rho + \varepsilon g} \right)^{t-1} \quad (28)$$

and:

$$W = \frac{c_1^{-\varepsilon}}{1 - \varepsilon} W^* \quad (29)$$

This final results demonstrates that it is not correct to believe that W^* , obtained by discounting money values of consumption at the social time preference rate, coincides with W , obtained by discounting $U(c_t)$ at the pure time preference rate ρ . For given ε , W^* automatically gives the same ranking as W only if $\varepsilon < 1$ and two consumption streams, with different growth rates, have the same initial value of consumption. Otherwise, inconsistencies can arise.

For example, Figure 10 shows the present value of the time streams of consumption shown in Figure 7, for $\varepsilon = 0.6$, using W^* , that is with money values discounted using the rate $\rho + \varepsilon g$ and with g set equal to the trend rate of growth. It can be seen that profile A dominates for all values of ρ whereas, using the same value of $\varepsilon = 0.6$, comparisons of W depend significantly on the value of ρ used, as illustrated in Figure 9 above.

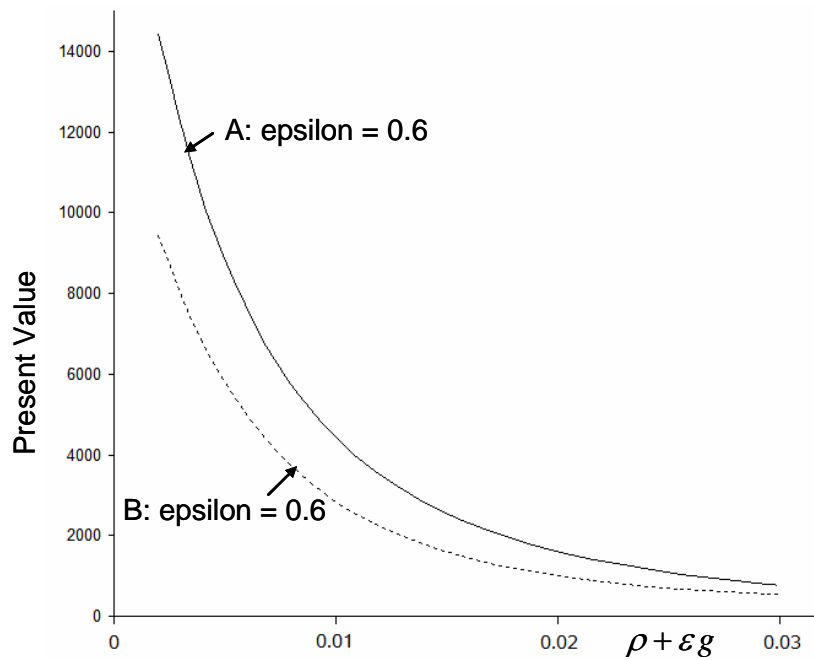


Figure 10: Comparisons Using the Social Time Preference Rate

Hence it is advisable to use the basic form of welfare function in (1), with an explicit form for $U(c_t)$, rather than discounting the stream c_t using the rate $\rho + \varepsilon g$.

6 The Choice of Unit of Analysis

The previous discussion has assumed that there are no relevant non-income differences between individuals and that population size is constant. Suppose instead that the number of individuals at time t is N_t and that individuals of age i have an equivalent adult size of s_i , for example because they may have special age-related needs. The equivalent size of the population at t is $P_t = \sum_i s_i N_{i,t}$ and the average equivalent size is $\bar{s}_t = P_t/N_t$. The question then arises as to the variable, or ‘welfare metric’ to enter the social welfare function. One approach is to write U , the weighting function, as a function of the ratio of average consumption to average equivalent size, $\bar{c}_t/\bar{s}_t = C_t/P_t$, where C_t denotes aggregate consumption in period t . It should be recognised that this is not equal to average consumption per equivalent person, the average value of c/s in the population at year t .⁸

Given a distinction between individuals and equivalent persons, a further decision must be made about the unit of analysis in a welfare function. This decision again involves value judgements. The question of choice of units has been considered in the literature on inequality measurement, but has received little attention in multi-period contexts; for an exception, see Creedy and Guest (2006).⁹ Statements about comparisons between households, in the context of inequality, can easily be converted to statements about comparisons between time periods.

One approach to defining a unit of analysis is to use the ‘adult equivalent person’. In the multiperiod context, there are P_t adult equivalent persons at time t , and so the social welfare function becomes:

$$W = \sum_{t=1} P_t \left(\frac{C_t}{P_t} \right)^{1-\varepsilon} \frac{(1+\rho)^{t-1}}{1-\varepsilon} \quad (30)$$

⁸The two terms are equal either if $c_{i,t}/s_{i,t}$ is constant for all i , or if $s_{i,t}$ and $c_{i,t}$ are uncorrelated.

⁹Major contributions in the context of inequality include Shorrocks (2004), Decoster and Ooghe (2002), Glewwe (1991) and Ebert (1997). The use of different units can lead to opposite conclusions about the effects on inequality of a tax policy change. Examples of such conflicts using tax microsimulation models are given by Decoster and Ooghe (2002) and Creedy and Scutella (2004).

giving a Euler equation for optimal growth at t of:

$$g_t = \frac{1}{\varepsilon} (r - \rho) + (p_t - n_t) \quad (31)$$

where g_t , p_t and n_t are respectively the proportional rates of change of C_t , P_t and N_t . In this way, the ‘income’ concept and the unit of analysis are treated consistently, ensuring that each individual’s contribution depends on the demographic structure of the time period to which they belong. An alternative approach is to treat the individual as the basic unit of analysis. As there are N_t individuals at time t , the social welfare function can be written as:

$$W = \sum_{t=1} N_t \left(\frac{C_t}{P_t} \right)^{1-\varepsilon} \frac{(1+\rho)^{t-1}}{1-\varepsilon} \quad (32)$$

For the optimal consumption path problem, the Euler equation is found to be:

$$g_t = \frac{1}{\varepsilon} \{r - \rho + (\varepsilon - 1) (p_t - n_t)\} \quad (33)$$

so that although the difference between the social welfare functions (30) and (32) concerns only the choice of weights in each period, that is a choice between P_t or N_t , the resulting optimal consumption paths can differ substantially. This is because the choice between individuals and adult equivalents as the basic unit of analysis can in principle lead to different conclusions about the effects of transferring consumption between time periods, which has implications for the path of optimal consumption.

7 Conclusions

This paper has provided an introduction to the evaluation of alternative time streams of consumption using the concept of time preference. The potential sensitivity of comparisons, especially to the choice of time preference rate and elasticity of marginal valuation, was stressed. The nature of time preference, based on an axiomatic approach, was examined. The analysis of individual optimisation over time then led to the concept of the social time preference rate, and a difficulty with using this rate was highlighted. Finally, complications introduced by non-income differences between individuals were

examined. Ultimately, evaluations cannot avoid value judgments, so the role of the economist is to examine the implications of adopting alternative value judgements. As argued by Varian (2006), ‘Exploring the implications of alternative assumptions is likely to lead to better policy than making a single blanket recommendation. At least at this stage of our understanding, exploration beats exhortation’.

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