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The Elasticity of Taxable Income: An Introduction*

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Abstract

This paper provides an introduction to the concept of the elasticity of taxable income with respect to the net-of-tax rate. This elasticity aims to capture all potential responses to income taxation in a single elasticity measure, without the need to specify the nature of the various adjustment processes involved or consider the details of tax regulations. These adjustments include, as well as labour supply changes, income shifting between sources which are taxed at different rates, and tax evasion through non-declaration of income. The use of such a 'reduced form' approach has proved to be very attractive. The elasticity of taxable income has the added attraction that, under certain assumptions, it is sufficient to obtain a measure of the excess burden of income taxation. Emphasis is placed on the assumptions behind the approach.

JEL Classification:

Keywords: Income taxation; Taxable income; Elasticity of taxable income; Excess burden of taxation.

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1 Introduction

This paper provides an introduction to the concept of the elasticity of taxable income with respect to the net-of-tax rate. This elasticity aims to capture all potential responses to income taxation in a single elasticity measure, without the need to specify the nature of the various adjustment processes involved or consider the details of tax regulations.[1] These adjustments include, as well as labour supply changes (both in terms of hours worked and intensity of effort), income shifting between sources which are taxed at different rates, and tax evasion through non-declaration of income. The use of such a 'reduced form' approach, contrasting with an attempt to model the structural form of the behaviour involved, has proved to be very attractive.

The elasticity of taxable income has the added attraction that, under certain assumptions, it is sufficient in order to obtain a measure of the excess burden of income taxation. This is both an elusive and central concept in public economics, and one which has proved to present a range of estimation difficulties.

It is therefore not surprising that many applications have been made in numerous countries since its introduction by Feldstein (1995). A wide-ranging survey of the literature is provided by Saez, Slemrod and Giertz (2009), so there is no need here to refer to other studies.[2] Rather, the aim of the present paper is to provide an introductory treatment of the basic concepts and results, stressing the assumptions involved in specifying a reduced form relationship and in obtaining welfare results.

First, the standard basic approach to labour supply modelling is briefly discussed in Section 2, in order to clarify the difficulties involved in attempting to extend such a structural approach to cover a wider range of tax responses. The reduced form approach is then introduced in Section 3. Emphasis is given to the use of the elasticity of taxable income in welfare measurement, and the sensitivity of results. In considering excess burdens and marginal welfare costs, emphasis in the literature has been on the effects of changes in a top tax rate. The appropriate measures for any income tax bracket are

[1] Some details may of course be needed at the estimation stage, as discussed below.

[2] This survey includes 111 references, and the list could easily be extended.

examined in Section 3. Section 5 turns to a brief consideration of estimation, making use of actual tax reforms which affect a (relatively) small group of individuals, and changes in marginal rates faced by individuals as a result of fiscal drag, or 'bracket creep'. Conclusions are in Section 6.

2 A Structural Approach

This section very briefly describes the standard approach to modelling labour supply, in terms of hours of work, by individuals who face highly nonlinear budget constraints. Subsection 2.1 considers utility maximisation where hours of work can be varied continuously, and Subsection 2.2 discusses the problems of extending this model to allow for tax shifting.

2.1 The 'Standard' Labour Supply Model

The standard approach to modelling labour supply, where hours of work can be varied continuously, assumes that an individual maximises a deterministic utility function $U(c, h)$, where c is net income (or consumption, with the price index normalised to unity) and h is the proportion of time spent in leisure (so that the endowment of time is 1 unit). The fixed wage rate is w and, faced with a piecewise-linear budget constraint, a tangency solution along any section is equivalent to a tangency along the linear constraint:

$$c = w(1-h)(1-\tau) + \mu \tag{1}$$

where τ is the tax rate and μ is 'virtual income' (that is, non-wage income equal to the intercept of the linear section extended to the c axis, where $h = 1$). For corner solutions, w becomes the 'virtual wage', given by the slope of the appropriate indifference curve at the corner. This constraint can be rewritten as:

$$c + wh(1-\tau) = w(1-\tau) + \mu \equiv M \tag{2}$$

where M is 'full income'. Gross earnings are $y = w(1-h)$.

Solving for c and h, and substituting into the direct utility function gives the indirect utility function V, and inverting this gives the expenditure function, $M(w(1-\tau), U)$, the minimum full income needed to attain utility U with net wage $w(1-\tau)$. Alternatively, the expenditure function can be expressed in terms of virtual income. Welfare changes are then expressed in terms of the expenditure function. For example, the equivalent variation resulting from a non-marginal increase in the tax rate from τ_0 to τ_1, involving a new utility level of U_1, is given by:

$$EV = \{M_1 - M(w(1-\tau_0), U_1)\} - \{M_0 - M_1\} \tag{3}$$

where M_i is full income under tax rate i.

The first term in curly brackets is the standard equivalent variation for the reduction in the 'price' of leisure, and the second term in curly brackets is the 'full income' effect of the tax change.[3] For small changes in τ, this can be written as:

$$EV = \frac{dW}{d\tau} = \frac{\partial M(w(1-\tau), U_1)}{\partial \tau} - \frac{dM}{d\tau} \tag{4}$$

A well-known problem arises from the fact that an individual's net wage and hours of work supplied are jointly determined, and depend on the particular range (or corner) of the budget constraint producing maximum utility. Hence any attempt to measure labour supply responses to taxes using a reduced form specification in which hours worked are regressed on the net wage and some measure of the tax rate are subject to serious endogeneity problems. Furthermore, given kinks and non-convexities in budget constraints, labour supply functions do not correspond to the simple smooth specifications used in such regressions.[4]

2.2 Under-Reporting of Income

The labour supply adjustment to a tax change may not be the only adjustment. For example, there is an incentive not to declare some of the income. To illustrate using

[3] However, the complexity (in particular the non-convexity) of actual budget constraints means that care needs to be taken, and (3) may need to be modified.

[4] More progress has been made in labour supply modelling in recent years using a structural approach involving discrete hours and estimation of the direct utility function, allowing for population heterogeneity. See Creedy and Duncan () and Creedy and Kalb ().

a simple specification, suppose that only $z < y$ is declared for tax purposes, but that there is a cost of concealing income, given by $A(y-z)$.[5] The budget constraint then becomes:

$$c = y - \tau z + \mu - A(y-z) \tag{5}$$

Maximising $U(c,h)$ subject to this constraint, income is concealed up to the point where the cost per unit is $A(y-z)/(y-z) = \tau$. Indeed, unless $A(y-z)$ is increasing in $y-z$, all income will be concealed unless, in addition, some energy needs also to be devoted or the individual suffers some feeling of guilt, in which case $y-z$ can be regarded as entering the direct utility function, with a negative effect. The utility function is thus written as $U(c,h,y-z)$.

The case where $A(y-z) = \delta(y-z)$, so that the cost per unit concealed is constant, is equivalent to an assumption that instead of not declaring the income component, $y-z$, it is declared under a different income schedule, attracting a tax rate of $\delta < \tau$. Again, an interior solution exists only if $y-z$ is included in the utility function, so that $U = U(c,h,y-z)$.

The possibilities of modelling taxable income responses to tax rate changes following this kind of structural approach are clearly severely limited. Even a fairly narrow range of income shifting options would give rise to a complex non-linear programming problem, where corner solutions and inequality constraints would be expected to play an important role.

3 A Reduced-Form Approach

Instead of trying to construct a necessarily highly complex structural model which includes labour supply adjustments and income concealment (or some simple form of shifting) as merely one of a number of possible responses to tax changes, this section considers a reduced-form approach. This aims to capture all possible responses to tax rate changes in a single elasticity measure, without attempting to model each form of

[5] A more extensive model would consider factors such as the probability of being audited and the size of a fine if evasion is detected.

response. The concept is that of the elasticity, η, of declared income with respect to the net-of-tax rate, $1-\tau$, defined as:

$$\eta = \frac{1-\tau}{z}\frac{dz}{d\left(1-\tau\right)} \tag{6}$$

Utility specifications giving rise to a functional relationship between declared income and the marginal tax rate are examined in Subsection 3.1. The case of changes in the top marginal rate of income tax, in a multi-rate structure, is considered in Subsection 3.2. This is followed in Subsection 3.3 by the derivation of welfare changes, the excess burden and the marginal welfare cost arising from the top marginal rate. The Income shifting and the sensitivity of results to parameter values are discussed in Subsection 3.4.

3.1 Utility Specification

In addition to consumption, or disposable income, c, total income, y, enters the utility function with a negative effect, reflecting the fact that some effort is required to obtain it. In addition, $y-z$ enters utility in view of the effort or psychic costs involved. Hence $U = U\left(c, y, y-z\right)$. Ignoring also the considerable complexity of actual budget constraints, a linear form may be adopted of the form:

$$c = y - \tau z + \mu - A\left(y-z\right) \tag{7}$$

Maximisation of utility subject to this linear constraint would give rise to a solution for z in terms of the tax rate, the cost of 'income shifting' and virtual income. The solution would be obtained by substituting for c in U using the budget constraint and solving the two equations $\partial U/\partial y = 0$ and $\partial U/\partial z = 0$. However, even for simple utility functions these equations become nonlinear.

With the emphasis only on the effect on declared income (from all sources) of a tax change, a common approach is to consider a yet more abbreviated form of this kind of model, in which utility is written simply as $U\left(c, z\right)$, with z entering negatively. Suppose that the tax rate, τ, applies to income measured above the threshold, z_T. For those with $z > z_T$, and ignoring any cost of 'shifting', the budget constraint is:

$$\begin{aligned} c_i &= \mu_{Vi} + z_i - \tau \left(z_i - z_T \right) \\ &= \left(\mu_{Vi} + \tau z_T \right) + z_i \left(1 - \tau \right) \end{aligned} \tag{8}$$

where μ_{Vi} depends on the nature of the tax and transfer structure, and the total income, y, of the individual: these components do not need to be considered explicitly. Hence, writing virtual income as $\mu_i = \mu_{Vi} + \tau z_T$, the budget constraint is written simply as:

$$c_i = \mu_i + z_i \left(1 - \tau \right) \tag{9}$$

To illustrate, consider the Cobb-Douglas case where $U = c^{\beta} \left(\xi - z \right)^{1-\beta}$, and ξ is a parameter; this simply allows for z to enter the utility function (separately from its contribution to c) with negative marginal utility, again reflecting the effort required to aquire it. Substituting for c using (9), z can be found by solving:

$$\frac{\partial U}{\partial z} = \frac{\beta U}{c} \left(1 - \tau \right) - \frac{\left(1 - \beta \right) U}{\xi - z} \tag{10}$$

giving:

$$z = \beta \xi - \left(1 - \beta \right) \left(\frac{\mu}{1 - \tau} \right) \tag{11}$$

In the absence of a tax, that is when $\tau = 0$, let $z = z_0 = \beta \xi - \left(1 - \beta \right) \mu$. Then declared income can be written as:

$$z = z_0 + \left(1 - \beta \right) \left(1 - \frac{1}{1 - \tau} \right) \tag{12}$$

and it is found that:

$$\eta = \frac{\frac{1}{1-\tau} + \tau \left(\frac{z_T}{\mu} \right)}{\frac{1}{1-\beta} \left(\frac{z_0}{\mu} \right) - \frac{\tau}{1-\tau}} \tag{13}$$

The elasticity is therefore a function of $1 - \tau$, z_0, τ_T, and virtual income, μ, and the latter is of course expected to vary among individuals. Clearly, η increases as τ increases. Of course, these results apply to tangency solutions. In general, with a tax structure containing a number of marginal rates, it would be necessary to consider corner solutions.

Considerable further simplification arises if income effects are not present. A specification which has received much attention is the quasi-linear form:[6]

$$U = c - \left(\frac{1}{1+\frac{1}{\varepsilon}}\right)\left(\frac{z}{z_0}\right)^{1+\frac{1}{\varepsilon}} \tag{14}$$

so that:

$$\frac{\partial U}{\partial z} = 1 - \tau - \left(\frac{z}{z_0}\right)^{\frac{1}{\varepsilon}} \tag{15}$$

and solving for z gives:

$$z = z_0 \left(1 - \tau\right)^{\varepsilon} \tag{16}$$

and the elasticity is constant at $\eta = \varepsilon$. It is the linear term in c, and the absence of μ from the term involving z, which ensures that income effects are zero. If ε is assumed to be the same for all individuals, then they necessarily all have the same elasticity, η, despite having different incomes.

It is seen below that the assumption of zero income effects is central in obtaining results regarding welfare changes. However, it should be recognised that the widespread and often implicit assumption of zero income effects is largely untested in this context.[7]

3.2 A Top Marginal Rate

Most attention in the literature has been given to the top marginal income tax rate in a multi-rate system. This is partly because the initial policy changes which were used to estimate the elasticity of taxable income involved changes to only the top rate. The revenue and welfare effects of such a top rate are considered here. Suppose income above a threshold, z_T, is taxed at the fixed rate, τ. If this is the top marginal rate, so that there are no income thresholds above z_T, the tax paid at that rate by an individual is given, for $z_i > z_T$, by $\tau\left(z_i - z_T\right)$ and total revenue collected by the top marginal

[6] The role of this form is pointed out by Saez *et al.* (2009, p.16, n.34). It is of course a simple matter to integrate backwards from a constant elasticity form to obtain the utility function. Alfred Marshall first used the assumption of additive utility with one good having constant marginal utility, when wishing to avoid income effects in disequilibrium trading.

[7] In the context of labour supply, some evidence for Australia is presented in Kalb and Ghantous (2006). Their results show that income effects are nontrivial for various demographic groups.

rate, R_T, is thus:

$$R_T = \tau \sum_{z_i > z_T} (z_i - z_T) = N_T \tau (\bar{z}_T - z_T) \tag{17}$$

where $\bar{z}_T$ is the arithmetic mean of those above the threshold, and N_T is the number of people above the threshold. It is important to recognise that $\tau (z_i - z_T)$ is not the total tax paid by person i, since the latter has to include tax paid at lower rates. However, changes in the top rate are of course expected to have no effect on revenue from the lower rates.

The effect on R_T, and thus on total revenue, of a change in τ is thus:

$$\frac{dR_T}{d\tau} = \frac{\partial R_T}{\partial \tau} + \frac{\partial R_T}{\partial \bar{z}_T} \frac{\partial \bar{z}_T}{\partial \tau} \tag{18}$$

The first term is a pure 'tax rate' effect while the second term is a 'tax base' effect of the tax rate change. Using $\frac{\partial R_T}{\partial \tau} = N_T (\bar{z}_T - z_T)$, $\frac{\partial R_T}{\partial \bar{z}_T} = N_T \tau$, and from the definition of η, $\frac{\partial \bar{z}_T}{\partial \tau} = -\frac{\eta \bar{z}_T}{1-\tau}$, the revenue change becomes:[8]

$$\frac{dR_T}{d\tau} = N_T (\bar{z}_T - z_T) \left\{ 1 - \eta \left(\frac{\bar{z}_T}{\bar{z}_T - z_T} \right) \left(\frac{\tau}{1-\tau} \right) \right\} \tag{19}$$

Let $f(z)$, $F(z)$ and $F_1(z) = \int_0^z u dF(u) \, / \int_0^\infty u dF(u)$ denote the density function, the distribution function and the first moment distribution function of z. The Lorenz curve is, for example, the relationship between $F(z)$ and $F_1(z)$, the proportion of people associated with a proportion of total income (cumulating from lowest to highest). In general it can be shown that:

$$\frac{\bar{z}_T}{\bar{z}_T - z_T} = \left[1 - \left(\frac{z_T}{\bar{z}} \right) \frac{1 - F(z_T)}{1 - F_1(z_T)} \right]^{-1} \tag{20}$$

where $\bar{z}$ is the arithmetic mean of the complete distribution of z. Define:

$$\alpha_T = \frac{\bar{z}_T}{\bar{z}_T - z_T} \tag{21}$$

so that (19) is more succinctly written as:

$$\frac{dR_T}{d\tau} = N_T (\bar{z}_T - z_T) \left\{ 1 - \eta \alpha_T \left(\frac{\tau}{1-\tau} \right) \right\} \tag{22}$$

[8] This is equivalent to the result stated by Saez *et al.* (2009, p. 5, equation 5).

The tax rate, τ^*, which maximises revenue from the top marginal rate, obtained by setting $dR_T/d\tau = 0$, is thus a simple function of α_T and the elasticity, η, whereby:

$$\tau^* = (1 + \alpha_T \eta)^{-1} \tag{23}$$

Thus the revenue change in (19) depends on the precise form of the distribution of declared income and the income threshold above which the tax rate of τ applies. In practice the various components of (20) can be obtained from information about the distribution of declared incomes. There is no need to make further special assumptions about the form of the distribution.

However, simply for illustrative purposes, suppose the distribution of z follows the lognormal distribution. Table 1 shows values of $\frac{\bar{z}_T}{\bar{z}_T - z_T}$ for a range of values of $z_T/\bar{z}$, where the variance of logarithms of declared income is equal to $\sigma^2 = 0.7$.

Table 1: Values of $\frac{\bar{z}_T}{\bar{z}_T - z_T}$ for Lognormal Distribution with $\sigma^2 = 0.7$

$z_T/\bar{z}$	$F(z_T)$	$F_1(z_T)$	α
1.000	0.662	0.338	2.042
1.500	0.817	0.526	2.384
2.000	0.894	0.659	2.655
2.500	0.935	0.751	2.879
3.000	0.958	0.815	3.071
3.500	0.972	0.860	3.238
4.000	0.981	0.892	3.386
4.500	0.987	0.916	3.519
5.000	0.990	0.934	3.640

If, instead, it is assumed that declared incomes above z_T are distributed as a Pareto Type I distribution, with density function, $f(z) = K/z^{1+\alpha}$. The Pareto distribution has been found to give a better approximation to empirical tails of taxable income distributions than the upper tail of the lognormal distribution (which is typically too 'thin' in the high income ranges). In this special case it can be shown that $\bar{z}_T/(\bar{z}_T - z_T) = \alpha$. Importantly, this term does not, unlike the general case, depend on the threshold income level itself, though of course the lower is z_T in relation to $\bar{z}$, the less likely it is that declared incomes above z_T can be approximated by the Pareto distribution. And of course there is nothing to be gained by assuming that α has this property, since it is easy enough to compute values from observed distributions.

3.3 Welfare Changes

Consider welfare changes arising from changes in the top marginal tax rate, for each individual, i, with $z > z_T$. If $E_i = E(\tau, U_i)$ denotes the expenditure function,[9] the welfare change for a small tax change, following (4) above, is:[10]

$$\frac{dW_i}{d\tau} = \frac{\partial E(\tau, U_i)}{\partial \tau} - \frac{d\mu_i}{d\tau} \tag{24}$$

Using Shephard's Lemma (the Envelope theorem), it is known that $\partial E(\tau, U_i)/\partial\tau = z_i^H$, where the superscript indicates that it is the Hicksian, or compensated, 'demand'. If income effects are ignored, Marshallian and Hicksian demands are equal for each individual. Furthermore, from the budget constraint defined above, $\mu_i = \mu_{Vi} + \tau z_T$, and so $d\mu_i/d\tau = z_T$ for all individuals above the threshold. Hence the welfare change is simply:[11]

$$\frac{dW_i}{d\tau} = z_i - z_T \tag{25}$$

Comparing this result with (18), this welfare change is equivalent to $\partial R_T/\partial\tau$ for each individual, that is the 'tax rate' effect on revenue of a change in τ. Adding these changes over all those above z_T gives the aggregate welfare change as:

$$\frac{dW}{d\tau} = N_T(\bar{z}_T - z_T) \tag{26}$$

[9] Here, 'expenditure' must be in terms of virtual income. On welfare changes and excess burdens see Creedy (1998).

[10] Strictly it would be necessary to consider whether equivalent or compensating variations are used (that is, depending on whether the utility level held constant is the post- or pre-change level). But the assumption that there are no income effects means that this can be ignored here.

[11] Alternatively, consider changes in indirect utility resulting from a change in τ,The indirect utility function, omitting the individual subscript, V, is defined as $Max_z U(\mu + (1-\tau)z, z)$. The total differential is $dV = \frac{\partial V}{\partial c}\frac{\partial c}{\partial \tau}d\tau + \frac{\partial V}{\partial c}\frac{\partial c}{\partial \mu}d\mu$. From the budget constraint, $\frac{\partial c}{\partial \tau} = -z$ and $\frac{\partial c}{\partial \mu} = 1$, and writing $\frac{\partial V}{\partial c} = u_c$, gives $dV = -u_c(zd\tau - d\mu) = -u_c(z - d\mu/d\tau)d\tau$. Using, as shown above, $d\mu/d\tau = \tau$, gives $dV = -u_c(z - z_T)d\tau$. With no income effects, and the quasi-linear utility function above, the term u_c is the same for everyone.

and the (aggregate) marginal excess burden, MEB, arising from the tax change is:[12]

$$MEB = N_T\left(\bar{z}_T - z_T\right)\eta\alpha_T\left(\frac{\tau}{1-\tau}\right) \tag{27}$$

The MEB is thus equal to the absolute value of the 'tax base' effect on tax revenue of a rate change.[13] The marginal welfare cost, MWC, defined as the marginal excess burden divided by the change in tax revenue, is:[14]

$$MWC = \frac{\eta\tau\alpha_T}{1-\tau-\eta\tau\alpha_T} \tag{28}$$

This expression is relevant only when the marginal tax rate is below the revenue-maximising rate in (23), so that $dR_T/d\tau > 0$.

3.4 Income Shifting between Tax Schedules

The above discussion has ignored the possibility of shifting some income to another schedule where it is taxed at a lower rate. Suppose a proportion, s, of the reduction in declared income attracts a tax rate of $t < \tau$. It can then be shown that:[15]

$$\frac{dR}{d\tau} = N\left(\bar{z} - z_T\right)\left\{1 - \eta\alpha\left(\frac{\tau - st}{1-\tau}\right)\right\} \tag{29}$$

and:

$$MWC = \frac{\eta\alpha\left(\tau - st\right)}{1-\tau-\eta\alpha\left(\tau - st\right)} \tag{30}$$

Again this expression is relevant only for parameter values for which $dR_T/d\tau > 0$, that is for values of τ which are less than τ^*, given by:

$$\tau^* = \frac{1 + st\alpha\eta}{1+\alpha\eta} \tag{31}$$

and which is higher than in (23).

[12] This is equivalent to the result given by Saez *et al.* (2009, p. 6, equation 6).

[13] The term $\tau/\left(1-\tau\right)$ is in fact the tax-inclusive marginal tax rate. That is, applying this rate to net income generates the same revenue as applying τ, the tax-exclusive rate, to the corresponding gross income.

[14] Saez *et al.* (2009, p. 6) call the MWC the 'marginal efficiency cost of funds (MECF)'. However, the 'marginal cost of funds', or MCF, is usually defined as $1+MWC$. On these concepts, see Creedy (1998, pp. 54-59).

[15] These results are given in Saez *et al.* (2009, p. 9).

Table 2: Marginal Welfare Cost of Top Marginal Tax Rate

				Elasticity of taxable income, η				
α	s	t	τ	0.2	0.4	0.6	0.8	1.0
1.6	0	0	0.3	0.159	0.378	0.699	1.215	2.182
1.6	0	0	0.4	0.271	0.744	1.778	5.818	
1.6	0	0	0.5	0.471	1.778	24.000		
1.6	0	0	0.6	0.923	24.000			
1.6	0.2	0.2	0.3	0.135	0.312	0.554	0.906	1.465
1.6	0.2	0.2	0.4	0.238	0.623	1.358	3.310	24.000
1.6	0.2	0.2	0.5	0.417	1.432	7.562		
1.6	0.2	0.2	0.6	0.812	8.615			
1.6	0.2	0.4	0.5	0.368	1.163	4.165		
1.6	0.2	0.4	0.6	0.712	4.952			
1.6	0.6	0.2	0.3	0.090	0.197	0.328	0.491	0.699
1.6	0.6	0.2	0.4	0.176	0.426	0.812	1.483	2.947
1.6	0.6	0.2	0.5	0.321	0.947	2.698	35.765	
1.6	0.6	0.2	0.6	0.623	3.310			
1.6	0.6	0.4	0.5	0.200	0.499	0.997	1.990	4.952
1.6	0.6	0.4	0.6	0.404	1.358	6.353		
1.6	0.8	0.2	0.3	0.068	0.147	0.238	0.344	0.471
1.6	0.8	0.2	0.4	0.147	0.344	0.623	1.049	1.778
1.6	0.8	0.2	0.5	0.278	0.771	1.880	6.716	
1.6	0.8	0.2	0.6	0.543	2.378			
1.6	0.8	0.4	0.5	0.130	0.299	0.528	0.855	1.358
1.6	0.8	0.4	0.6	0.289	0.812	2.049	8.615	
1.8	0.2	0.2	0.4	0.276	0.761	1.841	6.353	
1.8	0.2	0.2	0.5	0.495	1.962	155.255		
1.8	0.2	0.2	0.6	1.016				
1.8	0.2	0.4	0.5	0.433	1.530	9.776		
1.8	0.2	0.4	0.6	0.880	14.625			
1.8	0.6	0.3	0.4	0.152	0.359	0.656	1.119	1.941
1.8	0.6	0.3	0.5	0.299	0.855	2.238	11.755	
1.8	0.6	0.3	0.6	0.608	3.098			
1.8	0.6	0.4	0.5	0.230	0.598	1.281	2.981	14.625
1.8	0.6	0.4	0.6	0.479	1.841	34.715		
1.8	0.8	0.2	0.3	0.078	0.168	0.276	0.404	0.563
1.8	0.8	0.2	0.4	0.168	0.404	0.761	1.358	2.571
1.8	0.8	0.2	0.5	0.324	0.959	2.765	47.077	
1.8	0.8	0.2	0.6	0.656	3.808			

A feature of this framework, which may not be immediately obvious, is that the marginal welfare cost per dollar of revenue arising from the top marginal tax rate is highly sensitive to the elasticity of taxable income, as well as the relevant tax rates, t and τ. Illustrative examples are given in Table 2, for typical values of α. Where there is no entry in a cell, this means the system is on the 'wrong' side of the Laffer curve, and an increase in the marginal tax rate produces a reduction in revenue. This sensitivity must be borne in mind when considering empirical estimates of η.

4 Welfare Changes for All Income Tax Brackets

The discussion in Section 3 above concentrated on the top income tax bracket. However, the same approach can be applied to any tax bracket in a multi-rate system. Suppose that in the kth tax bracket the marginal tax rate is τ_k above the income threshold a_k, for $k = 1, ..., K$. For $a_k < z < a_{k+1}$ the tax paid at the rate τ_k is simply $\tau_k (z - a_k)$, and for $z > a_{k+1}$ the tax paid at that rate is $\tau_k (a_{k+1} - a_k)$. Hence if $F(z)$ denotes the distribution function of taxable income, the total tax paid at the rate τ_k, denoted R_k, is given by:

$$R_k = \tau_k \int_{a_k}^{a_{k+1}} (z - a_k)\, dF(z) + \tau_k (a_{k+1} - a_k) \int_{a_{k+1}}^{\infty} dF(z) \tag{32}$$

Expanding terms, and letting $F_1(z)$ denote the first moment distribution function of z, and N the total population size:

$$\frac{R_k}{N} = \tau_k \left[\bar{z} \left\{ F_1(a_{k+1}) - F_1(a_k) \right\} - a_k \left\{ F(a_{k+1}) - F(a_k) \right\} + (a_{k+1} - a_k) \left\{ 1 - F(a_{k+1}) \right\} \right] \tag{33}$$

where $\bar{z}$ is the arithmetic mean of the complete distribution of z. The number of individuals with income in the kth tax bracket, N_k, is:

$$N_k = N \left\{ F(a_{k+1}) - F(a_k) \right\} \tag{34}$$

The number of people with income above the threshold a_{k+1}, that is, the number above the kth tax bracket, N_{k+1}^{+}, is:

$$N_{k+1}^{+} = N \left\{ 1 - F(a_{k+1}) \right\} \tag{35}$$

Furthermore, if $\bar{z}_k$ denotes the arithmetic mean of z within the kth tax bracket, then:

$$\bar{z}_k = \frac{\bar{z}\left\{F_1\left(a_{k+1}\right) - F_1\left(a_k\right)\right\}}{\left\{F\left(a_{k+1}\right) - F\left(a_k\right)\right\}} \tag{36}$$

Thus revenue raised by the kth rate is:

$$R_k = \tau_k N_k\left(\bar{z}_k - a_k\right) + \tau_k N_{k+1}^{+}\left(a_{k+1} - a_k\right) \tag{37}$$

The change in tax revenue from a small change in τ_k is:

$$\frac{dR_k}{d\tau_k} = \frac{\partial R_k}{\partial \tau_k} + \frac{\partial R_k}{\partial \bar{z}_k}\frac{\partial \bar{z}_k}{\partial \tau_k} \tag{38}$$

The first term is the pure 'tax rate' effect while the second term is the 'tax base' effect of the tax rate change. Here:

$$\frac{\partial R_k}{\partial \tau_k} = N_k\left(\bar{z}_k - a_k\right) + N_{k+1}^{+}\left(a_{k+1} - a_k\right) \tag{39}$$

Furthermore, $\partial R_k/\partial \bar{z}_k = \tau_k N_k$, and if the elasticity of taxable income in the kth bracket is η_k:

$$\frac{\partial \bar{z}_k}{\partial \tau_k} = -\frac{\eta_k \bar{z}_k}{1-\tau_k} \tag{40}$$

The revenue change thus becomes:

$$\frac{dR_k}{d\tau_k} = N_k\left(\bar{z}_k - a_k\right)\left\{1 - \eta_k\left(\frac{\bar{z}_k}{\bar{z}_k - a_k}\right)\left(\frac{\tau_k}{1-\tau_k}\right)\right\} + N_{k+1}^{+}\left(a_{k+1} - a_k\right) \tag{41}$$

In considering the welfare change, allowance must be made for the income change of those above the threshold, a_{k+1}, though they do not experience a change in their effective marginal tax rate and there is no excess burden. For those with $a_k < z < a_{k+1}$, the welfare change is, as obtained above, given by $z_i - a_k$. Thus, writing $\alpha_k = \bar{z}_k/\left(\bar{z}_k - a_k\right)$, aggregate marginal excess burden is similar to the earlier result that:

$$MEB = N_k\left(\bar{z}_k - a_k\right)\frac{\eta_k \alpha_k \tau_k}{1-\tau_k} \tag{42}$$

However, the marginal welfare cost becomes:

$$MWC = \frac{\eta_k \alpha_k \tau_k}{1 - \tau_k - \eta_k \alpha_k \tau_k + D_k} \tag{43}$$

where:

$$D_k = \frac{N_{k+1}^{+}}{N_k}\left(\frac{a_{k+1}-a_k}{\bar{z}_k-a_k}\right)(1-\tau_k) \tag{44}$$

and substitution gives:

$$D_k = \frac{(1-\tau_k)(a_{k+1}-a_k)\{1-F(a_{k+1})\}}{\bar{z}\{F_1(a_{k+1})-F_1(a_k)\}-a_k\{F(a_{k+1})-F(a_k)\}} \tag{45}$$

The result for the top marginal rate is thus simply the special case where $a_{K+1}=\infty$ and $F(a_{K+1}) = F_1(a_{K+1}) = 1$, so that $D_K = 0$. If, as for example in New Zealand, there is no tax-free threshold in the income tax structure, so that $a_1 = 0$, it can be seen that $\alpha_1 = 1$ and $D_1 = [(1-\tau_1)\,a_2\{1-F(a_2)\}] / \{\bar{z}F_1(a_2)\}$.

For the top bracket it may be possible to use the Pareto special case where α_K is constant and independent of the threshold, but for lower tax brackets the value of α_k is expected to vary. In practice no special assumptions are needed as the various terms can easily be calculated using information about the distribution of declared income.

To give an idea of variations, consider a tax structure having four (positive) tax brackets, with thresholds of \$5000, \$15000, \$50000 and \$75000 and marginal rates of 0.15, 0.20, 0.30 and 0.35. Suppose that the distribution of taxable income is lognormally distributed with mean and variance of logarithms of 10 and 0.7 respectively. The marginal welfare cost is given in Table 3 for each marginal tax rate, for values of η_k varying from 0.1 to 0.9. The zero values for the top marginal rate appear because, for those values of η_k, a small increase in the tax rate actually reduces revenue. The effect of D_k in reducing the marginal welfare cost in the lower tax brackets – because it raises extra revenue from those above the relevant brackets, without an associated excess burden – can clearly be seen from these results. As anticipated, there are areas where results are sensitive to parameters. For example, if the variance of logarithms is instead 0.5, most values are similar to those in the table except that the marginal welfare cost in the top tax bracket for $\eta = 0.5$ becomes very high at 33.422.

Table 3: Marginal Welfare Cost for Tax Brackets

Bracket	$\bar{z}_k$	α_k	η_k	D_k	MWC
1	10201	1.961	0.1	3.884	0.006
			0.3		0.019
			0.5		0.032
			0.7		0.045
			0.9		0.059
2	27972	2.156	0.1	0.688	0.030
			0.3		0.095
			0.5		0.169
			0.7		0.255
			0.9		0.353
3	60445	5.787	0.1	1.302	0.095
			0.3		0.352
			0.5		0.765
			0.7		1.545
			0.9		3.555
4	115821	2.837	0.1	0	0.180
			0.3		0.846
			0.5		3.235
			0.7		
			0.9		

5 Estimation

The previous section has discussed a reduced-form approach to modelling behavioural responses to income taxation, in which all the potential types of adjustment (in addition to the familiar labour supply response) are summarised by the elasticity, η, of taxable income, z, with respect to the net-of-tax rate, $1 - \tau$. This approach appears to offer a great deal. Without any need to consider, for example, the highly complex nonlinear tax and transfer systems in operation and the associated corner solutions, the often complex rules regarding the use of different tax schedules (relating to different income sources), and the nature of individuals' preferences which are likely to display considerable heterogeneity, the responses are described by a single parameter. Furthermore, by assuming that income effects can be neglected, a simple expression can be obtain for the marginal welfare cost of taxation, applying to a group of individuals without any aggregation complexities.

This Section turns to estimation of the taxable income elasticity, using the commonly used reduced form discussed above. Letting z_{it} and z_{0it} denote declared income of person i at time t and the income which would be declared in the absence of taxation. Furthermore, τ_{it} is the marginal tax rate facing individual i at time t, equal to $dT(z_{it})/dz_{it}$ where $T(z)$ is the tax function. The relevant expression is thus:

$$z_{it} = z_{0it}(1 - \tau_{it})^{\eta} \tag{46}$$

where by assumption the elasticity η is the same for all individuals in the relevant population group considered. Taking logarithms:

$$\log z_{it} = \log z_{0it} + \eta \log(1 - \tau_{it}) \tag{47}$$

Given information about z_{it} and τ_{it} for a group of individuals, a simple regression of $\log z_{it}$ on $\log(1 - \tau_{it})$ and a constant, thereby omitting the unobservable z_{0it}, cannot be expected to provide a useful estimate of η, since the omitted variable is correlated with τ_{it}.

Subsection 5.1 considers estimation using information about declared incomes and marginal tax rates before and after a policy change. Some individuals also experience marginal rate changes as a result of 'fiscal drag', when the lack of adjustment to income tax thresholds during periods of inflation means that they move into higher tax brackets. The use of this kind of information to estimate the elasticity of taxable income is discussed in Subsection 5.2.

5.1 Policy Changes

One approach is to consider actual policy changes in the tax structure, and situations in which only a relatively small group of individuals are affected by the tax change, using information before and after the policy change. For example, suppose there is a change in only one marginal rate, say the top marginal income tax rate, which has no effect on those subject to lower rates. Let P_t denote the share of income of the affected group at time t, and their average marginal tax rate at t is τ_{Pt}. Let $t = 0$ and $t = 1$ denote pre- and post-change periods. Assuming that in the absence of the tax change the share of income in the relevant group would have remained constant, an estimate of η, denoted $\hat{\eta}$, can be obtained using:

$$\hat{\eta} = \frac{\log P_1 - \log P_0}{\log (1 - \tau_{P1}) - \log (1 - \tau_{P0})} \tag{48}$$

However, results have been found to be sensitive to the period and tax reform examined. Given a time series of cross-sections over a longer period, an alternative estimate can be obtained by running the regression:

$$\log P_t = \eta \log (1 - \tau_{Pt}) + \varepsilon_t \tag{49}$$

where ε_t is assumed to have the usual properties. However, in this case it is possible that income shares may change over time for non-tax related reasons. This difficulty may be overcome in some cases by including a time trend on the right hand side of (49).

A further approach involves using a difference-in-difference framework with panel data. However, the 'control' and 'treatment' groups cannot be identified in the usual way since in the present context they are defined by income. Suppose that the treatment group, T, comprises the top $P1$ percentile of the income distribution and the control group, C, is made up of individuals in the next $P2$ percentile. Being in the treatment group clearly depends on the behaviour and characteristics of taxpayers. Again, for a tax policy change from period 0 to period 1, the estimate, $\hat{\eta}$, can be obtained, where $E(.)$ denotes the respective sample average, using:

$$\hat{\eta} = \frac{\{E(\log z_{i1}|T) - E(\log z_{i0}|T)\} - \{E(\log z_{i1}|C) - E(\log z_{i0}|C)\}}{\{E(\log(1-\tau_{i1})|T) - E(\log(1-\tau_{i0})|T)\} - \{E(\log(1-\tau_{i1})|C) - E(\log(1-\tau_{i0})|C)\}} \tag{50}$$

This, as with the previous method, involves an assumption that without the policy change the incomes of the two groups would have grown at the same rate. In addition, the elasticity of taxable income is assumed to be the same for both groups. In practice, income changes occur for non-tax reasons, in particular the process of relative income changes from year to year may well display some 'regression towards the mean'. Thus SSG warn against using this approach with only two years of data, but if a long panel is available, they suggest regressing $\log(z_{it+1}/z_{it})$ on $\log\{(1-\tau_{it+1})/(1-\tau_{it})\}$, the coefficient on this being η, along with controls to handle income movements.

5.2 Fiscal Drag and Marginal Rate Changes

The difference-in-difference approach has also been applied in situations where there is no explicit policy change affecting the marginal tax rate faced by some individuals.[16] In particular, fiscal drag – whereby income tax thresholds are not adjusted in line with nominal income growth – gives rise to a general increase in average tax rates.[17] But for those who are initially close to the upper income threshold of a marginal tax bracket, fiscal drag can shift them into the next bracket and thus subject them to a higher marginal rate. Such individuals are regarded as being in the 'treatment' group. Those in the lower section of an income range do not face a higher marginal rate, as they do not cross a threshold, and are regarded as being in the 'control' group. The expression for $\hat{\eta}$ can thus be used in this context. This method can be used for each tax bracket, thereby allowing for variations in η with income (between brackets).[18]

Suppose the incomes of the members of the control group (those who do not move into a higher tax bracket) grow at the rate, θ_C, while those of the treatment group (those who move into the next bracket) grow at the rate, θ_T. The two rates are τ_1 and τ_0, and these apply to all members of the respective tax brackets. Hence the first term in the denominator of (50) is simply $\log\{(1-\tau_1)/(1-\tau_0)\}$ while the second term in the denominator is zero. Declared income is, as before, assumed to be generated by the reduced form expression, $z = z_0(1+\tau)^{\eta}$. Since z_0, the income that would be

[16] Saez (2009) has suggested another method using information about the bunching of individuals at the kinks of piecewise-linear budget constraints, created by increases in marginal rates at income thresholds. However, such kinks need not necessarily produce bunching, even if labour supply responses are significant, and modes in the distibution of income can occur at places other than kinks: see Creedy (2001).

[17] On fiscal drag, see Creedy and Gemmell (2006).

[18] In addition to income tax thresholds, the real value of other tax parameters (such as allowance and deduction limits) can fall during inflation, as examined by Onrubia and Sanz (2009).

declared in the absence of taxation, remains unchanged by assumption, the second term in the numerator of (50) becomes simply $\log(1+\theta_C)$ because there is no change in the tax rate facing members of that group. The first term in the numerator becomes $\eta \log\{(1-\tau_1)/(1-\tau_0)\} + \log(1+\theta_T)$. Thus, substituting in (50) gives:

$$\hat{\eta} = \eta + \frac{\log\{(1+\theta_T)/(1+\theta_C)\}}{\log\{(1-\tau_1)/(1-\tau_0)\}} \tag{51}$$

and so long as $\theta_C = \theta_T$, so that the incomes of all members of the tax bracket grow at the same rate, $\hat{\eta}$ provides an unbiased estimate of η. The bias arises because any difference in growth rates in the different income ranges is attributed to the effects of the the difference in marginal tax rates. Hence where $\theta_T > \theta_C$, the larger growth of the target group (those crossing the tax threshold) leads to a negative bias in estimating the elasticity of taxable income. Conversely, if the incomes in the higher ranges of the tax bracket grow more slowly than in the lower ranges, too much of the change is attributed to the higher marginal tax rate and there is a positive bias. Some illustrative examples are shown in Table 4, showing that the bias is higher, the smaller is the difference between the marginal tax rates and the larger is the proportional difference between growth rates. The bias can be substantial, bearing in mind the sensitivity of excess burdens to the value of η. Of course, in practice there is a dispersion of growth rates among individuals, so any bias arises from a systematic tendency for average growth rates to vary with income, in the absence of any difference in marginal tax rates faced by individuals. Any independent evidence regarding differences in relative income movements over time can therefore be helpful in obtaining some idea of the size of the second term in (51).

6 Conclusions

This paper has provided an introduction to the concept of the elasticity of taxable income, with respect to changes in the net-of-tax marginal tax rate. Emphasis has been placed on the assumptions underlying the approach, particularly with regard to welfare measurement, and the sensitivity of results to changes in some of the underlying

Table 4: Fiscal Drag and Bias in Estimation of Elasticity η

θ_C	$\theta_T - \theta_C$	τ_0	$\tau_1 - \tau_0$	Bias
0.10	0.01	0.35	0.05	-0.113
0.10	0.03	0.35	0.05	-0.336
0.05	0.01	0.35	0.05	-0.118
0.05	0.03	0.35	0.05	-0.352
0.10	0.01	0.35	0.15	-0.034
0.10	0.03	0.35	0.15	-0.103
0.05	0.01	0.35	0.15	-0.036
0.05	0.03	0.35	0.15	-0.107
0.10	0.01	0.30	0.05	-0.122
0.10	0.03	0.30	0.05	-0.363
0.05	0.01	0.30	0.05	-0.128
0.05	0.03	0.30	0.05	-0.380
0.10	0.01	0.30	0.15	-0.038
0.10	0.03	0.30	0.15	-0.112
0.05	0.01	0.30	0.15	-0.039
0.05	0.03	0.30	0.15	-0.117

parameters.

The attraction of a reduced form specification of taxable income in terms of the marginal tax rate, and very few other variables, is clear. It means that, if interest relates only to aggregate responses, and welfare and efficiency effects, there is no need to invest in the production of large scale tax models allowing for endogenous labour supply. Such models require large data sets and deal with the full complexity of tax and transfer systems and population heterogeneity, along with the estimation of preference functions. They need to overcome the complexities of producing welfare change measures. In contrast, the elasticity approach offers the simplicity of dealing with all adjustment margins in one single elasticity and produces a simple measure of marginal welfare cost per dollar.

Of course a very large proportion of empirical regression exercises over the whole range of economics involve reduced form equations, without modelling the structure, or considering aggregation problems explicitly. But most of these are more limited in the use made of results, particularly regarding welfare implications. The discussion here has shown that care needs to be exercised in using the elasticity measure. In particular, the marginal welfare cost of taxation is highly sensitive to the elasticity. Furthermore, in estimating the elasticity, care needs to be taken to avoid attributing some of the changes in declared income to marginal tax changes, when they may have arisen from other dynamic factors. Furthermore, although a reduced form equation is estimated, the changes in taxable income are typically attributed to supply-side factors only. In addition, some observed responses to tax changes may involve the timing of declarations, particularly in anticipation of announced changes taking effect.

While labour supply responses can be regarded as depending on individuals' preferences and their personal circumstances, it is important to recognise that many features involved in the other components of the elasticity of taxable income are substantially affected by the details of tax regulations. Hence policy changes affecting tax rules can have important implications for the elasticity of taxable income, and hence also for optimal tax rates. The availability of a reduced form approach to measurement does not mean that the details of the various income shifting mechanisms can be ignored.

References

[1] Creedy, J. (1998) *Measuring Welfare Changes and Tax Burdens.* Cheltenham: Edward Elgar.

[2] Creedy, J. (2001) Labour supply, welfare and the earnings distribution. *Australian Journal of Labour Economics,* 4, pp. 134-151.

[3] Creedy, J. and Gemmell, N. (2006) *Modelling Tax Revenue Growth.* Cheltenham: Edward Elgar.

[4] Feldstein, M. (1995) The effect of marginal tax rates on taxable income: a panel study of the 1986 Tax Reform Act. *Journal of Political Economy*, 103, ppp. 551-572.

[5] Kalb, G. and Ghantous, S. (2006) Tables with elasticities broken down by individual and household characteristics. Melbourne Institute, University of Melbourne.

[6] Onrubia, J. and Sanz, J.F. (2009) Reported taxable income and marginal tax rates: evidence for Spain based on fiscal drag. *Universidad Complutense de Madrid Working Paper.*

[7] Saez, E. (2009) Do tax filers bunch at kink points? Evidence, elasticity estimates and salience effects. *University of California at Berkeley Working Paper.*

[8] Saez, E., Slemrod, J.B. and Giertz, S.H. (2009) The elasticity of taxable income with respect to marginal tax rates: a critical review. *National Bureau of Economic Research Working Paper*, no. 15012.