

Platform Tying and its Effects^{*}

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Abstract

We study on-platform tying by a dominant platform that controls a primary access point and operates an adjacent specialised service. A rival service can reach users both through the dominant platform and through a direct channel. In a two-sided market where platforms charge sellers, tying improves the integrated service, but also shifts demand away from the rival. This reduces seller participation on the rival platform, weakening the rival also for users who access it directly. As a result, tying can reduce consumer surplus and welfare. We also compare tying with demotion and with a common-access policy, and show that our main insights carry over to a one-sided model in which network effects operate among consumers who pay directly for the specialised service.

JEL codes: D42, K21, L12, L41

Keywords: abuse of dominance, tying, self-preferencing, network effects, digital platforms.

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1 Introduction

In 2017, the European Commission (EC) fined Google EUR 2.42 billion for favouring its own price-comparison service in general search results: Google Shopping was displayed prominently, while rival comparison sites such as Kelkoo, Idealo, and PriceRunner were demoted to lower positions and lost substantial traffic.¹ In 2024, the EC fined Meta EUR 797 million for tying Facebook Marketplace to Facebook, thus giving it an automatic reach advantage over rival classified-ad platforms such as eBay, Vinted, and Subito. Similar concerns have also arisen, among others, in relation to Microsoft for tying Teams with Office 365, Meta for tying its AI assistant within WhatsApp, Apple for favouring iCloud over rival cloud storage services, Amazon for alleged *self-preferencing* on its Buy Box, or Booking.com’s promotion of its own services inside the booking funnel.

These cases differ in legal and technological details, but share a common structure: a firm that controls a primary access point favours its own adjacent service over rival alternatives, via defaults, ranking, prominence, or technical integration. Such tying conduct may create immediate user benefits, for example, by reducing inconvenience costs or improving the user experience. At the same time, it can divert the demand away from rivals and reduce their scale. The central question is whether and when the efficiency gains from integration might be outweighed by any indirect competitive harm.

In many digital markets, rivals retain a direct route to users outside the dominant ecosystem. Comparison shopping services sought access to Google Search, but in principle consumers could also access them directly on their websites. eBay and Vinted compete with the Facebook Marketplace within Meta’s social network, but also serve users through their websites.²

The existence of a direct channel is sometimes taken to weaken concerns about tying. If users can still reach the rival directly, the argument goes, the conduct within the dominant platform should not eliminate the rival’s competitive constraint. This paper shows that this reasoning can be misleading: when network effects are at play, the rival’s direct channel is not independent of the mediated channel. A conduct such as tying, that reduces rival traffic within the dominant platform, might affect the rival’s overall demand, including from its direct channel users.

Let us be more precise about how this mechanism works within our two-sided market (such as in *Google Shopping* or *Facebook Marketplace*) baseline model.³ In our model, a dominant platform controls a primary access point and operates an integrated specialised service, which faces a competing specialised service. A group of users can reach both services through the dominant platform, while other users can reach the rival directly. The specialised services charge sellers, but (for simplicity) not users. Seller participation affects consumer demand through indirect network effects: a larger seller base makes a service more attractive. Tying makes the integrated service easier to reach within the mediated channel, for instance through defaults, prominence or technical integration.

This is a source of *direct* benefit for consumers (we model tying as an increase in the utility they derive from using the integrated platform’s service), but it also shifts demand away from the rival. The loss of mediated demand reduces seller participation on the rival service, which in turn weakens the rival in both channels, imposing a negative externality on consumers on the latter channel. Thus, tying may reduce total consumer and seller surplus and may be socially undesirable even when it improves the user experience on the dominant platform— as we show

¹The Google Shopping decision was definitively upheld by the Court of Justice of the European Union (CJEU), on September 10, 2024.

²Similarly, communication and collaboration apps such as Zoom, Webex, or Slack; AI chatbots such as ChatGPT and Claude; or cloud services such as Dropbox, Proton Drive and many other specialized services are both intermediated by dominant platforms and reachable through their own apps or websites.

³With within-group network effects (more in line with *Microsoft Teams*, *Meta AI* or *Apple iCloud*) similar results hold, as we show in an extension.

in a Hotelling example we use to illustrate the general model and explore welfare effects.

The dominant platform’s incentive to tie depends on its ability to extract the rival’s rents through access fees. For simplicity, in the baseline model we assume away such fees (like in *Google Shopping* or *Facebook Marketplace*), and tying is always profitable. We then show that if fees are endogenous and allow it to extract all of the rival’s profits (perfect rent extraction), the incumbent ties only when joint platform profits rise. For intermediate cases, when access fees are positive but set exogenously to the sector (think of platforms that set the same fees for hundreds of different products and services) tying is profitable as long as fees are not too high. In this case of imperfect rent extraction, tying becomes an alternative means of capturing the rival’s rents, and there can exist substitutability between access fees and tying.

Whereas tying improves the platform’s own service, another form of self-preferencing, *demotion*, degrades the user’s experience with the rival directly, for instance by making it less visible or by adding interface frictions. Both practices can reduce the rival’s seller base and weaken its direct channel. But tying improves the integrated service and raises the utility for its consumers, whereas demotion reduces the convenience of the rival service and *lowers* the utility from using it — and hence lacks this offset. We show in our illustrative example that their welfare consequences are accordingly different: whereas the effects of tying are a priori ambiguous, we are unable to find situations where demotion is welfare-improving (in our simulations).

We also study a common (or non-discriminatory) access policy, in line with the EU Digital Markets Act (DMA) and analogous regulations,⁴ which aim to avoid preferential treatment within the gatekeeping platform environment. This policy differs from tying because it imposes an improvement in access to both the integrated and rival service. In our model, this expands seller participation in both services and strengthens the direct channel of the rival through the same network mechanism that makes tying potentially harmful. While still profitable for the platform, and therefore an incentive-compatible policy, non-discriminatory access eliminates possible anti-competitive effects.

Finally, we show that the main results continue to hold in a one-sided setting with consumer-side network effects (more in line with cases such as *Meta AI*): tying shifts mediated consumers towards the integrated service, reducing the rival’s scale and hence its value to users in the direct channel.

Related Literature Our paper is related to several strands of literature. First, it is related to papers on anti-competitive tying, in particular those motivated by imperfect rent extraction.⁵ Indeed, in our model, tying is always profitable when the exogenously given access price is sufficiently low, due to the platform design or access rules,⁶ a regulatory cap or a previous business decision by the incumbent.⁷

Other papers find an imperfect-rent extraction rationale for tying in two-sided markets.⁸ In Choi and Jeon (2021), perfect extraction would require negative prices, whereas prices are

⁴See e.g., the UK’s Digital Markets, Competition and Consumers Act 2024 (DMCCA).

⁵For instance, in Carlton and Waldman (2012), the good or service at issue does not yet exist and contingent contracts cannot be written. In de Cornière and Taylor (2024), there is downstream competition in the complementary-good or complementary-service market, and the input monopolist sets prices through secret bilateral contracts. See Fumagalli, Motta and Calcagno (2018) for a survey. Our paper also shares with Carlton and Waldman (2002) or Whinston (1990) the idea that tying may lead to (partial or total) foreclosure by depriving the rival of sales or profits. Note that in our model, tying is profitable whether or not it induces full exclusion.

⁶Like in many of the above-mentioned antitrust cases. For instance, in *Google Shopping* any website could be listed in the organic searches; in *Meta AI*, users could have third-party AI agents among their WhatsApp chats; As for *Microsoft Teams*, note that in a Windows environment, one could download any communication app.

⁷For instance, Motta (2023) argues that a platform such as Amazon or Apple might commit to favourable terms and conditions to attract third-party sellers or software developers to its marketplace or app store, but later might want to engage in tying or other conduct to extract more rents from them.

⁸Rochet and Tirole (2003, 2006), Armstrong (2006), Caillaud and Jullien (2003). See Jullien, Pavan and Rysman (2021) for a survey.

constrained to be non-negative.⁹ In Choi et al. (2026), consumers on the dominant platform are heterogeneous and some are left with surplus. Tying allows the dominant firm to leverage this unexploited surplus by converting those consumers into a quasi-installed base in the tied market, which is characterized by network externalities. Whereas in our paper and most of the literature, tying forces consumers to buy tied products or services, in de Cornière, Jerath and Taylor (2025), the platform imposes restrictions on sellers (third-party merchants or developers). To relax price competition, sellers would not symmetrically adopt the same ancillary service offered by the platform. The platform therefore ties its core service to the ancillary service to reduce seller differentiation. This increases its earnings and drives down prices for end consumers.

Importantly, we establish that a dominant incumbent maintains an incentive to engage in tying even when optimising access fees to extract all rival surplus. Under *perfect rent extraction*, tying remains profitable provided it increases joint industry profits.

Our result that in some circumstances (particularly when the rival has a large enough direct channel) access fees and tying might be substitutes, makes our paper also related to recent research that stresses that platforms can resort to different monetisation tools (Franck and Peitz, 2023; Bar-Isaac, 2026; Bar-Isaac and Shelegia, 2026), implying that policy restrictions on one such tool might be neutralised by a platform by using a substitutable tool.

One can also see the mechanism analysed here as a form of *platform envelopment* (see Eisenmann, Parker and Van Alstyne (2011)): by offering a better integrated service, the “platform enveloper” takes business and network effects away from the rival. A growing literature examines data-driven envelopment and ecosystem advantages (Condorelli and Padilla, 2024; Krämer and Shekhar, 2025; Rhodes, Zhou and Zhou, 2026). Our analysis departs from these works in two dimensions. First, existing models treat rivals as standalone single-product firms, whereas our rival relies on the dominant platform to reach a subset of its consumers. This intermediation enables conduct within the mediated channel to spill over into the rival’s direct channel. Second, while prior work generates consumer harm through market foreclosure (Condorelli and Padilla, 2024) or price–quality tradeoffs where price increases outweigh quality gains (Krämer and Shekhar, 2025; Rhodes, Zhou and Zhou, 2026), harm in our setting occurs even though the rival remains active, prices do not rise, and the integrated service strictly improves.

Our paper therefore also belongs to the large literature on hybrid platforms, and in particular *self-preferencing* and on the related practice of steering, in which a hybrid platform favours its own products or services through biased ranking, recommendation, or default design, or through imitation of the rivals’ products. This literature includes both theoretical and empirical contributions.¹⁰ Relative to this literature, we identify a mechanism based on network externalities (both cross-group and within-group) through which such conduct could have anti-competitive effects even when it generates a direct convenience benefit for consumers. A key difference is that we explicitly model a direct channel through which the rival reaches consumers outside the dominant platform.

This channel changes the welfare effects: through network effects, harm in the mediated channel spills over to consumers in the direct channel rather than remaining confined to the dominant platform.¹¹

There also exist a literature that models endogenous consumer arbitrage across sales channels

⁹See also Amelio and Jullien (2012) where tying is an indirect way to subsidise adoption when prices are non-negative.

¹⁰For theoretical contributions, see, for example, de Cornière and Taylor (2019); Padilla, Perkins and Piccolo (2022); Zenryo (2022); Hagiu, Teh and Wright (2022); Motta (2023); Anderson and Bedre Defolie (2024); Hervás-Drane and Shelegia (2025). For empirical contributions, see, for example, Farronato, Fradkin and MacKay (2023); Chen and Tsai (2024); Waldfogel (2024).

¹¹In a previous version of the paper, we showed that a similar effect arises when overall scale affects investment incentives. By reducing the rival’s expected scale and profits, tying weakens its incentive to invest, lowering its equilibrium quality in both the mediated and direct channels.

(e.g., Edelman and Wright, 2015; Wang and Wright, 2020; Hagiü and Wright, 2024) and is complementary to our framework. While these papers analyze how consumers move between platforms and direct channels in response to price parity, search costs, or platform leakage, we assume that two different groups of consumers are assigned to the mediated or the direct channel exogenously. While consumers theoretically arbitrage across sales channels, it is often the case that consumers face channel inertia due to search costs and platform lock-in. But apart from possible empirical justifications, this assumption gives us a simpler framework of analysis and it allows us to isolate the pure cross-channel network spillover from possible demand-reallocation effects, showing that conduct in the intermediate channel can harm consumers purely through network externalities. Moreover, it provides a conservative lower bound on harm: if consumers could freely arbitrage, the degradation of the rival’s direct channel quality would induce further migration toward the dominant platform, thereby compounding consumer harm.

The remainder of the paper is organised as follows. Section 2 presents the main model of tying in the presence of cross-group network externalities between sellers and consumers. Section 3 studies the effects of tying. Section 4 considers the case of demotion. Section 5 examines a common-access policy. Section 6 extends the analysis to a one-sided environment with within-consumer network effects. Section 7 concludes.

2 Baseline model: setup

Our environment is described in Figure 1. It is a two-sided market in which the *dominant platform* (platform 1) owns A_1 , the primary access point, and B_1 , its integrated specialised service. The *rival platform* (platform 2) operates B_2 , which competes with B_1 for consumers and sellers. Throughout, A_1 , B_1 and B_2 denote the services themselves, and platform i denotes their owner; since only B_1 and B_2 set fees, “platform i chooses r_i ” always refers to the fee set for service B_i . A group of consumers can reach both specialised services through the mediated channel controlled by A_1 . Another group of consumers can reach B_2 through a direct channel outside the dominant platform. As discussed in the literature review in the previous Section, this captures environments where consumers face costs of switching across channels, and allows us to isolate the effects of tying in a simple way.

Each specialised platform B_i charges a listing fee r_i , for $i = 1, 2$, to sellers, but does not directly charge consumers.¹² Seller participation affects consumer demand through indirect network effects: a larger seller base makes a platform more attractive to consumers. The rival’s direct channel also depends on its seller base.

In the baseline analysis, we assume for simplicity that B_2 does not pay an access fee to platform A_1 (in line with several of the antitrust cases mentioned in the Introduction). We relax this assumption in Sections 3.6 and 3.7.

2.1 Primitives

The functions f_{1M} and f_{2M} below describe consumer demand for B_1 and B_2 through the mediated channel. The function f_{2D} describes demand for B_2 through its direct channel.

Let s_1 and s_2 denote the masses of sellers active on B_1 and B_2 , respectively. The parameter b_i measures the convenience benefit associated with service B_i within the mediated channel. Equivalently, one can think of b_i as the inverse of the cost consumers incur to access service B_i while on the core platform A_1 . There is no such cost for consumers who access B_2 on the direct channel. Our analysis focuses on marginal changes in the platform’s convenience parameters. We interpret an increase in b_1 as the reduced-form effect of tying, which raises the convenience

¹²This simplification is not unrealistic, and fits several of the abovementioned antitrust cases. In some marketplace applications, though, consumers might also have to pay some commissions to the platform.

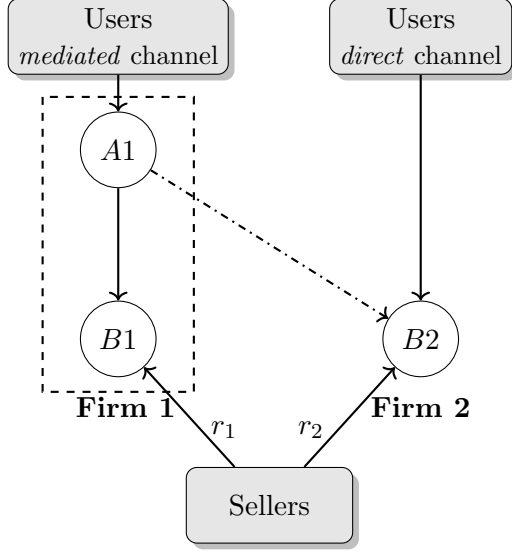


Figure 1: Market configuration

of accessing B_1 through the mediated channel. We interpret a decrease in b_2 as demotion of B_2 in the mediated channel.

Consumers. Consumer demand is given by

$$\begin{aligned} q_{1M} &= f_{1M}(b_1, b_2, s_1, s_2), \\ q_{2M} &= f_{2M}(b_1, b_2, s_1, s_2), \\ q_{2D} &= \alpha f_{2D}(s_2), \end{aligned} \tag{1}$$

where $\alpha > 0$ measures the relative size of the direct channel. We impose the following restrictions on demand primitives.

Assumption 1 (Demand primitives). *The demand functions f_{1M} , f_{2M} , and f_{2D} are continuously differentiable.*

(i) *For $i, j \in \{1, 2\}$, $i \neq j$, and for all relevant arguments,*

$$\frac{\partial f_{iM}}{\partial b_i} > 0, \quad \frac{\partial f_{iM}}{\partial b_j} < 0, \quad \frac{\partial f_{iM}}{\partial s_i} > 0, \quad \frac{\partial f_{iM}}{\partial s_j} < 0, \quad \frac{\partial f_{2D}}{\partial s_2} > 0.$$

(ii) *Own effects weakly dominate cross effects in the mediated channel. For $i, j \in \{1, 2\}$, $i \neq j$,*

$$\frac{\partial f_{iM}}{\partial b_i} \geq \left| \frac{\partial f_{jM}}{\partial b_i} \right|, \quad \frac{\partial f_{iM}}{\partial s_i} \geq \left| \frac{\partial f_{jM}}{\partial s_i} \right|.$$

Assumption 1 imposes standard monotonicity restrictions on demand. Part (i) requires that a higher own convenience benefit increases own demand in the mediated channel and reduces the rival's demand in that channel. It also captures indirect network effects from seller participation to consumer demand: an increase in seller participation on one platform raises demand for that platform and lowers demand for the competing platform in the mediated channel. Part (ii) requires own effects to weakly *dominate* cross effects.

Sellers. Sellers are heterogeneous in their fixed cost of participation, κ . The cost κ is distributed according to the cdf G , with density $g > 0$ in $[0, \bar{\kappa}]$. A seller active on platform i obtains payoff

$$\pi_i^S(\kappa) = mq_i - r_i - \kappa, \quad i = 1, 2,$$

where $q_1 = q_{1M}$ and $q_2 = q_{2M} + q_{2D}$, the parameter $m > 0$ denotes the gross seller margin per consumer and r_i is a lump-sum listing fee charged to active sellers on platform i . A seller participates on platform i if and only if $\pi_i^S(\kappa) \geq 0$. Hence aggregate seller participation satisfies

$$s_i = G(mq_i - r_i), \quad i = 1, 2.$$

Note that a seller's participation decision on one platform does not depend on whether it is also active on the other: a seller of type κ joins B_i whenever $m q_i - r_i \geq \kappa$, for $i = 1, 2$, so sellers multi-home and S_1^* and S_2^* are determined by two separate participation conditions.¹³

Given seller fees (r_1, r_2) , there is a fixed point (S_1, S_2) satisfying

$$\begin{aligned} S_1 &= G(m f_{1M}(b_1, b_2, S_1, S_2) - r_1), \\ S_2 &= G(m [f_{2M}(b_1, b_2, S_1, S_2) + \alpha f_{2D}(S_2)] - r_2). \end{aligned}$$

We assume that this system admits a locally unique solution. Equilibrium quantities are

$$\begin{aligned} Q_{1M}(b_1, b_2, r_1, r_2, \alpha) &= f_{1M}(b_1, b_2, S_1, S_2), \\ Q_{2M}(b_1, b_2, r_1, r_2, \alpha) &= f_{2M}(b_1, b_2, S_1, S_2), \\ Q_{2D}(b_1, b_2, r_1, r_2, \alpha) &= \alpha f_{2D}(S_2). \end{aligned}$$

For expositional convenience, let

$$g_1 \equiv g(m Q_{1M} - r_1), \quad g_2 \equiv g(m(Q_{2M} + Q_{2D}) - r_2).$$

We impose the following regularity conditions.

Assumption 2 (Regularity of seller-side feedbacks).

(i) For all relevant arguments,

$$1 - m g_1 \frac{\partial f_{1M}}{\partial s_1} > 0, \quad 1 - m g_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha m g_2 \frac{\partial f_{2D}}{\partial s_2} > 0,$$

$$(ii) \Delta_S \equiv \left(1 - m g_1 \frac{\partial f_{1M}}{\partial s_1}\right) \left(1 - m g_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha m g_2 \frac{\partial f_{2D}}{\partial s_2}\right) - m^2 g_1 g_2 \frac{\partial f_{1M}}{\partial s_2} \frac{\partial f_{2M}}{\partial s_1} > 0.$$

Assumption 2.(i) requires the feedback effect on the seller side not to be too large. Assumption 2.(ii) is a standard determinant condition, $\Delta_S > 0$, which guarantees well-defined comparative statics.

2.2 Platform payoffs and timing

Each platform i earns revenue from the seller fees paid by sellers active on B_i . The platform profits are $\Pi_i = r_i S_i$. (Sections 3.6 and 3.7 consider the case in which Platform 1 can also extract rents from transactions generated through the mediated channel by its rival.)

Throughout the analysis, we impose the following conditions.

¹³This is the empirically relevant case in the markets we have in mind: merchants list on Google Shopping and on comparison sites, and on Facebook Marketplace and on eBay or Vinted, at the same time. It also means that the two platforms do not compete for an exclusive seller base, so all the competitive effects we identify operate through consumer demand rather than through a contested seller side.

Assumption 3 (Regularity of the fee game). *For all relevant arguments,*

- (i) $\frac{\partial^2 \Pi_1}{\partial r_1^2} < 0, \frac{\partial^2 \Pi_2}{\partial r_2^2} < 0,$
- (ii) $\Delta_r \equiv \frac{\partial^2 \Pi_1}{\partial r_1^2} \frac{\partial^2 \Pi_2}{\partial r_2^2} - \frac{\partial^2 \Pi_1}{\partial r_1 \partial r_2} \frac{\partial^2 \Pi_2}{\partial r_2 \partial r_1} > 0.$

Assumption 4 (Super- and sub-modularity of fees in convenience benefit). *At the equilibrium, for $J, K \in \{1, 2\}$ with $J \neq K$:*

- (i) (own convenience) $\frac{\partial^2 \Pi_J}{\partial r_J \partial b_J} \geq 0;$
- (ii) (rival convenience) $\frac{\partial^2 \Pi_J}{\partial r_J \partial b_K} \leq 0.$

Assumption 3 is a standard assumption on profit concavity and ensures stability. Assumption 4 states that a platform's marginal return to raising its seller fee increases with the convenience of its own specialised service and decreases with that of its rival. Greater accessibility of B_J makes its mediated demand more valuable to sellers, relaxing platform J 's fee-setting trade-off; greater accessibility of the rival instead diverts mediated demand from B_J and may further reduce its direct demand through lower seller participation.

In Section 3, tying (an increase in b_1) invokes part (i) for platform 1 and part (ii) for platform 2. In Section 4, demotion (a decrease in b_2) invokes part (i) for platform 2 and part (ii) for platform 1.

The game. The timing is as follows. In the first stage, platforms simultaneously choose seller fees r_1 and r_2 . In the second stage, sellers decide whether to participate on each platform, and consumer demands are determined according to equation (1). Sellers and consumers have fulfilled expectations about seller participation and consumer demand on both sides of the market. The equilibrium concept is subgame perfect Nash equilibrium with fulfilled expectations.

3 Tying

We now characterise the comparative effects of tying—an increase in b_1 , due, for example, to better integration or lower inconvenience from accessing B_1 through the mediated channel—on equilibrium seller fees, seller participation, consumer demand across channels, and platform profits. We shall also discuss the role played by the direct channel, and illustrate the analysis with a specific demand example that will allow us to study welfare effects. Finally, we shall also explore the effects of the case where the dominant platform $A1$ charges a positive access fee to rival firm $B2$.

3.1 The effect of tying on seller fees

We first study how an increase in b_1 affects equilibrium seller fees. Throughout this section, all derivatives of Q_{1M} , Q_{2M} , and Q_{2D} are evaluated at the equilibrium seller-participation levels induced by $(r_1, r_2, b_1, b_2, \alpha)$.

The first-order conditions for an interior equilibrium in seller fees are

$$\frac{\partial \Pi_1}{\partial r_1} = S_1 + r_1 g_1 \left(m \frac{\partial Q_{1M}}{\partial r_1} - 1 \right) = 0, \quad (2)$$

$$\frac{\partial \Pi_2}{\partial r_2} = S_2 + r_2 g_2 \left[m \left(\frac{\partial Q_{2M}}{\partial r_2} + \frac{\partial Q_{2D}}{\partial r_2} \right) - 1 \right] = 0. \quad (3)$$

The terms S_i capture infra-marginal seller-fee revenue, whereas the remaining terms capture the loss of seller participation and the resulting change in the amount of transactions in the mediated channel.

Using the implicit function theorem, we sign the effect of b_1 on equilibrium fees (under Assumption 4):

$$\frac{dr_1^*}{db_1} = \frac{-\frac{\partial^2 \Pi_1}{\partial r_1 \partial b_1} \frac{\partial^2 \Pi_2}{\partial r_2^2} + \frac{\partial^2 \Pi_2}{\partial r_2 \partial b_1} \frac{\partial^2 \Pi_1}{\partial r_1 \partial r_2}}{\Delta_r}, \quad \frac{dr_2^*}{db_1} = \frac{-\frac{\partial^2 \Pi_2}{\partial r_2 \partial b_1} \frac{\partial^2 \Pi_1}{\partial r_1^2} + \frac{\partial^2 \Pi_1}{\partial r_1 \partial b_1} \frac{\partial^2 \Pi_2}{\partial r_2 \partial r_1}}{\Delta_r}. \quad (4)$$

where $\Delta_r > 0$ by Assumption 3. Thus, the sign of the expressions (4) is the same as the sign of the numerator. Here, tying affects equilibrium seller fees through two channels. The first is a *direct marginal profitability effect*: tying changes each platform's own incentive to raise its seller fee. The second is a *strategic feedback effect*: once one platform's fee incentive changes, the rival adjusts its own fee, which feeds back into the platform's optimal fee.

Under Assumption 4, tying raises platform 1's marginal incentive to increase its seller fee and lowers platform 2's. The reason is that a higher b_1 increases the value to sellers of participating on B_1 , making platform 1's seller base less sensitive to a fee increase. Conversely, tying reduces the attractiveness of B_2 by lowering its demand in the mediated channel and, through seller participation, weakening its demand in the direct channel too. Platform 2 therefore has a stronger incentive to lower its fee to retain sellers.

The equilibrium response also depends on the strategic interaction between seller fees. The following lemma summarizes the result.

Lemma 1 (Effect of tying on seller fees). *Under our maintained assumptions:*

- (i) *if r_1 and r_2 are strategic substitutes, then tying raises platform 1's equilibrium seller fee and lowers platform 2's equilibrium seller fee:*

$$\frac{dr_1^*}{db_1} > 0, \quad \frac{dr_2^*}{db_1} < 0;$$

- (ii) *if r_1 and r_2 are strategic complements, the same conclusion holds under the diagonal-dominance condition reported in the Appendix.*

Proof. See Appendix. □

The intuition is as follows. Tying strengthens platform 1's position and weakens platform 2's. Platform 1 therefore has an incentive to charge sellers more, while platform 2 has an incentive to cut its fee in order to preserve seller participation. Under strategic substitutability, the strategic-feedback terms in (4) reinforce the direct effect of tying, which pushes platform 1 toward a higher seller fee and platform 2 toward a lower seller fee. Under strategic complementarity, by contrast, the strategic-feedback terms work against the direct effect. The diagonal dominance condition (reported in the Appendix) ensures that the direct marginal effects dominate these countervailing feedbacks, and that the direct effect of tying determines the sign of the equilibrium fee response: r_1^* rises and r_2^* falls.

3.2 The effect of tying on seller participation

Differentiating the seller-entry conditions yields:

$$\frac{dS_1^*}{db_1} = g_1 \left(m \frac{dQ_{1M}^*}{db_1} - \frac{dr_1^*}{db_1} \right), \quad \frac{dS_2^*}{db_1} = g_2 \left(m \frac{dQ_{2M}^*}{db_1} + m \frac{dQ_{2D}^*}{db_1} - \frac{dr_2^*}{db_1} \right). \quad (5)$$

Two forces affect the response of seller participation. The first is an *indirect network effect*: tying changes consumer demand, which in turn affects seller participation. The second is a

strategic pricing effect: under the conditions in Lemma 1, tying raises platform 1's seller fee and lowers platform 2's seller fee.

On the consumer side, the response of demand reflects three forces: the direct effect of tying on mediated demand, and two cross-side network effects operating through seller participation on each platform:

$$\frac{dQ_{1M}^*}{db_1} = \underbrace{\frac{\partial f_{1M}}{\partial b_1}}_{\text{direct effect}} + \underbrace{\frac{\partial f_{1M}}{\partial s_1} \frac{dS_1^*}{db_1} + \frac{\partial f_{1M}}{\partial s_2} \frac{dS_2^*}{db_1}}_{\text{cross-side network effect}}, \quad (6)$$

$$\frac{dQ_{2M}^*}{db_1} = \underbrace{\frac{\partial f_{2M}}{\partial b_1}}_{\text{direct effect}} + \underbrace{\frac{\partial f_{2M}}{\partial s_1} \frac{dS_1^*}{db_1} + \frac{\partial f_{2M}}{\partial s_2} \frac{dS_2^*}{db_1}}_{\text{cross-side network effect}}, \quad (7)$$

$$\frac{dQ_{2D}^*}{db_1} = \underbrace{\alpha \frac{\partial f_{2D}}{\partial s_2} \frac{dS_2^*}{db_1}}_{\text{cross-side network effect}}. \quad (8)$$

Tying shifts demand toward platform 1 and away from platform 2, which tends to increase S_1^* and reduce S_2^* . The induced fee responses work in the opposite direction: under Lemma 1, platform 1 raises its seller fee, while platform 2 lowers it. To determine the net effect on seller participation, we solve the system simultaneously. Applying Cramer's rule yields:

$$\frac{dS_1^*}{db_1} = \frac{1}{\Delta_S} \left\{ g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \left(1 - mg_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha mg_2 \frac{\partial f_{2D}}{\partial s_2} \right) + mg_1 \frac{\partial f_{1M}}{\partial s_2} g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) \right\}, \quad (9)$$

$$\frac{dS_2^*}{db_1} = \frac{1}{\Delta_S} \left\{ \left(1 - mg_1 \frac{\partial f_{1M}}{\partial s_1} \right) g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) + mg_2 \frac{\partial f_{2M}}{\partial s_1} g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \right\}, \quad (10)$$

where $\Delta_S > 0$ by Assumption 2(ii).¹⁴ For the rest of the analysis, we introduce the following definition.

Definition 1 (Dominance of direct effects). *We say that direct demand effects of tying dominate strategic fee effects if*

$$m \frac{\partial f_{1M}}{\partial b_1} > \frac{dr_1^*}{db_1} > 0 \quad \text{and} \quad m \left| \frac{\partial f_{2M}}{\partial b_1} \right| > \left| \frac{dr_2^*}{db_1} \right|. \quad (11)$$

The intuition behind the condition in equation (11) is that an increase in b_1 makes platform 1 more attractive to consumers and platform 2 less attractive in the mediated channel. These changes affect sellers because seller participation depends on the consumer demand that sellers expect to reach. Holding fees fixed, tying raises the value of joining platform 1 and lowers the value of joining platform 2.

Equilibrium fee responses partly offset these forces. Platform 1 raises its seller fee, which discourages participation on B_1 . Platform 2 lowers its seller fee, which helps preserve participation on B_2 . The dominance condition requires the demand-side effect of tying to be stronger than this offsetting fee response. Were inequalities in (11) be reversed—so that on each platform the strategic fee response outweighed the direct demand effect—tying would reduce seller participation on platform 1 and raise it on platform 2: the practice would strengthen rather than weaken the rival, and it need not be profitable for the integrated platform. Our analysis focuses on the economically relevant case, in which tying expands platform 1's seller base and

¹⁴Stability conditions in Assumption 2(ii) ensure that the relevant terms in determining the sign of equations (9) and (10) are those in Definition 1.

its mediated demand. The condition is satisfied in a broad class of demand systems, including the Hotelling specification of Section 3.5.

Once seller participation moves in this direction, the effects on consumer demand are strengthened by cross-side network effects: more sellers on platform 1 further increase demand for B_1 , whereas fewer sellers on platform 2 further reduce demand for B_2 . This is true not only in the mediated channel. Because the direct channel demand for B_2 also depends on the mass of sellers on B_2 , the reduction in S_2^* lowers direct demand Q_{2D}^* as well. Thus a practice implemented inside the mediated channel spills over to the rival's direct channel through seller participation.

Under the condition in (11), $m \partial f_{1M} / \partial b_1 > dr_1^* / db_1$ and $m \partial f_{2M} / \partial b_1 < dr_2^* / db_1$. Then the signs in (9)–(10) are straightforward and, therefore, seller participation on platform 1, S_1 , increases and on platform 2, S_2 , decreases. Because in this case tying increases S_1^* and decreases S_2^* , demand for B_1 rises in the mediated channel too, whereas demand for B_2 falls in both the mediated and direct channels. The next proposition summarizes these results.

Proposition 1 (Effect of tying on seller participation and quantities). *Under Assumptions 1, 2, and condition (11), tying unambiguously leads to:*

$$(i) \quad dS_1^* / db_1 > 0, \quad dS_2^* / db_1 < 0.$$

$$(ii) \quad dQ_{1M}^* / db_1 > 0, \quad dQ_{2M}^* / db_1 < 0, \quad dQ_{2D}^* / db_1 < 0.$$

Proof. See Appendix. □

The effect on mediated and total quantities We have identified conditions under which tying increases mediated demand and seller participation for B_1 , while reducing demand for B_2 across both the mediated and direct channels. How tying affects total quantity, which may be viewed as a reduced-form proxy for consumer surplus, cannot be signed in general. Section 3.5 presents an illustrative example that identifies the key forces determining the net effects on consumer surplus and total surplus.

Even the effect on total quantity in the mediated channel is ambiguous. One might expect the dominance of own effects over cross effects in Assumption 1 to imply that the increase in Q_{1M}^* exceeds the reduction in Q_{2M}^* . That reasoning is incomplete, because it ignores equilibrium feedback through seller participation. If tying induces a sufficiently large reduction in S_2^* , the fall in Q_{2M}^* may dominate the increase in Q_{1M}^* , so quantity in the mediated channel may decline.

Let total quantity in the mediated channel be $Q_M \equiv Q_{1M} + Q_{2M}$. Then, summing (6)–(7), the effect of tying on the quantity in the mediated channel is given by

$$\frac{dQ_M^*}{db_1} = \underbrace{\left(\frac{\partial f_{1M}}{\partial b_1} + \frac{\partial f_{2M}}{\partial b_1} \right)}_{\text{direct effect (+)}} + \underbrace{\left(\frac{\partial f_{1M}}{\partial s_1} + \frac{\partial f_{2M}}{\partial s_1} \right) \frac{dS_1^*}{db_1}}_{\text{network-related effect on firm 1 (+)}} + \underbrace{\left(\frac{\partial f_{1M}}{\partial s_2} + \frac{\partial f_{2M}}{\partial s_2} \right) \frac{dS_2^*}{db_1}}_{\text{network-related effect on firm 2 (-)}}. \quad (12)$$

Under Assumption 1, the first term in brackets, which is the *direct net market expansion* of tying in the mediated channel, holding seller participation fixed, is positive; the second and third term in brackets, which are respectively the net effects on total mediated demand of an additional seller on B_1 and on B_2 , are respectively positive and negative.¹⁵

Under the conditions of Proposition 1, $dS_1^* / db_1 > 0$ and $dS_2^* / db_1 < 0$. Hence in (12) the first two terms are non-negative and the third is non-positive. Hence, the sign of dQ_M^* / db_1 cannot be determined unambiguously.

¹⁵Note that the equilibrium fee responses dr_1^* / db_1 and dr_2^* / db_1 do not enter (12) as separate terms: because f_{1M} and f_{2M} depend on fees only through the seller masses, every fee effect is already accounted for in the equilibrium participation responses dS_j^* / db_1 .

Tying raises total mediated quantity *unless* the rival's seller-base erosion is large enough to outweigh both the direct expansion and the gain on the integrated platform—which is when seller-side network effects are strong and the rival's loss of scale is most consequential.

Since we are interested in the effects of tying on consumer, seller, and total surplus, in Section 3.5 we study a specific functional form example to better understand the factors at play and how they interact.

3.3 The effect of tying on profits

We now consider the profit effects of tying. Since $\Pi_i^* = r_i^* S_i^*$, the equilibrium total derivatives with respect to b_1 are (using Lemma 1 and Proposition 1)

$$\begin{aligned}\frac{d\Pi_1^*}{db_1} &= S_1^* \frac{dr_1^*}{db_1} + r_1^* \frac{dS_1^*}{db_1} > 0, \\ \frac{d\Pi_2^*}{db_1} &= S_2^* \frac{dr_2^*}{db_1} + r_2^* \frac{dS_2^*}{db_1} < 0.\end{aligned}$$

Indeed, by Lemma 1 and Proposition 1, under the dominance condition in (11) we have $dr_1^*/db_1 > 0$, $dr_2^*/db_1 < 0$, $dS_1^*/db_1 > 0$, and $dS_2^*/db_1 < 0$: both the fee-level and the participation term are positive for platform 1, and both are negative for platform 2. The following summarises this discussion.

Proposition 2 (Profit effects of tying in the two-sided market). *Suppose the maintained assumptions and the dominance condition (11) hold. Then, tying is always profitable and harms the rival, i.e., $d\Pi_1^*/db_1 > 0$ and $d\Pi_2^*/db_1 < 0$.*

Proof. Follows from the discussion. □

3.4 The role of the direct channel

The direct channel, whose size is measured by α , plays a critical role. At $\alpha = 0$ there is no direct-channel harm: $Q_{2D}^* = 0$ throughout, so, trivially, $dQ_{2D}^*/db_1 = 0$. Tying still harms B_2 consumers through the mediated channel (that is, $dQ_{2M}^*/db_1 < 0$) and reduces seller participation on platform 2 ($dS_2^*/db_1 < 0$), but no consumers outside the dominant platform are affected.

For $\alpha > 0$, the response of direct-channel demand to tying is

$$\frac{dQ_{2D}^*}{db_1} = \alpha \frac{\partial f_{2D}}{\partial s_2} \times \frac{dS_2^*}{db_1},$$

where $\partial f_{2D}/\partial s_2 > 0$ by Assumption 1(i) and $dS_2^*/db_1 < 0$ by Proposition 1. The sign of dQ_{2D}^*/db_1 is therefore unambiguously negative.

The *magnitude* of the harm is the product of two terms. First, a *scale effect*: a larger α increases the mass of consumers in the direct channel exposed to any given fall in S_2^* . Holding $|dS_2^*/db_1|$ fixed, a marginal increase in α increases the harm by $(\partial f_{2D}/\partial s_2)|dS_2^*/db_1| > 0$. Second, a *participation-response effect*: $|dS_2^*/db_1|$ itself varies with α . When seller participation is interior, a larger direct channel raises the rival's total scale and thereby reinforces the feedback between seller participation and consumer demand on platform 2, so that a given loss of mediated demand translates into a larger contraction of S_2^* . (In the Hotelling specification of Section 3.5, this amplification dominates, and both terms increase in α .)

Since the cost κ has bounded support, seller participation eventually saturates: once the marginal seller's payoff $\kappa_2^* = m(Q_{2M}^* + Q_{2D}^*) - r_2^*$ reaches $\bar{\kappa}$, all sellers join platform 2, $g_2 = 0$, and $dS_2^*/db_1 = 0$. At that point, tying is absorbed entirely by platform 2's fee rather than by its seller base, insulating the direct channel from conduct in the mediated channel.

Lemma 2 (Role of the direct channel in tying harm). *Under our maintained assumptions and the dominance condition (11):*

- (i) *for every $\alpha > 0$ and $\kappa < \bar{\kappa}$, $dQ_{2D}^*/db_1 < 0$.*
- (ii) *if $m(Q_{2M}^* + Q_{2D}^*) - r_2^* \geq \bar{\kappa}$, so that all sellers join platform 2, then $dS_2^*/db_1 = 0$ and $dQ_{2D}^*/db_1 = 0$;*
- (iii) *the effect of tying on the direct channel is non-monotone in α .*

Proof. See Appendix. □

Lemma 2 shows that the effect of tying is non-monotonic in the size of the direct channel. Trivially, if there is no direct channel, only the mediated channel matters, and since own effects dominate cross effects, tying has a positive net effect on (mediated channel) users. As the size of the direct channel starts increasing from zero, tying has a negative effect on direct channel users, but eventually the direct channel will be so large that conduct in the mediated channel has no impact on platform $B2$. The intuition is that the direct channel both exposes consumers to harm and supports the rival's seller base. When not all sellers are on $B2$, a larger α exposes more consumers to a reduction in S_2^* and makes seller participation more responsive to the resulting loss of demand, increasing harm in the direct channel. But once the direct channel is large enough that all sellers join the rival regardless of mediated demand, this participation response disappears, insulating consumers in the direct channel from tying.

3.5 An illustrative example

We now provide a specific functional form example with two specialised platforms competing à la Hotelling on market B . We consider an uncovered market for the general model to encompass this illustration.

Consumers are uniformly distributed with unit density along the real line $\theta \in (-\infty, \infty)$, with platform B_1 located at $l_1 = 0$ and platform B_2 at $l_2 = 1$. Consumers may have access to market B either directly via platform B_2 , or through a mediated channel A_1 , in which case they can join either specialised platform.

On the direct channel (with size α), a consumer located at θ who accesses platform B_2 obtains utility

$$U_{2D}(\theta) = 1 + \omega s_2 - t|1 - \theta|, \quad (13)$$

where $t > 0$ is the transportation cost and θ captures consumers' heterogeneous preferences,¹⁶ and where s_2 is the mass of sellers active on platform B_2 with ωs_2 capturing the cross-network benefit generated by seller participation, $\omega > 0$. On the mediated channel, users of the primary platform A_1 can join either specialised platform. Their utility from joining platform B_J , $J = 1, 2$, is

$$U_{JM}(\theta) = 1 + b_J + \omega s_J - t|l_J - \theta|.$$

Figure 2 provides a visual representation of the consumer allocation in the mediated channel for two generic utility functions (the figure is drawn for a hypothetical case where $b_2 + \omega s_2 > b_1 + \omega s_1$). The cutoffs θ_1 and θ_2 are the marginal consumers who are indifferent between joining, respectively, B_1 or B_2 and taking the outside option, while θ_{12} is the consumer indifferent between the two specialised platforms.¹⁷ Hence, demands in the mediated channel are given by $q_{1M}(\cdot) = \theta_1 + \theta_{12}$, and $q_{2M}(\cdot) = \theta_2 - \theta_{12}$, that is

$$q_{JM}(b_J, b_{-J}, s_J, s_{-J}) = \frac{t + 2 + 3b_J - b_{-J} + 3\omega s_J - \omega s_{-J}}{2t}, \quad J = 1, 2.$$

¹⁶We assume that θ is i.i.d. and uncorrelated with whether a consumer is on the mediated or the direct channel.

¹⁷Formally, $\theta_1 = (1 + b_1 + \omega s_1)/t$, $\theta_2 = 1 + (1 + b_2 + \omega s_2)/t$, and $\theta_{12} = (t + b_1 - b_2 + \omega s_1 - \omega s_2)/2t$.

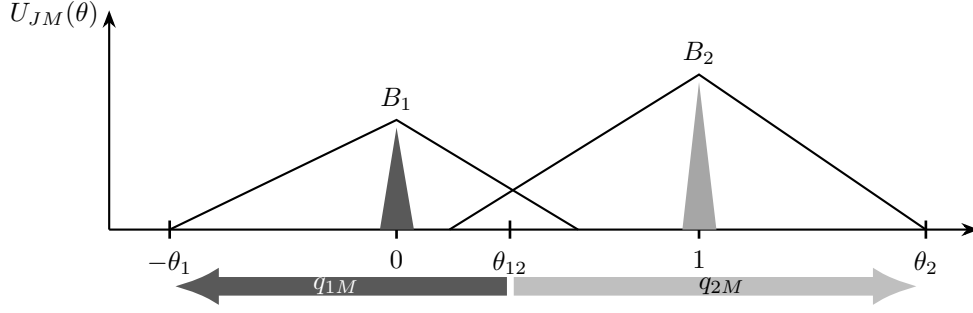


Figure 2: Mediated channel

The marginal consumers in the direct channel are symmetrically located around B_2 's position at 1 and they are defined by $U_{2D}(\theta) = 0$. Indicating these consumers with θ_D^- and θ_D^+ ,¹⁸ the demand in the direct channel is given by:

$$q_{2D}(s_2) = \alpha (\theta_D^+ - \theta_D^-) = \frac{2\alpha(1 + \omega s_2)}{t}.$$

One can verify that these functions satisfy all our maintained assumptions.

Sellers have mass M . We assume that κ is uniformly distributed in $[0, K]$. A seller with fixed cost κ obtains payoff $\pi_i^S(\kappa) = mq_i - r_i - \kappa$; hence seller participation on platform J is

$$s_J = M \frac{mq_J - r_J}{K}.$$

We can then find the equilibrium pair (S_1, S_2) . Using these expressions, we derive the demand functions of the two platforms on both the mediated and on the direct channel, Q_{JM} and Q_{2D} . (Formal details are in Appendix B.) We define consumer surplus as follows.

$$CS = \int_{-\theta_1}^{\theta_{12}} U_{1M}(\theta; S_1^*) d\theta + \int_{\theta_{12}}^{\theta_2} U_{2M}(\theta; S_2^*) d\theta + \alpha \int_{\theta_D^-}^{\theta_D^+} U_{2D}(\theta; S_2^*) d\theta. \quad (14)$$

The first two terms capture surplus in the mediated channel, where consumers choose between the two specialised platforms. The last term captures surplus in the direct channel and is weighted by the relative size α of that channel.

Seller surplus is

$$SS = \frac{M}{K} \int_0^{\kappa_1^*} (\kappa_1^* - \kappa) d\kappa + \frac{M}{K} \int_0^{\kappa_2^*} (\kappa_2^* - \kappa) d\kappa, \quad (15)$$

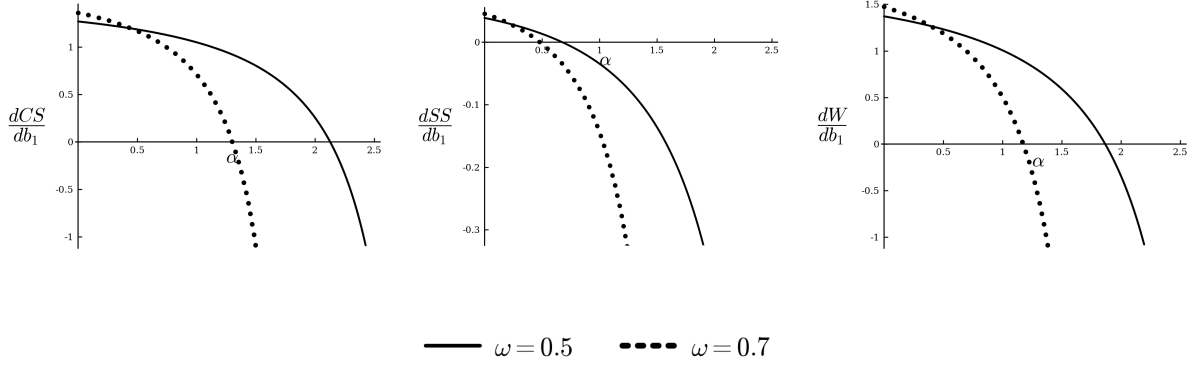
where

$$\kappa_1^* = mQ_{1M}^* - r_1^*, \quad \kappa_2^* = m(Q_{2M}^* + Q_{2D}^*) - r_2^*$$

Finally, total welfare is the sum of consumer surplus, seller surplus, and platform profits: $W = CS + SS + \Pi_1^* + \Pi_2^*$.

Figure 3 reports the effect of tying on consumer surplus, seller surplus, and total welfare for $b_1 = b_2 = 0.2$, $m = 0.4$, $t = 2$, $M = 5$, $K = 4$, and for two values of the intensity of cross-side network effects, $\omega = \{0.5, 0.7\}$. Panel (a) shows that tying increases consumer surplus only when the direct channel is sufficiently small. For low values of α , the surplus gain generated by the expansion of Q_{1M} exceeds the losses associated with the contractions of Q_{2M} and Q_{2D} . As α rises, however, the direct channel represents a larger share of platform 2's activity. The loss suffered by users who access B_2 directly therefore receives greater weight in aggregate consumer surplus. Consequently, dCS/db_1 decreases with α and eventually becomes negative.

¹⁸Formally, $\theta_D^- = 1 - (1 + \omega s_2)/t$, $\theta_D^+ = 1 + (1 + \omega s_2)/t$.



(a) Impact of tying on CS (b) Impact of tying on SS (c) Impact of tying on W

Figure 3: Welfare effects of tying ($b_1 = b_2 = 0.2$, $m = 0.4$, $t = 2$, $M = 5$ and $K = 4$)

Panel (b) shows that tying can also reduce seller surplus. Seller surplus depends on the total demand that sellers can reach through each platform. For small values of α , the effect is positive because tying expands demand on B_1 , increasing the value of participation for sellers on the tied platform. As α increases, however, the direct channel of B_2 becomes more important for seller revenues. Tying then reduces the total demand available to sellers on B_2 , and this loss eventually dominates the gain accruing to sellers on B_1 . The decline in seller surplus is stronger when ω is larger. Stronger cross-side network effects amplify the contraction on the rival platform: fewer consumers reduce sellers' incentives to participate, and lower seller participation further depresses consumer demand. Panel (c) reports the corresponding effect on total welfare.

The figure also illustrates the role of ω , which governs the strength of network effects on the seller side. A higher ω amplifies both the benefit from the expansion of B_1 's seller base and the harm from the reduction of platform 2's demand. When α is small, the former effect dominates because relatively few consumers in the direct channel are exposed to the reduction in S_2^* . As α increases, the latter effect becomes more important, as the reduction in B_2 's seller base affects a larger mass of consumers in the direct channel who receive no offsetting convenience benefit. As a result, a higher ω makes consumer surplus, seller surplus, and welfare more sensitive to α , lowering the value of α at which each becomes negative. Stronger network effects therefore narrow the range of direct-channel sizes over which tying is welfare-enhancing.

We summarise the numerical results obtained with the following remark.¹⁹

Remark 1. *In our real-line Hotelling example, there exist thresholds α^{CS} , α^{SS} and α^W such that $dCS/db_1 < 0$ for $\alpha > \alpha^{CS}$, $dSS/db_1 < 0$ for $\alpha > \alpha^{SS}$ and $dW/db_1 < 0$ for $\alpha > \alpha^W$, with i) $\alpha^{SS} < \alpha^W < \alpha^{CS}$, and ii) α^h , $h = \{CS, SS, W\}$ decreasing with ω .*

3.6 Partial rent extraction

The baseline model assumes that each platform earns revenues only from seller fees. We now allow platform $A1$ to extract rents from transactions mediated by its rival through a per-transaction access fee $a \geq 0$, which is assumed exogenous for now. This can be the case of uniform fees across different products and services.

Platform profits are given by

$$\Pi_1 = r_1 S_1 + a Q_{2M}, \quad \Pi_2 = r_2 S_2 - a Q_{2M}, \quad (16)$$

¹⁹We have also experimented with other parameter ranges obtaining qualitatively similar results.

where the access fee applies only to transactions mediated by platform 2. Throughout this subsection, the access fee is treated as an exogenous parameter. In the next one, we shall endogenize it.

The access fee creates an additional trade-off in the tying decision. On the one hand, tying continues to increase seller demand for platform 1, thereby raising seller-fee revenues as shown in the previous subsection. On the other hand, tying diverts transactions away from the rival platform, reducing the volume on which access-fee revenues are collected.

Differentiating equilibrium profits with respect to the tying parameter yields

$$\frac{d\Pi_1^*}{db_1} = S_1^* \frac{dr_1^*}{db_1} + r_1^* \frac{dS_1^*}{db_1} + a \frac{dQ_{2M}^*}{db_1}. \quad (17)$$

The first two terms coincide with those obtained in the baseline model where $a = 0$. The last term captures the direct opportunity cost of tying, which is negative (by Proposition 1). Since tying is profitable at $a = 0$ (Proposition 2), by continuity it is also profitable for sufficiently small a . When the access fee grows larger, the revenue losses from the fee revenues obtained by the rival reduces the profitability of tying.

Figure 4 shows that tying can become unprofitable when the access fee is positive and the direct channel is large enough. The figure plots the marginal effect of tying on each platform's profit against the direct-channel size α , for different values of a .

Panel (a) shows that a higher a lowers the profitability of tying for platform 1. For sufficiently higher α (which measures the importance of the direct channel) and a it shows that tying becomes unprofitable. Panel (b) shows that tying always harms platform 2 but the reduction in profits is mitigated the higher a and the higher α .

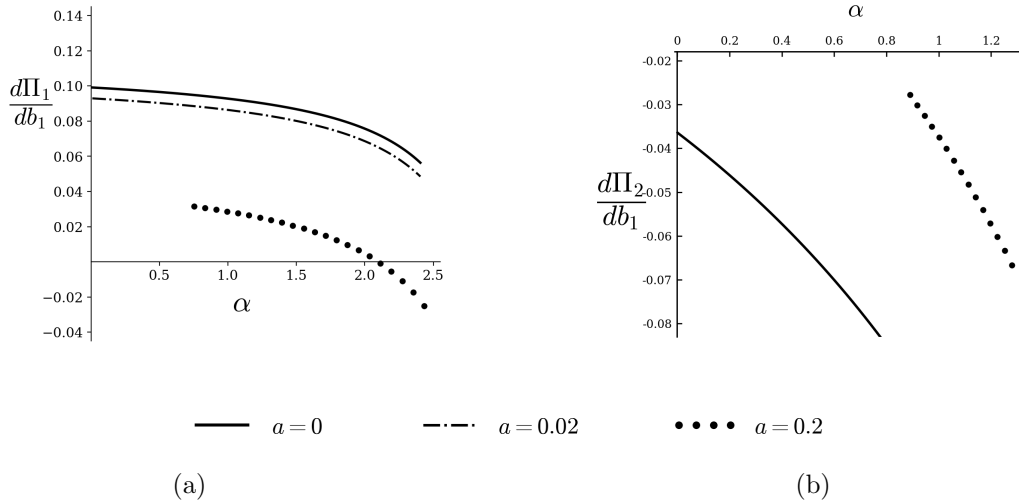


Figure 4: Effect of tying on platforms' profits ($b_1 = b_2 = 0.2$, $m = 0.4$, $t = 2$, $M = 5$, $K = 4$, $\omega = 0.5$).

3.7 Endogenous access fee

Consider now the case in which the access fee is endogenous. Then, the integrated platform 1 will set a at the highest level consistent with the rival remaining on the mediated channel. Let Π_2^{out} denote the profit platform 2 earns if it serves only its direct users. Then, platform A1 will set the fee $\bar{a}(b_1)$ at which platform 2's participation constraint binds:

$$\Pi_2^*(b_1, \bar{a}(b_1)) = \Pi_2^{\text{out}}. \quad (18)$$

Recall from (16) that Π_2^* is already net of the access-fee payment, $\Pi_2^* = r_2^* S_2^* - a Q_{2M}^*$; condition (18) therefore says that platform 2's total equilibrium profit, inclusive of the access-fee payment it makes to A1, is driven down to its outside option. Note that Π_2^{out} depends neither on the access fee a nor on the convenience parameter b_1 , because it is earned outside the mediated channel.

Define

$$\widehat{\Pi}_i(b_1) \equiv \Pi_i^*(b_1, \bar{a}(b_1)), \quad i = 1, 2,$$

that is, platform i 's equilibrium profit under perfect extraction, evaluated at the perfect-extraction fee $a = \bar{a}(b_1)$. By definition, $\widehat{\Pi}_2(b_1) = \Pi_2^{\text{out}}$ for every b_1 in the relevant neighborhood. Since Π_2^{out} is constant, $\widehat{\Pi}_2'(b_1) = 0$.

Here $\bar{a}(\cdot)$ is a function of b_1 : as b_1 changes, the access fee is continuously re-optimised so that platform 2's equilibrium profit remains at Π_2^{out} for every b_1 in the relevant neighborhood.

Because $\widehat{\Pi}_2'(b_1) = 0$, the increase in platform 1's profits from engaging in tying is equal to the effect of tying on the two platforms' joint profit:

$$\widehat{\Pi}'_1(b_1) = \widehat{\Pi}'_1(b_1) + \widehat{\Pi}'_2(b_1) = \frac{d}{db_1} [\Pi_1^*(b_1, \bar{a}(b_1)) + \Pi_2^*(b_1, \bar{a}(b_1))]. \quad (19)$$

The reason is the following. The access payment $a Q_{2M}^*$ is a transfer between platforms, and any change in the optimal a in response to a change in b_1 is also a transfer.²⁰ The transfer therefore cancels from the two platforms' joint profit, which by (16) reduces to $\Pi_1^* + \Pi_2^* = r_1^* S_1^* + r_2^* S_2^*$; consequently, $\widehat{\Pi}'_1(b_1)$ is positive whenever the right-hand side of (19) is.

Proposition 3 (Tying under endogenous fee). *Suppose the access fee is endogenous. Then, tying is profitable for the integrated platform only if it raises the two platforms' joint profit, $\Pi_1 + \Pi_2 = r_1 S_1 + r_2 S_2$.*

Proof. Follows from discussion. □

The intuition is that, once the integrated platform has extracted all the surplus that keeps the rival on the mediated channel, it becomes the residual claimant on that surplus. A marginal increase in b_1 can no longer benefit the integrated platform merely by shifting rents away from the rival: the rival is already held at Π_2^{out} . Tying is profitable only if it increases the joint profit generated by the two platforms, taking into account the induced changes in seller fees, seller participation, and quantities. If tying reduces joint platform profit, the loss is borne by the integrated platform through the lower access fee required to keep platform 2 on the mediated channel.

The role of the direct channel can then be understood as follows. In Figure 4 for the partial rent extraction, when α is close to zero, the direct channel is small and tying raises joint platform profit, so tying remains profitable. As α increases, the rival's direct channel business becomes more important. Tying then reduces not only platform 2's mediated quantity but also its direct-channel quantity through seller participation. At the same time, we have seen now that, Π_2^{out} rises with α , which reduces the rent that can be extracted through the mediated channel. This suggests that, as $\alpha \rightarrow \infty$, Π_2^{out} grows together with the rival's direct-channel activity, so the mediated-channel surplus available for platform A1 to extract reduces drastically; beyond some

²⁰To see why, note that

$$\frac{d\widehat{\Pi}_1}{db_1} = \left(S_1^* \frac{dr_1^*}{db_1} + r_1^* \frac{dS_1^*}{db_1} \right) + \underbrace{\bar{a} \frac{dQ_{2M}^*}{db_1}}_{\text{per-transaction}} + \underbrace{Q_{2M}^* \bar{a}'(b_1)}_{\text{fee adjustment}}, \quad (20)$$

$$\frac{d\widehat{\Pi}_2}{db_1} = \left(S_2^* \frac{dr_2^*}{db_1} + r_2^* \frac{dS_2^*}{db_1} \right) - \bar{a} \frac{dQ_{2M}^*}{db_1} - Q_{2M}^* \bar{a}'(b_1) = 0, \quad (21)$$

threshold $\bar{a}$, tying would no longer be marginally profitable under perfect rent extraction, and the integrated platform would not raise b_1 in equilibrium.

These results suggest that tying and rent extraction via access fee are, to some extent, substitutable instruments. Thus, as suggested in recent work (Franck and Peitz, 2023; Bar-Isaac, 2026), a policy which restricts access fees (such as caps) might incentivise a platform to using tying.

4 Demotion of the rival platform

We now consider an alternative conduct, which we call *demotion*. Formally, demotion is modelled as a marginal *decrease* in b_2 .²¹

The mechanism through which demotion propagates in the two-sided market is the mirror image of the tying mechanism. A lower b_2 contracts consumer demand for rival platform B_2 in the mediated channel, which in turn reduces sellers' incentive to participate on B_2 . Through the seller-side cross-network effect, the contraction of S_2 further depresses q_{2M} and, crucially, depresses q_{2D} through $\partial f_{2D}/\partial s_2 > 0$. Of course, the effects of demotion on the integrated platforms are of opposite sign: lowering b_2 shifts consumer demand toward platform B_1 and raises seller participation S_1 , further raising q_{1M} . This is the main mechanism for given fees. In the Appendix, we also show that, under conditions mirroring those in the tying case, a demotion (i.e., a reduction in b_2) leads to a higher fee for platform 1 sellers and a lower one for platform 2 sellers: $-dr_1^*/db_2 > 0$ and $-dr_2^*/db_2 < 0$.

Also in this case, we focus on the case where there is *dominance of direct effects over strategic effects* (see e.g., dominance condition in the Appendix).

We can then state the main result as follows, while relegating all algebra to the Appendix.

Proposition 4 (Effect of demotion on seller participation and quantities). *Suppose the maintained assumptions and the dominance condition hold. Then demotion unambiguously leads to:*

$$(i) -dS_1^*/db_2 > 0 \text{ and } -dS_2^*/db_2 < 0;$$

$$(ii) -dQ_{1M}^*/db_2 > 0, -dQ_{2M}^*/db_2 < 0, \text{ and } -dQ_{2D}^*/db_2 < 0.$$

Thus, there is contraction of platform 2's quantities in both channels.

Proof. See Appendix. □

The effect of demotion on mediated and total quantities As for tying, the net effect of demotion on mediated and total quantities cannot be established a priori. (See discussion following Proposition 1 above.)

The effect of demotion on profits We now sign the profit effects of demotion. They are obtained by the same product-rule decomposition as under tying. Since demotion is a decrease in b_2 , the relevant effect is the negative of the derivative with respect to b_2 . Hence platform 1's seller-fee revenue increases and platform 2's seller-fee revenue decreases. That is,

$$\frac{d\Pi_1^*}{db_2} = S_1^* \frac{dr_1^*}{db_2} + r_1^* \frac{dS_1^*}{db_2},$$

²¹Of course, in some environments, tying and demotion might take place simultaneously. For instance, if a search engine platform artificially changes the ranking of links, placing its own website ahead of the rival's, this can be seen as both tying and demotion. But in other environments they can be distinct. For instance, interoperability could be degraded, or users might be misled into thinking that use of the rival might create security issues. In any event, treating them as conceptually different can be insightful, as we shall see below.

and

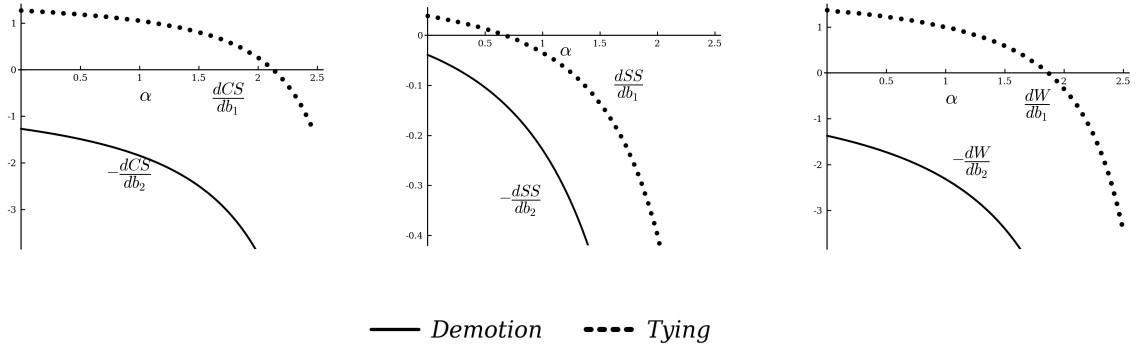
$$\frac{d\Pi_2^*}{db_2} = S_2^* \frac{dr_2^*}{db_2} + r_2^* \frac{dS_2^*}{db_2}.$$

Since $-dr_1^*/db_2 > 0$, $-dr_2^*/db_2 < 0$, $-dS_1^*/db_2 > 0$, and $-dS_2^*/db_2 < 0$, the effects of demotion on profits carry the same signs as those of tying: both the fee-level and the participation term raise platform 1's profit and lower platform 2's. Therefore, demotion is profitable for platform 1 and harms platform 2:

Proposition 5 (Profit effects of demotion in the two-sided market). *Suppose the maintained assumptions and the dominance condition hold. Demotion is always profitable for platform 1 and harms platform 2.*

Proof. Follows from the discussion. $\square$

Illustrative example. We conclude this section with a numerical illustration based on the same Hotelling environment introduced in Section 3.5. Figure 5 reports the welfare effects of demotion and, for comparison, those of tying already discussed in Section 3.5.²² Demotion always worsens consumer, seller, and total surplus; like tying, it shifts demand away from the rival in the mediated channel, but it does so by directly worsening the rival's accessibility, and this turns out to be always harmful.



(a) Consumer surplus

(b) Seller surplus

(c) Social welfare

Figure 5: Demotion *vs* tying ($b_1 = b_2 = 0.2$, $m = 0.4$, $t = 2$, $M = 5$, $K = 4$ and $\omega = 0.5$)

5 A common-access policy

We now turn to a policy that, rather than favouring one specialised service, marginally improves access to *both* services within the mediated environment. Such symmetric access obligations are the structure of many ex ante remedies imposed under the DMA in the EU. Formally, let

$$b_1 = b_{01} + b, \quad b_2 = b_{02} + b,$$

and study the effect of a marginal increase in b .

This non-discriminatory policy raises both b_1 and b_2 by the same amount. It therefore reduces frictions symmetrically for all services on the dominant platform, in contrast to tying,

²²Demotion is a marginal reduction in b_2 . To make it directly comparable with tying (a marginal increase in b_1), the figures plot its effect as minus the derivative with respect to b_2 , i.e. $-dCS/db_2$, $-dSS/db_2$ and $-dW/db_2$. The sign then reads as for tying: a positive value means the practice raises the welfare measure, a negative value means it lowers it.

which favours the integrated service alone. In the two-sided environment, the policy operates not only on consumers in the mediated channel but also, through cross-side network effects, on seller participation, and from there on demand in the direct channel.

The policy acts in a similar way as the tying mechanism in Section 2, but a common increase in b raises the attractiveness of *both* services in the mediated environment, which raises *both* consumer demands q_{1M} and q_{2M} at fixed seller masses. The induced expansion of seller participation on both platforms then feeds back positively into consumer demand.

The analysis is quite intuitive because a common increase in b is the sum of two shifts we have already studied. Since $b_1 = b_{01} + b$ and $b_2 = b_{02} + b$, the chain rule decomposes the policy response of any equilibrium object X^* into a *tying component* (its response to b_1) and a *raising- b_2 component* (its response to b_2 , the opposite of demotion),

$$\frac{dX^*}{db} = \frac{dX^*}{db_1} + \frac{dX^*}{db_2}. \quad (22)$$

By Lemmas 1 and 4 and Propositions 1 and 4, the two components carry opposite signs for every equilibrium fee, participation level, and mediated quantity: for a platform's own objects the own-shifter component is favourable and the rival-shifter component adverse. Whether the favourable component prevails is the content of the following condition, stated in the same reduced-form spirit as the dominance conditions for tying (Definition 1) and demotion.

Definition 2 (Own-shifter dominance under the common-access policy). *The own-shifter effects dominate the cross-shifter effects if, for each platform, the responses of its equilibrium seller fee, seller participation, and mediated-channel quantity to its own convenience parameter dominate, in absolute value, their responses to the rival's convenience parameter:*

$$\begin{aligned} \frac{dr_1^*}{db_1} &\geq \left| \frac{dr_1^*}{db_2} \right|, & \frac{dS_1^*}{db_1} &\geq \left| \frac{dS_1^*}{db_2} \right|, & \frac{dQ_{1M}^*}{db_1} &\geq \left| \frac{dQ_{1M}^*}{db_2} \right|, \\ \frac{dr_2^*}{db_2} &\geq \left| \frac{dr_2^*}{db_1} \right|, & \frac{dS_2^*}{db_2} &\geq \left| \frac{dS_2^*}{db_1} \right|, & \frac{dQ_{2M}^*}{db_2} &\geq \left| \frac{dQ_{2M}^*}{db_1} \right|. \end{aligned}$$

This is a reduced-form restriction on equilibrium comparative statics, the natural counterpart for the common-access policy of the dominance conditions used for tying and demotion. It is the equilibrium analogue of the “own effects dominate cross effects” structure of Assumption 1, to which it reduces when cross-effects are symmetric, as in the Hotelling specification of Section 3.5, where each inequality can be verified directly.

Proposition 6 (Effect of the policy on seller participation and quantities). *Under Assumptions 1 and 2 and the dominance conditions of Definition 2, the common-access policy:*

(i) *raises seller participation on both platforms,*

$$\frac{dS_1^*}{db} \geq 0, \quad \frac{dS_2^*}{db} \geq 0;$$

(ii) *expands platform 2's direct-channel quantity and expands both mediated quantities,*

$$\frac{dQ_{2D}^*}{db} \geq 0, \quad \frac{dQ_{1M}^*}{db} \geq 0, \quad \frac{dQ_{2M}^*}{db} \geq 0,$$

with strict expansion of the direct channel whenever $dS_2^/db > 0$.*

Proof. See Appendix. □

The contrast with tying (Proposition 1) is the central result of this section. As under both tying and demotion, seller participation and quantities rise for the integrated platform but fall for the rival. Under a policy that forces the dominant platform to improve accessibility (or reduce costs) symmetrically for both services it intermediates, seller participation and quantities instead rise for *both* platforms, and in particular the rival's direct channel expands too. In this dimension the policy is unambiguously preferable to tying.

The effect of the policy on profits We now ask the question of whether the dominant platform would willingly adopt such a policy: if the symmetric remedy lowered its profit, it might pre-empt the remedy by forgoing the convenience improvement altogether. In the baseline environment the platforms earn only seller-fee revenue, $\Pi_i^* = r_i^* S_i^*$, so the product rule gives the decomposition

$$\frac{d\Pi_1^*}{db} = \underbrace{S_1^* \frac{dr_1^*}{db}}_{\geq 0} + \underbrace{r_1^* \frac{dS_1^*}{db}}_{\geq 0}, \quad \frac{d\Pi_2^*}{db} = \underbrace{S_2^* \frac{dr_2^*}{db}}_{\geq 0} + \underbrace{r_2^* \frac{dS_2^*}{db}}_{\geq 0}, \quad (23)$$

where each fee response dr_i^*/db and each participation response dS_i^*/db is non-negative under Definition 2 and Proposition 6. We conclude the following.

Proposition 7 (Effect of the policy on profits). *Under Assumptions 1 and 2 and the dominance conditions of Definition 2:*

(i) *the integrated platform's profit weakly rises, $d\Pi_1^*/db \geq 0$;*

(ii) *the rival platform's profit weakly rises, $d\Pi_2^*/db \geq 0$;*

Proof. See Appendix. □

The policy is thus a Pareto improvement on the supply side: unlike tying, which raises the integrated platform's profit while lowering the rival's (Proposition 2), a non-discriminatory access improvement benefits *both* platforms. This matters for its implementability: because the integrated platform is not made worse off, it has no incentive to pre-empt the remedy by forgoing the convenience improvement in the first place.

Consumer surplus, seller surplus, and welfare The welfare consequences follow. The policy weakly raises every consumed quantity ($dQ_{1M}^*/db, dQ_{2M}^*/db, dQ_{2D}^*/db \geq 0$, Proposition 6), improves the convenience of *both* services (b_1, b_2 rise), and enlarges seller variety on *both* platforms (S_1^*, S_2^* rise). Because consumers pay no price to the platforms, none of these gains is offset by a higher consumer outlay, so under Assumption 1 consumer surplus does not fall. Sellers pay higher fees but serve a larger consumer base; since $dS_j^*/db \geq 0$ and $g_j > 0$, the marginal seller's net payoff rises, $mdq_j^*/db - dr_j^*/db = (1/g_j) dS_j^*/db \geq 0$, and so does the surplus of every infra-marginal seller (see the proof of Proposition 7). Combining these with Proposition 7(i)–(ii), total welfare $W = CS + SS + \Pi_1^* + \Pi_2^*$ increases with the policy.

6 Extension: a model with consumer-side network effects

In the baseline two-sided model of Sections 2–5 the network channel operated through seller participation: platforms charged fees to sellers and consumer demand depended on seller masses. We now consider the polar environment in which there are no sellers, platforms charge consumers directly, and network effects operate *among consumers* on each specialised service. The same primitive demand structure of Section 2, with seller masses s_j replaced by consumer-side total-usage levels n_j . Throughout the analysis, we point out the analogue between each one-sided assumption and the corresponding two-sided assumption introduced earlier; the proofs are relegated to Appendix A.

6.1 Setup

Consider a market in which there are no sellers and platforms charge consumers directly. Network effects operate *among consumers* on each specialised service: each consumer's payoff from

joining platform B_J rises with the total usage n_J of B_J . The dominant platform A_1 , its integrated service B_1 , and the rival service B_2 retain the same structure as in Section 2; what changes is the pricing instrument (consumer prices p_J rather than seller fees r_J) and the carrier of network externalities (own total usage n_J rather than seller participation s_J).

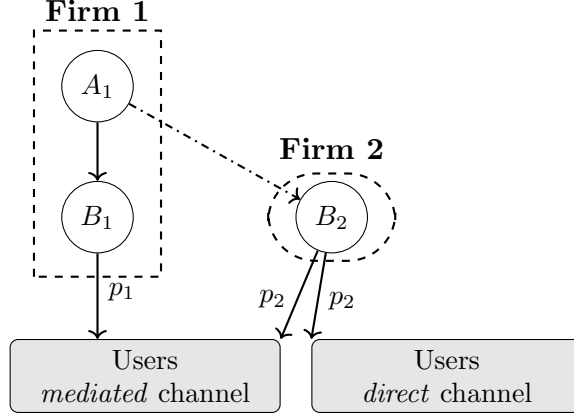


Figure 6: Market structure with consumer-side network effects

Figure 6 provides a graphical representation of our environment. Consumers have two access channels: (i) a *mediated* channel through the primary platform and (ii) a *direct* channel, which is available only for the rival specialised service.

The difference relative to the baseline model is twofold. Firstly, there are no sellers, but there exists a network externality in that the utility of B_2 users increases with the size of both the direct and mediated channels. Secondly, each platform B_J now charges its consumers a price p_J .

Let q_{1M} denote demand for B_1 through the mediated channel, q_{2M} demand for B_2 through the mediated channel, and q_{2D} demand for B_2 through the direct channel. We let demands be

$$\begin{aligned} q_{1M} &= f_{1M}(b_1, b_2, p_1, p_2, n_1, n_2), \\ q_{2M} &= f_{2M}(b_1, b_2, p_1, p_2, n_1, n_2), \\ q_{2D} &= \alpha f_{2D}(p_2, n_2). \end{aligned} \tag{24}$$

The variables p_1 and p_2 are the prices set by platforms 1 and 2, respectively. The variables n_1 and n_2 denote total usage levels of services B_1 and B_2 , with

$$n_1 = q_{1M}, \quad n_2 = q_{2M} + q_{2D}. \tag{25}$$

Hence, platform 2 sets a common price across its mediated and direct channels.

We adopt the standard fulfilled-expectations interpretation of network effects. For any price vector (p_1, p_2) , market demand is determined by a fixed point in total usage.

We provide details on the main assumptions in the Appendix, reflecting sign restrictions and stability conditions already made in Section 3.

Platforms. Platform i sets a single price p_i . Platform 2 charges the same price across the mediated and direct channels.²³

Timing. The game unfolds in two stages. In the first stage, platforms 1 and 2 simultaneously choose prices p_1 and p_2 . In the second stage, given these prices, consumers choose among the

²³Uniform pricing across channels is consistent with standard “most-favored-customer” or anti-steering rules on dominant platforms, and is the empirically relevant case for the policy debate.

available services and channels, and market quantities are determined as a fixed point of the demand system. The equilibrium concept is subgame perfect Nash equilibrium with fulfilled expectations.

Tying. We start by characterising market demand for given prices:

$$\begin{cases} n_1 = f_{1M}(b_1, b_2, p_1, p_2, n_1, n_2), \\ n_2 = f_{2M}(b_1, b_2, p_1, p_2, n_1, n_2) + \alpha f_{2D}(p_2, n_2). \end{cases} \quad (26)$$

Under Assumption 6, by the implicit-function theorem, there exists a locally unique pair of reduced-form fixed-point usage levels,

$$N_1 = \mathcal{N}_1(b_1, b_2, p_1, p_2, \alpha), \quad (27)$$

$$N_2 = \mathcal{N}_2(b_1, b_2, p_1, p_2, \alpha). \quad (28)$$

Substituting these quantities into the demand system yields the reduced-form equilibrium demands as follows:

$$\begin{aligned} Q_{1M}(b_1, b_2, p_1, p_2, \alpha) &= f_{1M}(b_1, b_2, p_1, p_2, N_1(\cdot), N_2(\cdot)), \\ Q_{2M}(b_1, b_2, p_1, p_2, \alpha) &= f_{2M}(b_1, b_2, p_1, p_2, N_1(\cdot), N_2(\cdot)), \\ Q_{2D}(b_1, b_2, p_1, p_2, \alpha) &= \alpha f_{2D}(p_2, N_2(\cdot)). \end{aligned}$$

By construction, $N_1 = Q_{1M}$ and $N_2 = Q_{2M} + Q_{2D}$. Under the stability and dominance conditions in the Appendix, we have: $\partial N_i / \partial p_i < 0$, $\partial N_i / \partial p_j > 0$, $\partial N_1 / \partial b_1 > 0$ and $\partial N_2 / \partial b_1 < 0$.

6.2 The effect of tying on prices and quantities

The effect of tying on prices. We now consider the effect of tying, represented by an increase in b_1 . Platforms choose p_1 and p_2 to maximize profits. In the Appendix, we show that, under the regularity condition on the price game and the maintained restriction that tying raises platform 1's marginal profitability of increasing p_1 while lowering platform 2's marginal profitability of increasing p_2 , the equilibrium price response has the following sign: $dp_1^*/db_1 \geq 0$ and $dp_2^*/db_1 \leq 0$, which resembles the related effect in the two-sided market where prices are set on the seller side (Lemma 1). This conclusion is immediate when prices are strategic substitutes. When prices are strategic complements, it holds under the diagonal-dominance condition stated in the Appendix.

The intuition is as follows. Tying improves platform 1's position in the mediated channel and weakens platform 2's. Platform 1 therefore has an incentive to raise its price, while platform 2 has an incentive to cut its price to retain demand. These price adjustments partially offset the direct reallocation of demand induced by tying. Since platform 2 charges a common price across channels, the price cut also benefits its consumers in the direct channel.

The effect of tying on quantities. To isolate these forces, consider first the benchmark without network effects. In this case demand does not depend on usage levels, so the only channels are the direct demand effect of tying and the induced price response.

Lemma 3. *Suppose that there are no network effects:*

$$\frac{\partial f_{JM}}{\partial n_J} = \frac{\partial f_{JM}}{\partial n_{-J}} = \frac{\partial f_{2D}}{\partial n_2} = 0, \quad J = 1, 2.$$

Then:

- (i) *tying always raises quantity in the direct channel: $dQ_{2D}^*/db_1 > 0$;*

(ii) if direct effects outweigh strategic pricing effects, tying increases quantity in the mediated channel, with:

$$\frac{dQ_{1M}^*}{db_1} > 0, \quad \frac{dQ_{2M}^*}{db_1} < 0;$$

if instead strategic pricing effects dominate, the two inequalities are reversed.

The lemma isolates the benchmark without network effects. Tying directly reallocates mediated demand toward platform 1 and away from platform 2. The induced price responses partly offset this reallocation: platform 1 raises its price, which limits the expansion of Q_{1M} , while platform 2 lowers its price, which mitigates the contraction of Q_{2M} and raises demand in the direct channel Q_{2D} . Thus, unlike in the two-sided model analysed above, the direct channel weakly buffers platform 2 when network effects are absent.

In the mediated channel, the overall effects of tying are generally ambiguous. Tying increases Q_{1M} only if the direct convenience effect dominates the offsetting price response, and it reduces Q_{2M} only if the direct demand loss dominates platform 2's price cut. Aggregate mediated demand rises only when the market-expansion effect of tying exceeds the induced effect on prices.²⁴

This distinction is particularly transparent when comparing covered and uncovered markets. In a covered market, tying primarily reallocates mediated users across firms, so total mediated demand is fixed. In an uncovered market, tying may also expand demand by attracting marginal consumers, but this effect must be weighed against the equilibrium price responses.

We now study the effect of tying in the presence of within-consumer side network effects. Consider the following

$$Q_{1M}^* = f_{1M}(b_1, b_2, p_1^*, p_2^*, Q_{1M}^*, Q_{2M}^* + Q_{2D}^*), \quad (29)$$

$$\begin{aligned} Q_{2M}^* &= f_{2M}(b_1, b_2, p_1^*, p_2^*, Q_{1M}^*, Q_{2M}^* + Q_{2D}^*), \\ Q_{2D}^* &= \alpha f_{2D}(p_2^*, Q_{2M}^* + Q_{2D}^*). \end{aligned} \quad (30)$$

Differentiating the above terms with respect to b_1 shows that the total effect of tying on each channel demand can be decomposed into three parts: a *direct effect*, a *strategic price effect*, and a *network-feedback effect*. In the mediated channels, the direct effect captures the immediate reallocation of demand induced by a higher b_1 . The strategic effect captures the equilibrium price responses of the two firms. Under our maintained assumptions, $SE_{1M} < 0$, $SE_{2M} > 0$, and $SE_{2D} > 0$ denoting the strategic price effects. Thus the induced price responses mitigate the demand expansion of firm 1 in the mediated channel, mitigate the contraction of firm 2 in the mediated channel, and support firm 2's direct demand.

Tying therefore has the following effects:

$$\frac{dQ_{1M}^*}{db_1} = \frac{\partial f_{1M}}{\partial b_1} + \underbrace{\frac{\partial f_{1M}}{\partial p_1} \frac{dp_1^*}{db_1} + \frac{\partial f_{1M}}{\partial p_2} \frac{dp_2^*}{db_1}}_{\equiv SE_{1M}} + \underbrace{\frac{\partial f_{1M}}{\partial n_1} \frac{dQ_{1M}^*}{db_1} + \frac{\partial f_{1M}}{\partial n_2} \left(\frac{dQ_{2M}^*}{db_1} + \frac{dQ_{2D}^*}{db_1} \right)}_{\text{network-related effect}}, \quad (31)$$

$$\frac{dQ_{2M}^*}{db_1} = \frac{\partial f_{2M}}{\partial b_1} + \underbrace{\frac{\partial f_{2M}}{\partial p_1} \frac{dp_1^*}{db_1} + \frac{\partial f_{2M}}{\partial p_2} \frac{dp_2^*}{db_1}}_{\equiv SE_{2M}} + \underbrace{\frac{\partial f_{2M}}{\partial n_1} \frac{dQ_{1M}^*}{db_1} + \frac{\partial f_{2M}}{\partial n_2} \left(\frac{dQ_{2M}^*}{db_1} + \frac{dQ_{2D}^*}{db_1} \right)}_{\text{network-related effect}}, \quad (32)$$

$$\frac{dQ_{2D}^*}{db_1} = \underbrace{\alpha \frac{\partial f_{2D}}{\partial p_2} \frac{dp_2^*}{db_1}}_{\equiv SE_{2D}} + \underbrace{\alpha \frac{\partial f_{2D}}{\partial n_2} \left(\frac{dQ_{2M}^*}{db_1} + \frac{dQ_{2D}^*}{db_1} \right)}_{\text{network-related effect}}. \quad (33)$$

²⁴ Absent network effects, the direct channel can partly offset foreclosure and, under suitable conditions, tying may increase total quantity.

The three expressions decompose the equilibrium response of channel demand into two forces. The first is the *network-adjusted direct effect*: the effect of tying holding prices fixed, while allowing the induced quantity feedbacks generated by network effects to operate. The second is the *network-adjusted strategic effect*: the effect of the equilibrium price responses induced by tying, propagated through the same quantity-feedback system. The full derivations are reported in the Appendix; here we discuss the main implications.

For platform 1's mediated channel, the network-adjusted direct effect is positive. Tying raises the attractiveness of the integrated service, and the resulting reduction in platform 2's activity further increases platform 1's mediated demand, since demand for one platform falls with usage of the rival platform. For platform 2's mediated channel, the corresponding direct effect is negative: tying lowers the relative attractiveness of platform 2, and the induced decline in its activity reinforces the contraction in mediated demand.

The direct channel of platform 2 is where network effects present a main difference relative to the two-sided market setting. Without network effects, this channel is affected by tying only through platform 2's equilibrium price response, so platform 2's price cut tends to expand direct demand. With network effects, however, the loss of mediated demand reduces platform 2's total usage, which weakens the value of its direct channel. Thus the network-adjusted direct effect on Q_{2D} is negative.

The strategic price response works in the opposite direction in the direct channel. Platform 2's price cut tends to increase Q_{2D} , whereas the network-induced reduction in platform 2's usage tends to reduce it. The overall effect is therefore ambiguous. The next result provides sufficient conditions under which the direct network-adjusted effects dominate the strategic price response.

Proposition 8 (Effect of tying on equilibrium quantities). *If the network-adjusted direct demand effects of tying dominate the network-adjusted strategic pricing effects for each of quantity, then tying leads to:*

- (i) an increase in platform 1's mediated quantity, $dQ_{1M}^*/db_1 > 0$;
- (ii) a decrease in platform 2's mediated quantity, $dQ_{2M}^*/db_1 < 0$;
- (iii) a decrease in platform 2's direct quantity, $dQ_{2D}^*/db_1 < 0$.

Proposition 8 implies that network effects do not change the sign of the direct shock generated by tying in the mediated environment: they amplify the expansion of platform 1's mediated demand and the contraction of platform 2's mediated demand. What they do change is the direct channel. Once direct demand depends positively on total platform activity, the loss suffered by platform 2 in the mediated channel spills over negatively to its direct channel. Thus, under network effects, tying no longer has an unambiguously positive effect on platform 2's direct quantity. Whether Q_{2D}^* rises or falls depends on the balance between two opposing forces: the positive strategic effect of the induced price cut and the negative direct effect operating through the contraction in platform 2's total activity.

Recall that in models with differentiated products, total quantities are not valid proxies for the overall effect on consumer surplus. To identify the effects at stake, the illustrative model below (see also Appendix B2) quantifies them for a specific functional form and derives the welfare implications.

The effect of tying on profitability As in the two-sided model, we obtain

$$\frac{d\Pi_1^*}{db_1} = p_1^* \left(\frac{dQ_{1M}^*}{db_1} - \frac{\partial Q_{1M}}{\partial p_1} \frac{dp_1^*}{db_1} \right), \quad (34)$$

$$\frac{d\Pi_2^*}{db_1} = p_2^* \left(\frac{dQ_{2M}^*}{db_1} + \frac{dQ_{2D}^*}{db_1} - \left(\frac{\partial Q_{2M}}{\partial p_2} + \frac{\partial Q_{2D}}{\partial p_2} \right) \frac{dp_2^*}{db_1} \right). \quad (35)$$

These expressions are much sharper. Since $dp_1^*/db_1 > 0$, $\partial Q_{1M}/\partial p_1 < 0$, the second term inside the bracket in (34) is positive. Hence a sufficient condition for platform 1's profit to increase is simply $dQ_{1M}^*/db_1 > 0$, which holds when network-adjusted direct effects are stronger than network-adjusted strategic effects, that is conditions in Proposition 8. Similarly, since $\partial Q_{2M}/\partial p_2 < 0$, $\partial Q_{2D}/\partial p_2 < 0$ and $dp_2^*/db_1 < 0$, the network-adjusted price-adjustment term in (35) is negative. Therefore a sufficient condition for platform 2's profit to decrease is $dQ_{2D}^*/db_1 + dQ_{2M}^*/db_1 \leq 0$, that is overall demand decreases.

Therefore, as in Section 3, tying is unambiguously profitable for platform 1, but it harms platform 2's profitability.

The role of the direct channel To isolate the role of the direct channel, consider the limit $\alpha \rightarrow \infty$. From the fixed-point equation $n_2 = f_{2M}(b_1, b_2, p_1, p_2, n_1, n_2) + \alpha f_{2D}(p_2, n_2)$, the mediated component remains bounded while the direct component diverges with α . Dividing by α and taking the limit, $n_2/\alpha \rightarrow f_{2D}(p_2, n_2)$, which pins down p_2^* from firm 2's direct-channel problem alone. Hence

$$\lim_{\alpha \rightarrow \infty} \frac{dp_2^*}{db_1} = 0, \quad \lim_{\alpha \rightarrow \infty} \frac{d(Q_{2M}^* + Q_{2D}^*)}{db_1} = 0.$$

Firm 1's problem decouples: it maximises $\Pi_1 = p_1 f_{1M}$ at the unchanged p_2^* . Recall that $dp_1^*/db_1 > 0$. Substituting into the equilibrium quantity equation,

$$\frac{dQ_{1M}^*}{db_1} = \frac{\frac{\partial f_{1M}}{\partial b_1} + \frac{\partial f_{1M}}{\partial p_1} \frac{dp_1^*}{db_1}}{1 - \frac{\partial f_{1M}}{\partial n_1}},$$

which is positive if the direct demand effect dominates the induced price increase. Under this condition, tying expands platform 1's mediated demand while platform 2's quantities are insensitive to tying; total output rises. A sufficiently large direct channel therefore insulates the rival from the conduct, and any harm from tying becomes confined to the dominant platform's side of the market.

An illustrative model. We now provide an illustrative example with two specialised platforms competing à la Hotelling on market B . The welfare results and economic logic closely mirror those of the two-sided model; only the transmission channel differs. There, tying reduces the rival's mediated demand, lowering seller participation on B_2 and, through cross-side network effects, its value to direct users. Here, with no sellers, the spillover is direct: tying lowers B_2 's total usage n_2 , which directly reduces its value to consumers. Technical details are relegated to Appendix B2.

Figure 7 illustrates the impact of tying on consumer surplus and welfare and how it changes with the size of the direct channel. Panel (a) shows that tying raises consumer surplus only when the direct channel is small. As in the baseline setting with two-sided network effects, for low α , the gain from the expansion of Q_{1M} dominates the losses from the contractions of Q_{2M} and Q_{2D} . As α increases, direct users of B_2 account for a larger share of activity, so their loss receives greater weight. Hence dCS/db_1 declines with α and eventually becomes negative. Panel (b) shows the analogous pattern for total welfare.

The figure highlights the role of network externalities. As in the two-sided model, a higher externality amplifies whichever force dominates: for small α the direct channel is negligible and ρ mainly magnifies the beneficial reallocation of demand toward the more prominent B_1 in the mediated channel, so the curves for the higher ρ lie above. For larger α the dominant force becomes the loss of usage on B_2 , whose value to its direct users falls more sharply the stronger are network effects. A higher ρ therefore shifts dCS/db_1 and dW/db_1 downward, so that the adverse effects of tying arise at lower values of α .

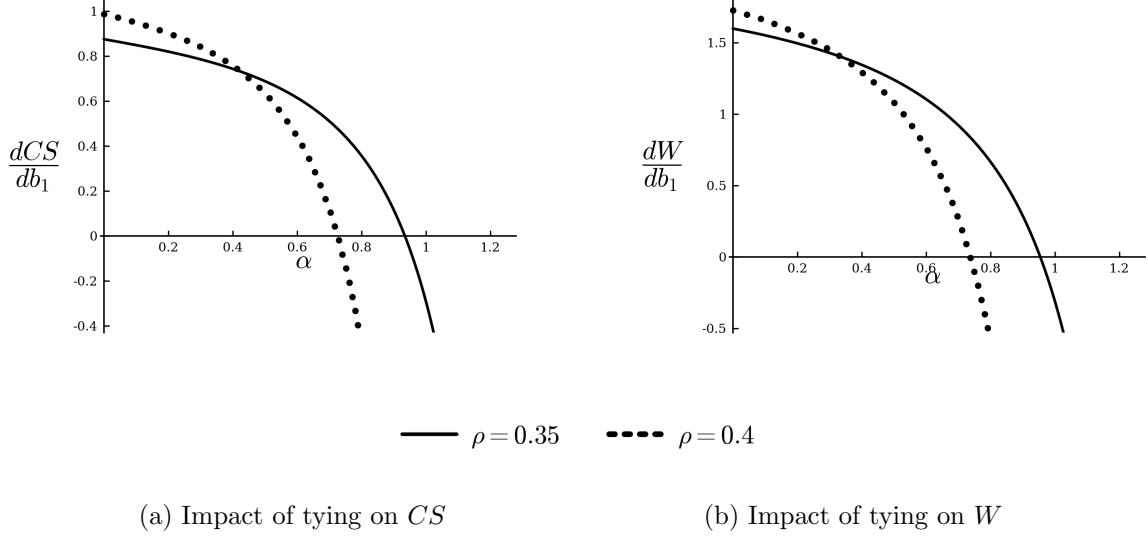


Figure 7: Welfare effects of tying - consumer-side network effects ($b_1 = b_2 = 0.2$, $t = 1.6$, and $\rho \in \{0.35, 0.4\}$)

7 Conclusions

Digital companies often integrate into their core platforms new services, or make more visible and accessible services they already offer, typically in competition with other companies. This practice, which is a form of tying, and of self-preferencing, reduces the inconvenience costs of the core platform users who also consume those services, thus providing a direct short-term benefit to them.

However, it might also have a detrimental impact. In our baseline two-sided model, tying tilts demand in the mediated channel (that is, of consumers on the core platform) away from the rivals and towards the platform's own service, thus reducing the attractiveness for third-party sellers to be on the rival platform. To the extent that the rival also serves consumers via a direct channel, this will decrease rival's consumer demand in that channel and will feed back negatively on its seller participation. If the direct channel is important enough, this negative externality might lead to a detrimental net welfare effect.

A similar mechanism might also take place in other environments, as we show in a one-sided version of the model with network externalities: tying shifts demand away from the rival, and this scale effect impacts negatively on consumers in the direct channel, and feeds back into the mediated channel by reinforcing the initial effect. Likewise, it might also affect investment decisions (not formalised in the current version of the paper): the fall in rival's demand in the mediated channel will reduce the incentive of the rival to invest in product improvement. To the extent that it also sells via a direct channel, the lower investment will be harmful to consumers accessing directly the rival's services. Again, tying's beneficial effect on consumers in the intermediated channel should be traded-off with this source of negative effects.²⁵

Tying is a practice that *reduces* inconvenience costs of the users of the services of the dominant core platform, and as such it might give rise to a trade-off between beneficial and anti-competitive effects. Instead, another form of self-preferencing, demotion, which consists of *increasing* the inconvenience costs of *rivals* (for instance, by making them less visible and more cumbersome to use, or degrading their interoperability with the core platform) will tend to have

²⁵We have assumed that the rival is already in the market. However, one could extend the model by having an initial stage where the rival decides whether to enter by paying a fixed sunk cost. Since tying by the integrated platform is optimal (due to imperfect rent extraction) even when the entrant is already in the market, the expectation of lower profits might deter entry, even absent any pre-commitment on tying.

unambiguous effects, decreasing consumer and total surplus both directly and indirectly.

We have also studied a non-discriminatory remedy to tying, often used by antitrust agencies. For instance, in *Google Shopping*, the European Commission initially considered a remedy whereby Google committed to giving access to its ‘Shopping unit’ (that is, the box where links and pictures of merchants appear prominently on Google’s Search Engine Results Pages) not only to its own service, but also to selected rival Comparison Shopping Services. In *Facebook Marketplace*, to comply with the European Commission’s abuse decision, Meta launched a commitment program consisting in the inclusion of listings from other platforms (e.g., eBay) directly within Marketplace.²⁶

This type of remedy is aimed at avoiding asymmetric treatment between the integrated and the independent service within the dominant platform’s ecosystem. In our model, it amounts to increasing the benefit for (or reducing the inconvenience costs incurred by) consumers in the mediated channel for both the dominant firm and its rival’s specialised platforms. We show that this policy unambiguously increases welfare and raises both platforms’ profits relative to the status quo, so the dominant platform has no incentive to pre-empt the remedy by abandoning integration altogether. It does not follow, however, that the remedy is self-enforcing: the platform strictly prefers asymmetric treatment to symmetric access, so the obligation still has to be imposed.

Our setting could be modified to account for other real-world situations, or to explore other issues. For instance, we could use our theoretical framework to analyse the acquisition of adjacent platforms to enrich the ecosystem, rather than assuming that the specialised service is already offered by the core platform and examining the impact of giving it increased prominence and making it accessible for users at a lower inconvenience cost.

References

- Amelio, Andrea and Bruno Jullien. 2012. “Tying and freebies in two-sided markets.” *International Journal of Industrial Organization* 30(5):436–446.
- Anderson, Simon and Özlem Bedre Defolie. 2024. “Hybrid platform model: Monopolistic competition and a dominant firm.” *The RAND Journal of Economics* 55(4):684–718.
- Armstrong, Mark. 2006. “Competition in two-sided markets.” *The RAND Journal of Economics* 37(3):668–691.
- Bar-Isaac, Heski. 2026. “Antitrust and Platforms: From “of” to “in”.” *Mimeo* .
- Bar-Isaac, Heski and Sandro Shelegia. 2026. “Monetizing Steering.” *Journal of Political Economy: Microeconomics* . Forthcoming.
- Caillaud, Bernard and Bruno Jullien. 2003. “Chicken & Egg: Competition among intermediation service providers.” *The RAND Journal of Economics* 34(2):309–328.
- Carlton, Dennis W and Michael Waldman. 2002. “The strategic use of tying to preserve and create market power in evolving industries.” *The RAND Journal of Economics* .
- Carlton, Dennis W and Michael Waldman. 2012. “Upgrades, switching costs and the leverage theory of tying.” *The Economic Journal* 122(561):675–706.
- Chen, Nan and Hsin-Tien Tsai. 2024. “Steering via algorithmic recommendations.” *The RAND Journal of Economics* 55(4):501–518.

²⁶In the same vein, in response of the European Commission’s scrutiny for self-preferencing its own products, assigning to them the Buy Box with higher frequency in its Marketplace platform, Amazon introduced a second Buy Box in its European marketplace, displaying an alternative seller’s offer alongside the default one.

- Choi, Jay Pil and Doh-Shin Jeon. 2021. “A leverage theory of tying in two-sided markets with nonnegative price constraints.” *American Economic Journal: Microeconomics* 13(1):283–337.
- Choi, Jay Pil, Doh-Shin Jeon, and Michael Dennis Whinston. 2026. “Tying with Network Effects.” *American Economic Review* 116(1):332–374.
- Condorelli, Daniele and Jorge Padilla. 2024. “Data-driven envelopment with privacy-policy tying.” *The Economic Journal* 134(658):515–537.
- de Cornière, Alexandre and Greg Taylor. 2019. “A model of biased intermediation.” *The RAND Journal of Economics* 50(4):854–882.
- de Cornière, Alexandre and Greg Taylor. 2024. “Anticompetitive bundling when buyers compete.” *American Economic Journal: Microeconomics* 16(1):293–328.
- de Cornière, Alexandre, Kinshuk Jerath and Greg Taylor. 2025. “Seller-Side Tying of Platform Services.” *TSE Working Paper* .
- Edelman, Benjamin and Julian Wright. 2015. “Price Coherence and Excessive Intermediation.” *The Quarterly Journal of Economics* 130(3):1283–1328.
- Eisenmann, Thomas, Geoffrey Parker and Marshall Van Alstyne. 2011. “Platform envelopment.” *Strategic Management Journal* 32(12):1270–1285.
- Farronato, Chiara, Andrey Fradkin and Alexander MacKay. 2023. “Self-preferencing at Amazon: evidence from search rankings.” *AEA Papers and Proceedings* 113:239–243.
- Franck, Jens-Uwe and Martin Peitz. 2023. “Market power of digital platforms.” *Oxford Review of Economic Policy* 39(1):34–46.
- Fumagalli, Chiara, Massimo Motta and Claudio Calcagno. 2018. *Exclusionary practices: The economics of monopolisation and abuse of dominance*. Cambridge University Press.
- Hagiu, Andrei and Julian Wright. 2024. “Marketplace Leakage.” *Management Science* 70(3):1529–1553.
- Hagiu, Andrei, Tat-How Teh and Julian Wright. 2022. “Should platforms be allowed to sell on their own marketplaces?” *The RAND Journal of Economics* 53(2):297–327.
- Hervas-Drane, Andres and Sandro Shelegia. 2025. “Retailer-led marketplaces.” *Management Science* 71(11):9650–9669.
- Jullien, Bruno, Alessandro Pavan and Marc Rysman. 2021. Two-sided markets, pricing, and network effects. In *Handbook of Industrial Organization*, ed. Kate Ho, Ali Hortaçsu and Alessandro Lizzeri. Vol. 4 Elsevier pp. 485–592.
- Krämer, Jan and Shiva Shekhar. 2025. “Regulating digital platform ecosystems through data sharing and data siloing: Consequences for innovation and welfare.” *MIS Quarterly* 49(1):123–154.
- Motta, Massimo. 2023. “Self-preferencing and foreclosure in digital markets: Theories of harm for abuse cases.” *International Journal of Industrial Organization* 90:102974.
- Padilla, Jorge, Joe Perkins and Salvatore Piccolo. 2022. “Self-preferencing in markets with vertically integrated gatekeeper platforms.” *The Journal of Industrial Economics* 70(2):371–395.

- Rhodes, Andrew, Jidong Zhou and Junjie Zhou. 2026. “Digital ecosystems and data regulation.” *Available at SSRN 5138129* .
- Rochet, Jean-Charles and Jean Tirole. 2003. “Platform competition in two-sided markets.” *Journal of the European Economic Association* 1(4):990–1029.
- Rochet, Jean-Charles and Jean Tirole. 2006. “Two-sided markets: A progress report.” *The RAND Journal of Economics* 37(3):645–667.
- Rosen, J. B. 1965. “Existence and Uniqueness of Equilibrium Points for Concave N-Person Games.” *Econometrica* 33(3):520–534.
- Waldfoegel, Joel. 2024. “Amazon self-preferencing in the shadow of the digital markets act.” *National Bureau of Economic Research Working Paper No. 32299*.
- Wang, Chengsi and Julian Wright. 2020. “Search Platforms: Showrooming and Price Parity Clauses.” *The RAND Journal of Economics* 51(1):32–58.
- Whinston, Michael D. 1990. “Tying, foreclosure, and exclusion.” *American Economic Review* 89(2):837—859.
- Zenryo, Yusuke. 2022. “Platform encroachment and own-content bias.” *The Journal of Industrial Economics* 70(3):684–710.

Appendix A

Two-sided model: proofs

Proof of Lemma 1

Differentiating the first-order conditions (2) and (3) with respect to b_1 and applying Cramer's rule yields

$$\begin{aligned}\frac{dr_1^*}{db_1} &= \frac{-\partial^2 \Pi_1 / \partial r_1 \partial b_1 \cdot \partial^2 \Pi_2 / \partial r_2^2 + \partial^2 \Pi_2 / \partial r_2 \partial b_1 \cdot \partial^2 \Pi_1 / \partial r_1 \partial r_2}{\Delta_r}, \\ \frac{dr_2^*}{db_1} &= \frac{-\partial^2 \Pi_2 / \partial r_2 \partial b_1 \cdot \partial^2 \Pi_1 / \partial r_1^2 + \partial^2 \Pi_1 / \partial r_1 \partial b_1 \cdot \partial^2 \Pi_2 / \partial r_2 \partial r_1}{\Delta_r},\end{aligned}$$

where $\Delta_r > 0$ is the standard regularity determinant of the fee game. Under Assumption 4 $\partial^2 \Pi_1 / \partial r_1 \partial b_1 \geq 0$ and $\partial^2 \Pi_2 / \partial r_2 \partial b_1 \leq 0$. Under strategic substitutes both terms in the numerator of dr_1^* / db_1 are non-negative and both terms in the numerator of dr_2^* / db_1 are non-positive, so $dr_1^* / db_1 \geq 0$ and $dr_2^* / db_1 \leq 0$.

Under strategic complementarity, the same conclusion follows under a *diagonal-dominance condition* on the fee-game Jacobian:

$$\frac{-\partial^2 \Pi_1 / \partial r_1^2}{\partial^2 \Pi_2 / \partial r_2 \partial r_1} > \frac{\partial^2 \Pi_1 / \partial r_1 \partial b_1}{-\partial^2 \Pi_2 / \partial r_2 \partial b_1} > \frac{\partial^2 \Pi_1 / \partial r_1 \partial r_2}{-\partial^2 \Pi_2 / \partial r_2^2},$$

with the inequalities evaluated at the equilibrium.

The condition above means that each platform's own-fee curvature ($\partial^2 \Pi_i / \partial r_i^2$, the slope of its own best-response, which is large in absolute value when own-fee marginal profitability falls steeply) outweighs the cross-fee strategic response ($\partial^2 \Pi_i / \partial r_i \partial r_j$, the slope of i 's best response in j 's fee).²⁷

Proof of Proposition 1

We proceed by steps.

Part (i).

Definition 1 implies

$$m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} > 0 \quad m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} < 0.$$

Under Assumption 1 and Assumption 2, we can immediately sign (9) and (10):

$$\frac{dS_1^*}{db_1} = \underbrace{\frac{1}{\Delta_S}}_{(+)} \left\{ \underbrace{g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right)}_{(+)\text{ by Definition 1}} \underbrace{\left(1 - m g_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha m g_2 \frac{\partial f_{2D}}{\partial s_2} \right)}_{(+)\text{ by Assumption 2(i)}} + \underbrace{m g_1 \frac{\partial f_{1M}}{\partial s_2}}_{(-)\text{ by Assumption 1(i)}} \underbrace{g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right)}_{(-)\text{ by Definition 1}} \right\}$$

and

$$\frac{dS_2^*}{db_1} = \underbrace{\frac{1}{\Delta_S}}_{(+)} \left\{ \underbrace{\left(1 - m g_1 \frac{\partial f_{1M}}{\partial s_1} \right)}_{(+)\text{ by Assumption 2(ii)}} \underbrace{g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right)}_{(-)\text{ by Definition 1}} + \underbrace{m g_2 \frac{\partial f_{2M}}{\partial s_1}}_{(-)\text{ by Assumption 1(i)}} \underbrace{g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right)}_{(+)\text{ by Definition 1}} \right\},$$

where $\Delta_S > 0$ by Assumption 2(ii).

²⁷This is the same diagonal strict concavity condition of Rosen (1965).

Part (ii)

Substituting (9)–(10) into (6)–(8) yields

$$\frac{dQ_{2D}^*}{db_1} = \alpha \frac{\partial f_{2D}}{\partial s_2} \frac{1}{\Delta_S} \left\{ \left(1 - mg_1 \frac{\partial f_{1M}}{\partial s_1} \right) g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) + mg_2 \frac{\partial f_{2M}}{\partial s_1} g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \right\}, \quad (\text{A-1})$$

$$\begin{aligned} \frac{dQ_{1M}^*}{db_1} = & \frac{\partial f_{1M}}{\partial b_1} + \frac{\partial f_{1M}}{\partial s_1} \frac{1}{\Delta_S} \left\{ g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \left(1 - mg_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha mg_2 \frac{\partial f_{2D}}{\partial s_2} \right) \right. \\ & \left. + mg_1 \frac{\partial f_{1M}}{\partial s_2} g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) \right\} \\ & + \frac{\partial f_{1M}}{\partial s_2} \frac{1}{\Delta_S} \left\{ \left(1 - mg_1 \frac{\partial f_{1M}}{\partial s_1} \right) g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) + mg_2 \frac{\partial f_{2M}}{\partial s_1} g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \right\}, \end{aligned} \quad (\text{A-2})$$

$$\begin{aligned} \frac{dQ_{2M}^*}{db_1} = & \frac{\partial f_{2M}}{\partial b_1} + \frac{\partial f_{2M}}{\partial s_1} \frac{1}{\Delta_S} \left\{ g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \left(1 - mg_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha mg_2 \frac{\partial f_{2D}}{\partial s_2} \right) \right. \\ & \left. + mg_1 \frac{\partial f_{1M}}{\partial s_2} g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) \right\} \\ & + \frac{\partial f_{2M}}{\partial s_2} \frac{1}{\Delta_S} \left\{ \left(1 - mg_1 \frac{\partial f_{1M}}{\partial s_1} \right) g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) + mg_2 \frac{\partial f_{2M}}{\partial s_1} g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \right\}, \end{aligned} \quad (\text{A-3})$$

where $\Delta_S > 0$ by assumption.

The total effect of tying on each of the above quantities is determined by two objects:

$$m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1}, \quad m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1},$$

whose signs are established by Definition 1: the first is positive and the second negative. Under the maintained assumptions, both braces in (A-1) are then negative (as shown in the proof of part (i)), and since $\alpha \partial f_{2D}/\partial s_2 > 0$ and $\Delta_S > 0$, we obtain $dQ_{2D}^*/db_1 < 0$. In (A-2), the first brace is positive and is multiplied by $\partial f_{1M}/\partial s_1 > 0$, while the second brace is negative and is multiplied by $\partial f_{1M}/\partial s_2 < 0$; adding the direct effect $\partial f_{1M}/\partial b_1 > 0$ yields $dQ_{1M}^*/db_1 > 0$. Symmetrically, in (A-3) the first brace (positive) is multiplied by $\partial f_{2M}/\partial s_1 < 0$ and the second brace (negative) by $\partial f_{2M}/\partial s_2 > 0$; adding $\partial f_{2M}/\partial b_1 < 0$ yields $dQ_{2M}^*/db_1 < 0$.

Proof of Lemma 2

Part (i). From (1), direct-channel demand depends on b_1 only through S_2^* , so

$$\frac{dQ_{2D}^*}{db_1} = \alpha \frac{\partial f_{2D}}{\partial s_2} \frac{dS_2^*}{db_1}.$$

By Assumption 1(i), $\partial f_{2D}/\partial s_2 > 0$. If seller participation on platform 2 is interior, then $g_2 > 0$ and Proposition 1 applies, so $dS_2^*/db_1 < 0$. Hence $dQ_{2D}^*/db_1 < 0$ for every $\alpha > 0$. Since $\partial f_{2D}/\partial s_2$ and dS_2^*/db_1 are bounded on any compact set of parameters, the product converges to zero as α tends to 0.

Part (ii). Suppose the equilibrium is such that $\kappa_2^* = m(Q_{2M}^* + Q_{2D}^*) - r_2^* = \bar{\kappa}$, so that $S_2^* = MG(\bar{\kappa}) = M$. Two observations are needed.

First, this configuration cannot be an interior equilibrium. If $\kappa_2^* > \bar{\kappa}$ then $g_2 = 0$ and $\partial \Pi_2 / \partial r_2 = S_2^* + r_2 g_2(\cdot) = M > 0$, so platform 2 strictly gains from raising r_2 . It does so until $\kappa_2^* = \bar{\kappa}$. At that point the left derivative of Π_2 with respect to r_2 equals $M > 0$ and the right derivative equals $M - r_2^* M g(\bar{\kappa}^-)(\cdot)$; when the latter is non-positive, $r_2^* = m q_2^* - \bar{\kappa}$ is a local maximum and the equilibrium sits at this kink with full participation. The first-order condition (3) therefore holds as an inequality, and the comparative statics of Proposition 1, which are derived from it, do not apply.

Secondly, at such a kink equilibrium the participation constraint $\kappa_2^* = \bar{\kappa}$ holds identically in a neighbourhood of b_1 , because platform 2 adjusts r_2 to preserve it. Hence $S_2^* = M$ for all b_1 in that neighbourhood, so

$$\frac{dS_2^*}{db_1} = 0 \quad \text{and, from Part (i),} \quad \frac{dQ_{2D}^*}{db_1} = 0.$$

Differentiating $r_2^* = m q_2^* - \bar{\kappa}$ and using $s_2 = M$ gives

$$\frac{dr_2^*}{db_1} = m \left(\frac{\partial f_{2M}}{\partial b_1} + \frac{\partial f_{2M}}{\partial s_1} \frac{dS_1^*}{db_1} \right) < 0,$$

which is negative by Assumption 1(i) and $dS_1^*/db_1 > 0$. Tying is thus absorbed entirely by the rival's fee.

Part (iii). By Part (i), $|dQ_{2D}^*/db_1|$ converges to zero as α tends to 0 and is strictly positive at every $\alpha > 0$ at which participation is interior. By Part (ii) it equals zero at every α at which participation is complete. Suppose there exists $\hat{\alpha} < \infty$ such that participation is complete for all $\alpha \geq \hat{\alpha}$. Then $|dQ_{2D}^*/db_1|$ is zero in the limit $\alpha \rightarrow 0$, strictly positive on $(0, \hat{\alpha})$, and zero on $[\hat{\alpha}, \infty)$, and is therefore non-monotone in α . Such an $\hat{\alpha}$ exists in the specification of Section 3.5, in which Q_{2D}^* increases without bound in α while $\bar{\kappa}$ is finite.

Proof of Proposition 4

Let us start by identifying the effect of demotion on equilibrium fees. Following the same steps as in Section 2, we determine the effect of demotion on seller fees. The argument below is the symmetric counterpart of Lemma 1.

Lemma 4 (Effect of demotion on seller fees). *Under Assumptions 1, 2, and 4:*

(i) *if r_1 and r_2 are strategic substitutes, then demotion raises platform 1's equilibrium seller fee and lowers platform 2's equilibrium seller fee:*

$$-\frac{dr_1^*}{db_2} > 0, \quad -\frac{dr_2^*}{db_2} < 0;$$

(ii) *if r_1 and r_2 are strategic complements, the same conclusion holds under the diagonal-dominance condition analogous to that in Lemma 1.*

The proof is as follows. Differentiating the first-order conditions (2) and (3) with respect to b_2 and applying Cramer's rule yields

$$\frac{dr_1^*}{db_2} = \frac{-\frac{\partial^2 \Pi_1}{\partial r_1 \partial b_2} \frac{\partial^2 \Pi_2}{\partial r_2^2} + \frac{\partial^2 \Pi_2}{\partial r_2 \partial b_2} \frac{\partial^2 \Pi_1}{\partial r_1 \partial r_2}}{\Delta_r},$$

$$\frac{dr_2^*}{db_2} = \frac{-\frac{\partial^2 \Pi_2}{\partial r_2 \partial b_2} \frac{\partial^2 \Pi_1}{\partial r_1^2} + \frac{\partial^2 \Pi_1}{\partial r_1 \partial b_2} \frac{\partial^2 \Pi_2}{\partial r_2 \partial r_1}}{\Delta_r},$$

where $\Delta_r > 0$ by assumption. Under Assumption 4,

$$\frac{\partial^2 \Pi_1}{\partial r_1 \partial b_2} \leq 0, \quad \frac{\partial^2 \Pi_2}{\partial r_2 \partial b_2} \geq 0.$$

Thus, an increase in b_2 weakly lowers platform 1's marginal profitability of raising r_1 and weakly raises platform 2's marginal profitability of raising r_2 . Equivalently, a demotion of platform 2, corresponding to a reduction in b_2 , raises platform 1's marginal incentive to increase its seller fee and lowers platform 2's marginal incentive to increase its seller fee.

Under strategic substitutes,

$$\frac{\partial^2 \Pi_1}{\partial r_1 \partial r_2} \leq 0, \quad \frac{\partial^2 \Pi_2}{\partial r_2 \partial r_1} \leq 0.$$

Together with own second-order conditions, this implies that both terms in the numerator of dr_1^*/db_2 are non-positive and both terms in the numerator of dr_2^*/db_2 are non-negative. Since $\Delta_r > 0$,

$$-\frac{dr_1^*}{db_2} \geq 0, \quad -\frac{dr_2^*}{db_2} \leq 0.$$

This proves part (i).

Under strategic complements,

$$\frac{\partial^2 \Pi_1}{\partial r_1 \partial r_2} \geq 0, \quad \frac{\partial^2 \Pi_2}{\partial r_2 \partial r_1} \geq 0.$$

The strategic-feedback terms then work against the direct marginal effects. The same sign pattern follows under the diagonal-dominance condition

$$\frac{-\partial^2 \Pi_1 / \partial r_1^2}{\partial^2 \Pi_2 / \partial r_2 \partial r_1} > \frac{-\partial^2 \Pi_1 / \partial r_1 \partial b_2}{\partial^2 \Pi_2 / \partial r_2 \partial b_2} > \frac{\partial^2 \Pi_1 / \partial r_1 \partial r_2}{-\partial^2 \Pi_2 / \partial r_2^2},$$

with all inequalities evaluated at the equilibrium and with the middle ratio well defined.

This condition is the demotion analogue of the diagonal-dominance condition used in the proof of Lemma 1. It requires each platform's own-fee curvature to dominate the cross-fee feedback induced by the rival's response. Hence the direct effect of a change in b_2 on each platform's marginal fee incentive is not reversed by strategic complementarity. Consequently,

$$-\frac{dr_1^*}{db_2} \geq 0, \quad -\frac{dr_2^*}{db_2} \leq 0.$$

This proves part (ii) of Lemma 4

Next, we look at the effect of demotion on seller participation. Differentiating the seller-entry conditions with respect to b_2 yields, in analogy with (5),

$$\frac{dS_1^*}{db_2} = g_1 \left(m \frac{dQ_{1M}^*}{db_2} - \frac{dr_1^*}{db_2} \right), \quad \frac{dS_2^*}{db_2} = g_2 \left(m \frac{dQ_{2M}^*}{db_2} + m \frac{dQ_{2D}^*}{db_2} - \frac{dr_2^*}{db_2} \right). \quad (\text{A-4})$$

The total derivatives of consumer quantities with respect to b_2 are

$$\frac{dQ_{1M}^*}{db_2} = \frac{\partial f_{1M}}{\partial b_2} + \frac{\partial f_{1M}}{\partial s_1} \frac{dS_1^*}{db_2} + \frac{\partial f_{1M}}{\partial s_2} \frac{dS_2^*}{db_2}, \quad (\text{A-5})$$

$$\frac{dQ_{2M}^*}{db_2} = \frac{\partial f_{2M}}{\partial b_2} + \frac{\partial f_{2M}}{\partial s_1} \frac{dS_1^*}{db_2} + \frac{\partial f_{2M}}{\partial s_2} \frac{dS_2^*}{db_2}, \quad (\text{A-6})$$

$$\frac{dQ_{2D}^*}{db_2} = \alpha \frac{\partial f_{2D}}{\partial s_2} \frac{dS_2^*}{db_2}. \quad (\text{A-7})$$

To state the result cleanly, we mirror Definition 1 from the tying analysis.

Definition 3 (Dominance of direct effects under demotion). *We say that the direct demand effects of demotion dominate the strategic fee effects if*

$$m \left| \frac{\partial f_{1M}}{\partial b_2} \right| > \left| \frac{dr_1^*}{db_2} \right|, \quad m \frac{\partial f_{2M}}{\partial b_2} > \frac{dr_2^*}{db_2}$$

Proposition 1 yields a 2×2 linear system in $(dS_1^*/db_2, dS_2^*/db_2)$ with the same coefficient matrix (determinant $\Delta_S > 0$) but right-hand-side brackets $g_J(m \partial f_{JM}/\partial b_2 - dr_J^*/db_2)$. By Assumption 1(i), $\partial f_{1M}/\partial b_2 < 0$ and $\partial f_{2M}/\partial b_2 > 0$; by Lemma 4, $dr_1^*/db_2 \leq 0$ and $dr_2^*/db_2 \geq 0$.

Applying Cramer's rule:

$$\frac{dS_1^*}{db_2} = \frac{1}{\Delta_S} \left\{ \underbrace{\left(1 - mg_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha mg_2 \frac{\partial f_{2D}}{\partial s_2}\right)}_{(+)\text{ by Assumption 2(i)}} \underbrace{g_1 \left(m \frac{\partial f_{1M}}{\partial b_2} - \frac{dr_1^*}{db_2}\right)}_{(-)\text{ by Definition 3}} + \underbrace{mg_1 \frac{\partial f_{1M}}{\partial s_2}}_{(-)\text{ by Assumption 1(i)}} \underbrace{g_2 \left(m \frac{\partial f_{2M}}{\partial b_2} - \frac{dr_2^*}{db_2}\right)}_{(+)\text{ by Definition 3}} \right\} < 0, \quad (\text{A-8})$$

$$\frac{dS_2^*}{db_2} = \frac{1}{\Delta_S} \left\{ \underbrace{\left(1 - mg_1 \frac{\partial f_{1M}}{\partial s_1}\right)}_{(+)\text{ by Assumption 2(i)}} \underbrace{g_2 \left(m \frac{\partial f_{2M}}{\partial b_2} - \frac{dr_2^*}{db_2}\right)}_{(+)\text{ by Definition 3}} + \underbrace{mg_2 \frac{\partial f_{2M}}{\partial s_1}}_{(-)\text{ by Assumption 1(i)}} \underbrace{g_1 \left(m \frac{\partial f_{1M}}{\partial b_2} - \frac{dr_1^*}{db_2}\right)}_{(-)\text{ by Definition 3}} \right\} > 0, \quad (\text{A-9})$$

since both terms in each expression carry the same sign and $\Delta_S > 0$. This establishes part (i).

For part (ii), substitute $dS_1^*/db_2 < 0$ and $dS_2^*/db_2 > 0$ into (A-5)–(A-7):

$$\begin{aligned} \frac{dQ_{1M}^*}{db_2} &= \frac{\partial f_{1M}}{\partial b_2} + \frac{\partial f_{1M}}{\partial s_1} \frac{dS_1^*}{db_2} + \frac{\partial f_{1M}}{\partial s_2} \frac{dS_2^*}{db_2} < 0 \\ \frac{dQ_{2M}^*}{db_2} &= \frac{\partial f_{2M}}{\partial b_2} + \frac{\partial f_{2M}}{\partial s_1} \frac{dS_1^*}{db_2} + \frac{\partial f_{2M}}{\partial s_2} \frac{dS_2^*}{db_2} > 0 \\ \frac{dQ_{2D}^*}{db_2} &= \alpha \frac{\partial f_{2D}}{\partial s_2} \frac{dS_2^*}{db_2} > 0, \end{aligned}$$

where all terms in each expression share the same sign, so $-dQ_{1M}^*/db_2 > 0$, $-dQ_{2M}^*/db_2 < 0$, and $-dQ_{2D}^*/db_2 < 0$.

Proof of Proposition 5

Writing the equilibrium profits as $\Pi_J^* = r_J^* S_J^*$, where all objects are the equilibrium values as functions of b_2 , the product rule gives

$$\frac{d\Pi_1^*}{db_2} = S_1^* \frac{dr_1^*}{db_2} + r_1^* \frac{dS_1^*}{db_2}, \quad (\text{A-10})$$

$$\frac{d\Pi_2^*}{db_2} = S_2^* \frac{dr_2^*}{db_2} + r_2^* \frac{dS_2^*}{db_2}, \quad (\text{A-11})$$

where dS_J^*/db_2 are the full equilibrium derivatives, incorporating both platforms' fee responses. By Lemma 4 and Proposition 4, under Definition 3: $dr_1^*/db_2 \leq 0$, $dr_2^*/db_2 \geq 0$, $dS_1^*/db_2 < 0$, and $dS_2^*/db_2 > 0$. Hence in (A-10) both terms are non-positive, with the second strictly negative, and in (A-11) both terms are non-negative, with the second strictly positive. Thus, $d\Pi_1^*/db_2 < 0$ and $d\Pi_2^*/db_2 > 0$ and, therefore, $d\Pi_1^*/d(-b_2) > 0$ and $d\Pi_2^*/d(-b_2) < 0$.

Proof of Proposition 6

Since $b_1 = b_{01} + b$ and $b_2 = b_{02} + b$ with b_{01}, b_{02} held fixed, $db_1/db = db_2/db = 1$, so for any equilibrium object X^* the chain rule gives (22), wherein the first term being the *tying component* and the second the *raising- b_2 component*.

The signs of the two components are established elsewhere in the paper: the b_1 component by Lemma 1 (fees) and Proposition 1 (participation and quantities), and the b_2 component by Lemma 4 and Proposition 4, read with reversed sign because those results are stated for a *decrease* in b_2 . Collecting them:

	component in b_1	component in b_2
r_1^* :	$dr_1^*/db_1 > 0$	$dr_1^*/db_2 < 0$
r_2^* :	$dr_2^*/db_1 < 0$	$dr_2^*/db_2 > 0$
S_1^* :	$dS_1^*/db_1 > 0$	$dS_1^*/db_2 < 0$
S_2^* :	$dS_2^*/db_1 < 0$	$dS_2^*/db_2 > 0$
Q_{1M}^* :	$dQ_{1M}^*/db_1 > 0$	$dQ_{1M}^*/db_2 < 0$
Q_{2M}^* :	$dQ_{2M}^*/db_1 < 0$	$dQ_{2M}^*/db_2 > 0$.

For each platform's own objects the own-shifter component is favourable and the rival-shifter component adverse; Definition 2 states precisely that the former dominates the latter in absolute value.

Part (i): seller participation. By (22) and the participation inequalities of Definition 2,

$$\frac{dS_1^*}{db} = \underbrace{\frac{dS_1^*}{db_1}}_{>0} + \underbrace{\frac{dS_1^*}{db_2}}_{<0} \geq 0, \quad \frac{dS_2^*}{db} = \underbrace{\frac{dS_2^*}{db_1}}_{<0} + \underbrace{\frac{dS_2^*}{db_2}}_{>0} \geq 0,$$

since in each expression the favourable component dominates the adverse one. By the same argument applied to fees,

$$\frac{dr_1^*}{db} = \frac{dr_1^*}{db_1} + \frac{dr_1^*}{db_2} \geq 0, \quad \frac{dr_2^*}{db} = \frac{dr_2^*}{db_1} + \frac{dr_2^*}{db_2} \geq 0,$$

which we use in the proof of Proposition 7.

Part (ii): quantities. For the direct channel, $Q_{2D}^* = \alpha f_{2D}(S_2^*)$, so

$$\frac{dQ_{2D}^*}{db} = \alpha \frac{\partial f_{2D}}{\partial s_2} \frac{dS_2^*}{db} \geq 0,$$

since $\alpha > 0$ and $\partial f_{2D}/\partial s_2 > 0$ (Assumption 1) and $dS_2^*/db \geq 0$ by part (i), with strict inequality whenever $dS_2^*/db > 0$. For the mediated quantities, (22) and the quantity inequalities of Definition 2 give

$$\frac{dQ_{1M}^*}{db} = \underbrace{\frac{dQ_{1M}^*}{db_1}}_{>0} + \underbrace{\frac{dQ_{1M}^*}{db_2}}_{<0} \geq 0, \quad \frac{dQ_{2M}^*}{db} = \underbrace{\frac{dQ_{2M}^*}{db_1}}_{<0} + \underbrace{\frac{dQ_{2M}^*}{db_2}}_{>0} \geq 0.$$

Proof of Proposition 7

In the baseline setting $\Pi_J^* = r_J^* S_J^*$, so the product rule gives the decomposition (23),

$$\frac{d\Pi_J^*}{db} = S_J^* \frac{dr_J^*}{db} + r_J^* \frac{dS_J^*}{db}, \quad J = 1, 2.$$

By the proof of Proposition 6, under Definition 2 the fee responses $dr_J^*/db \geq 0$ and the participation responses $dS_J^*/db \geq 0$; and $r_J^*, S_J^* \geq 0$ at an interior equilibrium. Hence both terms are

non-negative for $J = 1$ and for $J = 2$, giving $d\Pi_1^*/db \geq 0$ (part i) and $d\Pi_2^*/db \geq 0$ (part ii). Adding the two yields $d(\Pi_1^* + \Pi_2^*)/db \geq 0$ (part iii).

Seller surplus (used in the welfare discussion of Section 5). A type- κ seller on platform J earns $mq_J^* - r_J^* - \kappa$ and participates iff $\kappa \leq \hat{\kappa}_J \equiv mq_J^* - r_J^*$, with $S_J^* = G(\hat{\kappa}_J)$, $q_1 = Q_{1M}^*$, $q_2 = Q_{2M}^* + Q_{2D}^*$. Hence $SS_J = \int_0^{\hat{\kappa}_J} (\hat{\kappa}_J - \kappa) g(\kappa) d\kappa$, and since the integrand vanishes at $\kappa = \hat{\kappa}_J$,

$$\frac{dSS_J}{db} = \frac{d\hat{\kappa}_J}{db} G(\hat{\kappa}_J) = S_J^* \left(m \frac{dq_J^*}{db} - \frac{dr_J^*}{db} \right).$$

Differentiating $S_J^* = G(\hat{\kappa}_J)$ gives $dS_J^*/db = g_J(m dq_J^*/db - dr_J^*/db)$, so $m dq_J^*/db - dr_J^*/db = (1/g_J) dS_J^*/db$ and

$$\frac{dSS_J}{db} = \frac{S_J^*}{g_J} \frac{dS_J^*}{db} \geq 0,$$

since $S_J^* \geq 0$, $g_J > 0$, and $dS_J^*/db \geq 0$ (Proposition 6). Thus seller surplus on each platform weakly rises.

Welfare. Consumers pay no monetary price and every consumed quantity weakly rises (Proposition 6), while the convenience of both services and seller variety on both platforms increase. Under the own-dominates-cross demand structure of Assumption 1, consumer surplus is therefore non-decreasing. Combined with the non-decreasing seller surplus above and the non-negative profit effect of parts (i)–(ii), total welfare $W = CS + SS + \Pi_1^* + \Pi_2^*$ is non-decreasing in b .

One-sided model: comparative statics and proofs

Assumptions used in Section 6.1

Assumption 5 (Consumer-demand primitives). *For $i, j \in \{1, 2\}$, $i \neq j$, and for all relevant arguments,*

$$\begin{aligned} \frac{\partial f_{iM}}{\partial b_i} &> 0, & \frac{\partial f_{iM}}{\partial b_j} &< 0, & \frac{\partial f_{iM}}{\partial p_i} &< 0, & \frac{\partial f_{iM}}{\partial p_j} &> 0, \\ \frac{\partial f_{iM}}{\partial n_i} &> 0, & \frac{\partial f_{iM}}{\partial n_j} &< 0, & \frac{\partial f_{2D}}{\partial p_2} &< 0, & \frac{\partial f_{2D}}{\partial n_2} &> 0. \end{aligned}$$

Moreover, own effects weakly dominate cross effects in the mediated channel:

$$\frac{\partial f_{iM}}{\partial b_i} \geq \left| \frac{\partial f_{jM}}{\partial b_i} \right|, \quad \left| \frac{\partial f_{iM}}{\partial p_i} \right| \geq \frac{\partial f_{jM}}{\partial p_i}, \quad \frac{\partial f_{iM}}{\partial n_i} \geq \left| \frac{\partial f_{jM}}{\partial n_i} \right|.$$

Assumption 5 is the consumer-network analogue of the seller-side demand assumptions in Section 2. It says that own convenience, lower own prices, and own usage increase demand, whereas rival convenience, lower rival prices, and rival usage reduce it.

Assumption 6 (Stability of consumer-side network feedbacks). *For all relevant arguments,*

$$1 > \frac{\partial f_{1M}}{\partial n_1} + \left| \frac{\partial f_{1M}}{\partial n_2} \right|, \quad 1 > \frac{\partial f_{2M}}{\partial n_2} + \alpha \frac{\partial f_{2D}}{\partial n_2} + \left| \frac{\partial f_{2M}}{\partial n_1} \right|,$$

and

$$\Delta_n \equiv \left(1 - \frac{\partial f_{1M}}{\partial n_1} \right) \left(1 - \frac{\partial f_{2M}}{\partial n_2} - \alpha \frac{\partial f_{2D}}{\partial n_2} \right) - \frac{\partial f_{1M}}{\partial n_2} \frac{\partial f_{2M}}{\partial n_1} > 0.$$

Assumption 6 rules out explosive fulfilled-expectations feedbacks and guarantees local invertibility of the fixed-point system. It is a local regularity condition, not a global existence or uniqueness result.

Assumption 7 (Regularity of the price game). *At the equilibrium,*

$$\frac{\partial^2 \Pi_1}{\partial p_1^2} < 0, \quad \frac{\partial^2 \Pi_2}{\partial p_2^2} < 0,$$

and

$$\Delta_p \equiv \frac{\partial^2 \Pi_1}{\partial p_1^2} \frac{\partial^2 \Pi_2}{\partial p_2^2} - \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} > 0.$$

Assumption 8 (Super-modularity and sub-modularity). *At the equilibrium,*

$$\frac{\partial^2 \Pi_1}{\partial p_1 \partial b_1} \geq 0, \quad \frac{\partial^2 \Pi_2}{\partial p_2 \partial b_1} \leq 0.$$

Assumption 8 states that tying raises firm 1's marginal profitability of increasing its price and lowers firm 2's marginal profitability of increasing its price.

Reduced-form comparative statics

The Jacobian of (26) with respect to (n_1, n_2) is

$$J_n = \begin{pmatrix} 1 - \frac{\partial f_{1M}}{\partial n_1} & -\frac{\partial f_{1M}}{\partial n_2} \\ -\frac{\partial f_{2M}}{\partial n_1} & 1 - \frac{\partial f_{2M}}{\partial n_2} - \alpha \frac{\partial f_{2D}}{\partial n_2} \end{pmatrix}.$$

Its determinant is $\Delta_n > 0$ by Assumption 6. Therefore, by the implicit-function theorem, the fixed-point usage levels are locally differentiable.

Differentiating the fixed-point system with respect to b_1 gives

$$\frac{\partial N_1}{\partial b_1} = \frac{\left(1 - \frac{\partial f_{2M}}{\partial n_2} - \alpha \frac{\partial f_{2D}}{\partial n_2}\right) \frac{\partial f_{1M}}{\partial b_1} + \frac{\partial f_{1M}}{\partial n_2} \frac{\partial f_{2M}}{\partial b_1}}{\Delta_n} > 0,$$

and

$$\frac{\partial N_2}{\partial b_1} = \frac{\left(1 - \frac{\partial f_{1M}}{\partial n_1}\right) \frac{\partial f_{2M}}{\partial b_1} + \frac{\partial f_{2M}}{\partial n_1} \frac{\partial f_{1M}}{\partial b_1}}{\Delta_n} < 0.$$

Thus, holding prices fixed, tying raises firm 1's total usage and lowers firm 2's total usage.

Similarly,

$$\frac{\partial N_1}{\partial p_1} < 0, \quad \frac{\partial N_2}{\partial p_1} > 0, \quad \frac{\partial N_1}{\partial p_2} > 0, \quad \frac{\partial N_2}{\partial p_2} < 0.$$

These signs follow directly from Assumptions 5 and 6. In particular, no additional cross-price dominance condition is needed for the signs with respect to p_2 .

Finally, holding prices fixed, an increase in the relative size of the direct channel shifts usage toward firm 2:

$$\frac{\partial N_1}{\partial \alpha} < 0, \quad \frac{\partial N_2}{\partial \alpha} > 0,$$

for any given $f_{2D}(p_2, n_2) > 0$. Since $Q_{2D} = \alpha f_{2D}(p_2, N_2)$ we also have $\partial Q_{2D} / \partial \alpha > 0$. Moreover,

$$\frac{\partial Q_{2M}}{\partial \alpha} = \frac{\partial f_{2M}}{\partial n_1} \frac{\partial N_1}{\partial \alpha} + \frac{\partial f_{2M}}{\partial n_2} \frac{\partial N_2}{\partial \alpha} > 0.$$

Thus, for given prices, a larger direct channel increases firm 2's total usage and reduces firm 1's usage.

Price comparative statics under tying

Differentiating the two first-order conditions with respect to b_1 gives

$$\begin{pmatrix} \frac{\partial^2 \Pi_1}{\partial p_1^2} & \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} & \frac{\partial^2 \Pi_2}{\partial p_2^2} \end{pmatrix} \begin{pmatrix} \frac{dp_1^*}{db_1} \\ \frac{dp_2^*}{db_1} \end{pmatrix} = - \begin{pmatrix} \frac{\partial^2 \Pi_1}{\partial p_1 \partial b_1} \\ \frac{\partial^2 \Pi_2}{\partial p_2 \partial b_1} \end{pmatrix}.$$

Applying Cramer's rule yields

$$\frac{dp_1^*}{db_1} = \frac{-\frac{\partial^2 \Pi_1}{\partial p_1 \partial b_1} \frac{\partial^2 \Pi_2}{\partial p_2^2} + \frac{\partial^2 \Pi_2}{\partial p_2 \partial b_1} \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2}}{\Delta_p},$$

and

$$\frac{dp_2^*}{db_1} = \frac{-\frac{\partial^2 \Pi_2}{\partial p_2 \partial b_1} \frac{\partial^2 \Pi_1}{\partial p_1^2} + \frac{\partial^2 \Pi_1}{\partial p_1 \partial b_1} \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1}}{\Delta_p}.$$

By Assumptions 7 and 8, $\Delta_p > 0$, $\partial^2 \Pi_1 / \partial p_1^2 < 0$ and $\partial^2 \Pi_2 / \partial p_2^2 < 0$. Moreover, $\partial^2 \Pi_1 / \partial p_1 \partial b_1 \geq 0$ and $\partial^2 \Pi_2 / \partial p_2 \partial b_1 \leq 0$

If prices are strategic substitutes,

$$\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \leq 0, \quad \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} \leq 0,$$

then the numerator of dp_1^* / db_1 is non-negative and the numerator of dp_2^* / db_1 is non-positive. Hence

$$\frac{dp_1^*}{db_1} \geq 0, \quad \frac{dp_2^*}{db_1} \leq 0.$$

If prices are strategic complements, the cross-price terms work against the direct marginal effects. The same sign pattern follows under the sufficient diagonal-dominance condition

$$\frac{-\frac{\partial^2 \Pi_1}{\partial p_1^2}}{\frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1}} > \frac{\frac{\partial^2 \Pi_1}{\partial p_1 \partial b_1}}{-\frac{\partial^2 \Pi_2}{\partial p_2 \partial b_1}} > \frac{\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2}}{-\frac{\partial^2 \Pi_2}{\partial p_2^2}},$$

with all terms evaluated at the equilibrium and with the middle ratio well defined. The right-hand inequality makes the numerator of dp_1^* / db_1 positive, while the left-hand inequality makes the numerator of dp_2^* / db_1 negative. Thus, as in the baseline model in Section 2, we have:

$$\frac{dp_1^*}{db_1} > 0, \quad \frac{dp_2^*}{db_1} < 0.$$

Proof of Lemma 3

Suppose there are no network effects:

$$\frac{\partial f_{JM}}{\partial n_J} = \frac{\partial f_{JM}}{\partial n_{-J}} = \frac{\partial f_{2D}}{\partial n_2} = 0, \quad J = 1, 2.$$

Then

$$Q_{1M}^* = f_{1M}(b_1, b_2, p_1^*, p_2^*), \quad Q_{2M}^* = f_{2M}(b_1, b_2, p_1^*, p_2^*),$$

and $Q_{2D}^* = \alpha f_{2D}(p_2^*)$.

Define the strategic pricing effects as

$$SE_{1M} \equiv \frac{\partial f_{1M}}{\partial p_1} \frac{dp_1^*}{db_1} + \frac{\partial f_{1M}}{\partial p_2} \frac{dp_2^*}{db_1} \leq 0$$

$$SE_{2M} \equiv \frac{\partial f_{2M}}{\partial p_1} \frac{dp_1^*}{db_1} + \frac{\partial f_{2M}}{\partial p_2} \frac{dp_2^*}{db_1} \geq 0,$$

and

$$SE_{2D} \equiv \alpha \frac{\partial f_{2D}}{\partial p_2} \frac{dp_2^*}{db_1} \geq 0$$

where the sign follows from using $dp_1^*/db_1 \geq 0$, $dp_2^*/db_1 \leq 0$, and Assumption 5. The direct-channel derivative is

$$\frac{dQ_{2D}^*}{db_1} = SE_{2D} = \alpha \frac{\partial f_{2D}}{\partial p_2} \frac{dp_2^*}{db_1} \geq 0.$$

It is strictly positive whenever $dp_2^*/db_1 < 0$.

For the mediated channels,

$$\frac{dQ_{1M}^*}{db_1} = \frac{\partial f_{1M}}{\partial b_1} + SE_{1M},$$

and

$$\frac{dQ_{2M}^*}{db_1} = \frac{\partial f_{2M}}{\partial b_1} + SE_{2M}.$$

Therefore,

$$\frac{dQ_{1M}^*}{db_1} > 0 \iff \frac{\partial f_{1M}}{\partial b_1} > -SE_{1M},$$

and

$$\frac{dQ_{2M}^*}{db_1} < 0 \iff -\frac{\partial f_{2M}}{\partial b_1} > SE_{2M}.$$

These two inequalities are the precise sense in which the direct effects of tying dominate the strategic-pricing effects in the mediated channel.

Finally,

$$\frac{d(Q_{1M}^* + Q_{2M}^*)}{db_1} = \frac{\partial f_{1M}}{\partial b_1} + \frac{\partial f_{2M}}{\partial b_1} + SE_{1M} + SE_{2M}.$$

Thus total mediated demand increases if and only if

$$\frac{\partial f_{1M}}{\partial b_1} + \frac{\partial f_{2M}}{\partial b_1} > -(SE_{1M} + SE_{2M}).$$

This proves the lemma.

Proof of Proposition 8

Differentiating (29)–(30) with respect to b_1 and solving the resulting linear system yields

$$\begin{aligned} \frac{dQ_{1M}^*}{db_1} = \frac{1}{\Delta_n} & \left\{ \underbrace{\left(1 - \frac{\partial f_{2M}}{\partial n_2} - \alpha \frac{\partial f_{2D}}{\partial n_2}\right) \frac{\partial f_{1M}}{\partial b_1} + \frac{\partial f_{1M}}{\partial n_2} \frac{\partial f_{2M}}{\partial b_1}}_{\text{network-adjusted direct effect}} \right. \\ & \left. + \underbrace{\left(1 - \frac{\partial f_{2M}}{\partial n_2} - \alpha \frac{\partial f_{2D}}{\partial n_2}\right) S_{1M} + \frac{\partial f_{1M}}{\partial n_2} (S_{2M} + S_{2D})}_{\text{network-adjusted strategic-pricing effect}} \right\}, \end{aligned}$$

$$\begin{aligned} \frac{dQ_{2M}^*}{db_1} = \frac{1}{\Delta_n} & \left\{ \underbrace{\left(1 - \alpha \frac{\partial f_{2D}}{\partial n_2}\right) \left[\frac{\partial f_{2M}}{\partial n_1} \frac{\partial f_{1M}}{\partial b_1} + \left(1 - \frac{\partial f_{1M}}{\partial n_1}\right) \frac{\partial f_{2M}}{\partial b_1} \right]}_{\text{network-adjusted direct effect}} \right. \\ & \left. + \underbrace{\left(1 - \alpha \frac{\partial f_{2D}}{\partial n_2}\right) \left[\frac{\partial f_{2M}}{\partial n_1} S_{1M} + \left(1 - \frac{\partial f_{1M}}{\partial n_1}\right) S_{2M} \right] + \left[\left(1 - \frac{\partial f_{1M}}{\partial n_1}\right) \frac{\partial f_{2M}}{\partial n_2} + \frac{\partial f_{1M}}{\partial n_2} \frac{\partial f_{2M}}{\partial n_1} \right] S_{2D}}_{\text{network-adjusted strategic-pricing effect}} \right\}, \end{aligned}$$

and

$$\begin{aligned} \frac{dQ_{2D}^*}{db_1} = \frac{1}{\Delta_n} & \left\{ \underbrace{\alpha \frac{\partial f_{2D}}{\partial n_2} \left[\frac{\partial f_{2M}}{\partial n_1} \frac{\partial f_{1M}}{\partial b_1} + \left(1 - \frac{\partial f_{1M}}{\partial n_1}\right) \frac{\partial f_{2M}}{\partial b_1} \right]}_{\text{network-adjusted direct effect}} \right. \\ & \left. + \underbrace{\alpha \frac{\partial f_{2D}}{\partial n_2} \left[\frac{\partial f_{2M}}{\partial n_1} S_{1M} + \left(1 - \frac{\partial f_{1M}}{\partial n_1}\right) S_{2M} \right] + \left[\left(1 - \frac{\partial f_{1M}}{\partial n_1}\right) \left(1 - \frac{\partial f_{2M}}{\partial n_2}\right) - \frac{\partial f_{1M}}{\partial n_2} \frac{\partial f_{2M}}{\partial n_1} \right] S_{2D}}_{\text{network-adjusted strategic-pricing effect}} \right\}. \end{aligned}$$

The network-adjusted direct effect is positive for Q_{1M}^* , negative for Q_{2M}^* , and negative for Q_{2D}^* . The strategic-pricing blocks have the opposite signs: they dampen the increase in Q_{1M}^* and mitigate the contractions of Q_{2M}^* and Q_{2D}^* . Therefore, if the network-adjusted direct effects dominate the corresponding network-adjusted strategic-pricing effects for each of the quantities, then

$$\frac{dQ_{1M}^*}{db_1} > 0, \quad \frac{dQ_{2M}^*}{db_1} < 0, \quad \frac{dQ_{2D}^*}{db_1} < 0.$$

This proves Proposition 8.

Appendix B: Illustrative models

B1. The two-sided model

This appendix provides the additional formal details used in the numerical exercise of Section 3.5. The primitive demand functions derived in Section 3.5 satisfy Assumption 1. For the mediated-channel demands, for $J = 1, 2$,

$$\frac{\partial q_{JM}}{\partial b_J} = \frac{3}{2t} > 0, \quad \frac{\partial q_{-J,M}}{\partial b_J} = -\frac{1}{2t} < 0,$$

and

$$\frac{\partial q_{JM}}{\partial s_J} = \frac{3\omega}{2t} > 0, \quad \frac{\partial q_{-J,M}}{\partial s_J} = -\frac{\omega}{2t} < 0.$$

Thus, in the mediated channel, own effects dominate cross effects. Moreover, for the direct-channel demand,

$$\frac{\partial q_{2D}}{\partial s_2} = \frac{2\alpha\omega}{t} > 0.$$

Hence the sign restrictions in Assumption 1 hold for $t > 0$, $\omega > 0$, and $\alpha > 0$.

Given the channel demands, the total demands relevant for seller participation are

$$q_1 = q_{1M}, \quad q_2 = q_{2M} + q_{2D}.$$

Since sellers' fixed costs are uniformly distributed on $[0, K]$, seller participation satisfies

$$s_J = M \frac{mq_J - r_J}{K}, \quad J = 1, 2.$$

Throughout the numerical exercise we focus on interior equilibria, so that

$$0 < s_J < M, \quad J = 1, 2.$$

Substituting the channel demands into the seller-participation equations gives the following linear system:

$$s_1 = \frac{M}{K} \left[\frac{m(t+2+3b_1-b_2+3\omega s_1-\omega s_2)}{2t} - r_1 \right], \quad (\text{B-1})$$

and

$$s_2 = \frac{M}{K} \left[\frac{m(t+2+3b_2-b_1+4\alpha-\omega s_1+(3+4\alpha)\omega s_2)}{2t} - r_2 \right]. \quad (\text{B-2})$$

Solving this system yields the seller-participation levels as functions of the seller fees. For platform B_1 ,

$$S_1(r_1, r_2) = \frac{\left[\frac{M}{K} \left(\frac{m(t+2+3b_1-b_2)}{2t} - r_1 \right) \right] \left(1 - \frac{(3+4\alpha)Mm\omega}{2tK} \right) - \frac{Mm\omega}{2tK} \left[\frac{M}{K} \left(\frac{m(t+2+3b_2-b_1+4\alpha)}{2t} - r_2 \right) \right]}{\left(1 - \frac{3Mm\omega}{2tK} \right) \left(1 - \frac{(3+4\alpha)Mm\omega}{2tK} \right) - \left(\frac{Mm\omega}{2tK} \right)^2}. \quad (\text{B-3})$$

For platform B_2 ,

$$S_2(r_1, r_2) = \frac{\left(1 - \frac{3Mm\omega}{2tK} \right) \left[\frac{M}{K} \left(\frac{m(t+2+3b_2-b_1+4\alpha)}{2t} - r_2 \right) \right] - \frac{Mm\omega}{2tK} \left[\frac{M}{K} \left(\frac{m(t+2+3b_1-b_2)}{2t} - r_1 \right) \right]}{\left(1 - \frac{3Mm\omega}{2tK} \right) \left(1 - \frac{(3+4\alpha)Mm\omega}{2tK} \right) - \left(\frac{Mm\omega}{2tK} \right)^2}. \quad (\text{B-4})$$

The equilibrium channel quantities then follow by substituting $S_1(r_1, r_2)$ and $S_2(r_1, r_2)$ in the demand functions:

$$Q_{1M}(r_1, r_2) = \frac{t + 2 + 3b_1 - b_2 + 3\omega S_1(r_1, r_2) - \omega S_2(r_1, r_2)}{2t},$$

$$Q_{2M}(r_1, r_2) = \frac{t + 2 + 3b_2 - b_1 + 3\omega S_2(r_1, r_2) - \omega S_1(r_1, r_2)}{2t},$$

and

$$Q_{2D}(r_1, r_2) = \frac{2\alpha(1 + \omega S_2(r_1, r_2))}{t}.$$

Total demand faced by platform B_2 is therefore

$$Q_2(r_1, r_2) = Q_{2M}(r_1, r_2) + Q_{2D}(r_1, r_2),$$

while total demand faced by platform B_1 is

$$Q_1(r_1, r_2) = Q_{1M}(r_1, r_2).$$

The platforms' profits are $\Pi_1(r_1, r_2) = r_1 S_1(r_1, r_2)$ and $\Pi_2(r_1, r_2) = r_2 S_2(r_1, r_2)$, respectively.

We then solve for the equilibrium seller fees (r_1^*, r_2^*) and then obtain the equilibrium seller participation.²⁸

$$S_J^* = S_J(r_1^*, r_2^*), \quad J = 1, 2,$$

and

$$Q_{1M}^* = Q_{1M}(r_1^*, r_2^*), \quad Q_{2M}^* = Q_{2M}(r_1^*, r_2^*), \quad Q_{2D}^* = Q_{2D}(r_1^*, r_2^*).$$

Assumption 2 requires local stability of the seller-participation fixed point. In the illustrative example, the two diagonal stability conditions reduce to

$$1 - \frac{3Mm\omega}{2tK} > 0, \quad 1 - \frac{(3 + 4\alpha)Mm\omega}{2tK} > 0.$$

The determinant condition is

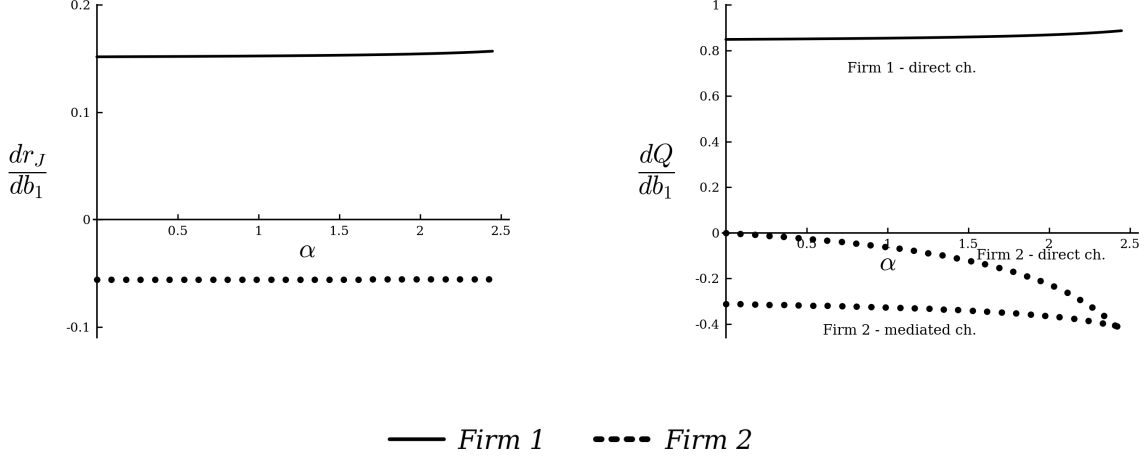
$$\left(1 - \frac{3Mm\omega}{2tK}\right) \left(1 - \frac{(3 + 4\alpha)Mm\omega}{2tK}\right) - \left(\frac{Mm\omega}{2tK}\right)^2 > 0. \quad (\text{B-5})$$

These conditions are checked numerically over the full range of α reported in the figures.

The expressions for consumer surplus, seller surplus, and total welfare are defined in Section 3.5.²⁹ Figure B-1 shows that tying increases platform B_1 's equilibrium seller fee and reduces platform B_2 's equilibrium seller fee. It also expands B_1 's mediated quantity, while reducing B_2 's quantity in both the mediated and the direct channel.

²⁸In the numerical exercise, we also check that the equilibrium is robust to a deviation in which platform B_2 withdraws from the mediated channel and serves only those consumers that are in the direct channel.

²⁹The impact of tying on consumer surplus, seller surplus, and total welfare is reported in the main text. Here, for completeness, we report the corresponding effects on equilibrium seller fees and quantities. The full algebraic expressions obtained after substitution are omitted for brevity. All reported derivatives are evaluated at an interior equilibrium and only over the range of α for which our maintained assumptions hold and there is an interior equilibrium..



(a) Impact of tying on seller fees

(b) Impact of tying on quantities

Figure B-1: Impact of tying on seller fees and quantities ($b_1 = b_2 = 0.2$, and $t = 2$)

B2. The model with consumer-side network effects

We now adapt the illustrative Hotelling environment of Section 3.5 to the model with consumer-side network effects. Platforms now charge consumer prices and network effects operate through total usage.

On the mediated channel, a consumer located at θ obtains

$$U_{JM}(\theta) = 1 + b_J + \rho n_J - t|l_J - \theta| - p_J, \quad J = 1, 2.$$

On the direct channel, a consumer who accesses B_2 directly obtains

$$U_{2D}(\theta) = 1 + \rho n_2 - t|1 - \theta| - p_2.$$

The term ρn_J captures the consumer-side network benefit from joining a platform with total usage n_J , with $\rho \geq 0$.

The cutoffs are therefore

$$\theta_1 = \frac{1 + b_1 - p_1 + \rho n_1}{t}, \quad \theta_2 = 1 + \frac{1 + b_2 - p_2 + \rho n_2}{t},$$

and

$$\theta_{12} = \frac{t + b_1 - b_2 - p_1 + p_2 + \rho n_1 - \rho n_2}{2t},$$

and mediated demands are $q_{1M} = \theta_1 + \theta_{12}$, and $q_{2M} = \theta_2 - \theta_{12}$, while direct-channel demand is

$$q_{2D} = \frac{2\alpha(1 - p_2 + \rho n_2)}{t}.$$

Usage levels are determined by fulfilled expectations, that is $n_1 = q_{1M}$, $n_2 = q_{2M} + q_{2D}$. Substituting the demand expressions above gives the fixed-point system

$$n_1 = \frac{t + 2 + 3b_1 - b_2 - 3p_1 + p_2 + 3\rho n_1 - \rho n_2}{2t},$$

and

$$n_2 = \frac{t + 2 + 3b_2 - b_1 + p_1 - 3p_2 + 3\rho n_2 - \rho n_1}{2t} + \frac{2\alpha(1 - p_2 + \rho n_2)}{t}.$$

Solving this linear system yields the reduced-form usage levels

$$N_1 = N_1(b_1, b_2, p_1, p_2, \alpha; \rho), \quad N_2 = N_2(b_1, b_2, p_1, p_2, \alpha; \rho).$$

The reduced-form channel quantities are then

$$Q_{1M}(p_1, p_2) = \frac{t + 2 + 3b_1 - b_2 - 3p_1 + p_2 + 3\rho N_1 - \rho N_2}{2t},$$

$$Q_{2M}(p_1, p_2) = \frac{t + 2 + 3b_2 - b_1 + p_1 - 3p_2 + 3\rho N_2 - \rho N_1}{2t},$$

and

$$Q_{2D}(p_1, p_2) = \frac{2\alpha(1 - p_2 + \rho N_2)}{t}.$$

The fixed-point system is locally stable whenever the feedback from usage to demand is sufficiently weak. In the numerical exercise, we verify this condition over the full range of α reported in the figures.

The platform profit functions are

$$\Pi_1(p_1, p_2) = p_1 Q_{1M}(p_1, p_2),$$

and

$$\Pi_2(p_1, p_2) = p_2(Q_{2M}(p_1, p_2) + Q_{2D}(p_1, p_2)).$$

The equilibrium prices (p_1^*, p_2^*) solve the first-order conditions of this price-setting game. Equilibrium quantities are then

$$Q_{1M}^* = Q_{1M}(p_1^*, p_2^*), \quad Q_{2M}^* = Q_{2M}(p_1^*, p_2^*), \quad Q_{2D}^* = Q_{2D}(p_1^*, p_2^*).$$

Consumer surplus is computed by integrating the utility of all consumers who join a platform:

$$CS = \int_{-\theta_1}^{\theta_{12}} U_{1M}(\theta; N_1^*, p_1^*) d\theta + \int_{\theta_{12}}^{\theta_2} U_{2M}(\theta; N_2^*, p_2^*) d\theta + \alpha \int_{\theta_D^-}^{\theta_D^+} U_{2D}(\theta; N_2^*, p_2^*) d\theta,$$

where all cutoffs are evaluated at the equilibrium usage levels and prices. The direct-channel cutoffs are

$$\theta_D^- = 1 - \frac{1 - p_2^* + \rho N_2^*}{t}, \quad \theta_D^+ = 1 + \frac{1 - p_2^* + \rho N_2^*}{t}.$$

A numerical simulation on the welfare effects of tying is reported in Figure 7.