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RISK, CAPITAL AND PROFIT IN INSURANCE

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RISK, CAPITAL AND PROFIT IN INSURANCE

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Summary

An insurer remains solvent as long as assets exceed liabilities in value. Both assets and liabilities fluctuate in value in an unforeseen manner. An insurer with no margin in the value of its assets over its liabilities is therefore exposed to failure.

The chance of failure is reduced if such a margin is created. This margin may be recognised as **net assets**, or **capital**. Capital is not a costless commodity. As will be seen in Section 6, it needs to be financed partly from premiums paid by policy owners. The components of premium providing this finance will appear as **profit margins**.

Thus, there is a fundamental relationship between solvency, capital and profit.

Policy owners, taken in the aggregate, suffer a loss, consisting of unpaid claims, when their insurer becomes insolvent. They will therefore be willing to contribute some level of profit margin to support capital which protects against insolvency.

There is, however, a limit to the level of capital they will willingly support. This level will be reached when the incremental cost of capital (in terms of premium margin) exceeds the value provided by it in increased policy owner security.

In a free and rational insurance market, levels of capitalisation and profit margins should settle, at least on average over a medium term, at values which reflect this balance between security and the cost of providing it.

One may argue that these levels of capitalisation and profit are related to the US concept of **fair pricing**. The quasi-legal framework of Rate Hearings has developed a methodology for pricing insurance which permits insurers to charge profit margins providing an assessed fair return on the capital deemed necessary to support the business underwritten. Arguably, these levels of capitalisation and profit might be those which would arise in a free market equilibrium.

In order to understand fully the relationship between solvency, capital and profit, it is necessary first to understand:

- the sources of risk which threaten solvency;
- the relation between capital and solvency;
- market levels of capitalisation;
- the relation between capital and profit margins.

Section 3 of the paper examines the sources of solvency risk. In this context, risk means relative uncertainty in the quantum of net assets remaining after present assets, together with investment return generated by them, have been used to pay all liabilities in respect of losses incurred to date.

This risk is decomposed into three major components:

- liability risk;
- asset risk;
- asset-liability interaction risk.

Asset risk is decomposed further into:

- asset mismatch risk (relating to the mismatch between assets and liabilities);
- asset performance risk for various asset sectors;
- asset mismatch-performance interaction risk.

Section 4 examines the effect of risk (as defined above) on the capital, and therefore total assets, required to achieve a particular level of security, i.e. particular chance of remaining solvent.

It is found that required assets increase with increasing liability risk. However, the effect of increased asset risk on required capitalisation is more complex. The increased asset risk increases the expected investment return, hence decreasing the discounted value of liabilities. But this is accompanied by increased volatility of investment returns and so, in some circumstances, increased risk that assets will be insufficient to meet liabilities.

It is found that the net result of these two effects depends on the level of security required. When security is high (low), increased asset risk requires an increased (decreased) volume of assets.

Section 5 relates an insurer's equilibrium capital base to the probability of solvency it provides. It is seen that the market equilibrium leads to, at least in first approximation, equal probabilities of solvency across insurers. It follows from this that the equilibrium capitalisation of an insurer depends on the risks, both liability and asset risks, to which that insurer is subject. The precise relationship is given in Section 5.

Section 6 examines the relation between insurer's capital base and its profit margin. Some results from the literature are reviewed, and used to conclude that the equilibrium profit margin is approximated by the Myers-Cohn margin calculated on the basis of the equilibrium capitalisation discussed in Section 5.

This result addresses one of the unsolved problems of insurance pricing, namely the capital base which should be allowed for in fair pricing of the Myers-Cohn (or similar) type? The suggestion made here is that it is the base which would emerge in competitive equilibrium, with Section 5 indicating how that capital base would vary across insurers with different distributions of underwriting by line of business.

With some qualifications, as discussed in Section 6, conclusions of this sort can be extrapolated to hypothetical single line insurers to determine the capital base, and hence profit margin, associated with a particular line. This is a form of solution to the problem of **allocation of capital by line.**

This suggestion is taken up in Section 7. Formulas given there indicate how the market average capitalisation ratio can be transformed to that applicable to a single line by reference to the asset and liability risks of:

- that single line; and
- the market in aggregate;

respectively. It is seen that, at least to first approximation, the equilibrium capitalisation of a single line of business increases with the total risk of that line.

The results of Sections 5 and 7 are illustrated by means of a numerical example, based on real data, in Section 8. The capitalisation of the market in aggregate is estimated, as are the associated asset and liability risks. The asset and liability risks in respect of individual lines are also estimated, and used to estimate the equilibrium capitalisation of those lines.

The example inevitably involves some assumptions about areas of the market for which numerical evidence is lacking or deficient. The sensitivity of the results to these assumptions is tested. It is found that, while variation of the assumptions causes some variation in the results, the general shape of capitalisation across the different lines of business remains much the same.

1. Introduction

An insurer remains solvent as long as assets exceed liabilities in value. Both assets and liabilities fluctuate in value in an unforeseen manner. An insurer with no margin in the value of its assets over its liabilities is therefore exposed to failure.

The chance of failure is reduced if such a margin is created. This margin may be recognised as **net assets**, or **capital**. Capital is not a costless commodity. As will be seen in Section 6, it needs to be financed partly from premiums paid by policy owners. The components of premium providing this finance will appear as **profit margins**.

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There is, however, a limit to the level of capital they will willingly support. This level will be reached when the incremental cost of capital (in terms of premium margin) exceeds the value provided by it in increased policy owner security.

In a free and rational insurance market, levels of capitalisation and profit margins should settle, at least on average over a medium term, at values which reflect this balance between security and the cost of providing it.

In order to understand fully the relationship between solvency, capital and profit, it is necessary first to understand:

- the sources of risk which threaten solvency;
- the relation between capital and solvency;
- market levels of capitalisation;
- the relation between capital and profit margins.

These subjects are examined in Sections 3, 4, 5 and 6 respectively.

The calculation of equilibrium capitalisation of insurers with different distributions of underwriting by line of business, and the associated profit margins, addresses one of the unsolved problems of insurance pricing. This concerns the amount of capital which should be regarded as supporting a line of business ("allocation of capital") in so-called "fair-pricing" of that line. This subject is pursued in Section 7. A numerical example is given in Section 8.

2. Basic framework and notation

In describing the financial structure of the insurance company analysed in this paper, consider the sequence of **accident years** to which business underwritten relates. Let accident year be denoted by i. It is convenient to speak in terms of years, but any other unit of time may be substituted without changing any of the reasoning which follows.

The experience of accident year i will emerge over the years i, $i+1$, etc. These will be referred to as **development years** 0, 1, etc. (of accident year i). In the following, various quantities will be indexed by subscripts i and/or j to indicate the accident and/or development periods to which they relate.

Note that the **calendar period** corresponding to (i, j) is $i+j$, which will be denoted by k. Sometimes quantities will be suffixed with this time index.

Sometimes only i or j will occur as a suffix. This will indicate that the other time index has been integrated out. For example, for some generic variable X,

$$X_i = \sum_{j=0}^{\infty} X_{ij}$$

Let

P_{ij}	=	gross premium income;
a_{ij}	=	acquisition costs expressed as a proportion of gross premium;
C_{ij}	=	paid losses (stochastic quantity);
e_{ij}	=	loss adjustment expenses expressed as a proportion of paid losses;
τ	=	tax rate.

A few derived quantities are now defined:

$$\eta_i = \sum_{j=0}^{\infty} \left[P_{ij} (1-a_{ij}) - C_{ij} (1+e_{ij})\right] / P_i ;$$

$$\gamma_{ij} = E\left[C_{ij}\right] ;$$

with E denoting the expectation operator.

As a matter of convenience, losses payable in each future period are assumed paid **at the end of that period.**

Acquisition costs are assumed payable at the time of receipt of the premium to which they relate. Reinsurance is ignored.

Let K denote the most recent **past** value of k. Any occurrence of $k > K$ will tacitly refer to past accident periods $i \le K$, e.g.

$$C_k = \sum_{\substack{i+j=k \\ i \le K}} C_{ij} .$$

Now consider the insurance company's **balance sheet** at the end of period K. This will take the following highly simplified form.

ASSETS
Investments

LIABILITIES
Technical liabilities

NET ASSETS

It will be convenient to consider investments as being of two types, according to whether or not they involve defined cash flows. If they do, they will be referred to here (somewhat imprecisely, but succinctly) as **fixed interest investments**. Those without defined cash flows, e.g. shares, property, will be referred to, again with some imprecision, as **volatile investments**.

Let

A^F	=	market value of fixed interest investments;
A^V	=	market value of volatile investments;
A	=	$A^F + A^V$ = market value of all investments.

Consider now the defined cash flows of investments A^F. Again as a matter of convenience (as with paid losses), asset cash flows payable in each future year will be assumed paid at the end of that year. Let

G_k	=	cash flow **due to be generated** by fixed interest assets A^F in future period k (a non-stochastic quantity);
F_k	=	corresponding cash flow **actually generated** (a stochastic quantity, due to credit risk);
ϕ_k	=	$E[F_k]$.

Consider future **rates of return** (RORs) on the different investment types. Let

i_k^V	=	ROR on volatile investments A^V in period k, including all income and all capital appreciation, realised or unrealised.

These are stochastic quantities conditioned by the information available at end of period K.

Define

$$r_k^V = 1 + i_k^V,$$

$$R_{kl}^V = r_{k+1}^V\, r_{k+2}^V \ldots r_l^V,$$

with the convention that $R_{kk}^V = 1$.

Also define

$$\rho_{kl}^V = E\left[R_{kl}^V\right],$$

with $\rho_{k-1,k}^V = E\left[r_k^V\right]$ abbreviated to just ρ_k^V.

The ρ_{kl}^V are accumulation factors calculated at risk-adjusted RORs applicable to volatile investments.

Now let

r_k^F = expected ROR in respect of a 1-year fixed interest investment made at the start of period k, risk adjusted to reflect the chance of default;

and define i_k^F, R_{kl}^F, ρ_{kl}^F similarly to i_k^V, R_{kl}^V, ρ_{kl}^V.

Note that the accumulation factors ρ_{kl}^F connect fixed interest asset values to their associated cash flows:

$$\mathrm{A}^F \rho_{KT}^F = \sum_{k>K} \phi_k \rho_{kT}^F \tag{2.1}$$

where $T = \max\{k\colon \phi_k, C_k \neq 0\}$, and provided that the i_k^F are stochastically independent of default risk. It will be assumed that $T < \infty$ throughout this paper.

A sometimes more convenient form of (2.1) is:

$$\mathrm{A}^F = \sum_{k>K} \mathrm{v}_k^F \phi_k, \tag{2.2}$$

with

$$\mathrm{v}_k^F = \rho_{kT}^F / \rho_{KT}^F\,. \tag{2.3}$$

Similarly, define

$$\nu_k^V = \rho_{kT}^V / \rho_{KT}^V \,.$$

It will be assumed that:

$$Cov\left[C_k, X_l\right] = 0 \textit{ for all } k,\ l > K, \tag{2.4}$$

where X_l can equal i_l^F, i_l^V or F_l, i.e. assets and liabilities are stochastically independent.

3. Solvency risk and its sources

3.1 Solvency risk

Let

L_T = accumulated value of liabilities at end of period T.

This quantity will be stochastic because of uncertainty in both liability cash flows and asset values. It will be discussed further in Section 3.2.

For the purpose of the present sub-section, simply let

$$\Lambda_T = E\left[L_T\right]. \tag{3.1}$$

Similarly, define

A_T = value of assets accumulated to end of period T.

Again this is a stochastic quantity, whose value will depend on the stochastic RORs included in the accumulation. This will be discussed in Section 3.3.

Define

$S_T = A_T - L_T$ = surplus of assets over liabilities accumulated to end of period T,

and

$$\Sigma_T = E\left[S_T\right] = E\left[A_T\right] - \Lambda_T. \tag{3.2}$$

Considering just liabilities incurred up to the end of period K, the insurer will remain solvent if $S_T \geq 0$. However S_T is stochastic, and unknown at that time. The insurer will be regarded as solvent if $\Sigma_T \geq 0$.

3.2 Definition of liability and asset values

The liability L must be in some sense the accumulated value of the sequence of paid losses $\{C_k\}$. A question arises over the definition of the accumulation factors when risky assets are held.

The answer to this question depends on investment strategy. For example, if the strategy were to cover the liability existing at end of period K by realising only volatile assets until these were exhausted, and then realising fixed interest assets, the discount factors would be different from those applicable to the reverse situation in which fixed interest were realised first, and then volatile.

Strategies such as this are also difficult to deal with analytically, because the value of assets, as actually realised to pay claims, depends on the cost of those claims.

The present paper will adopt a simple strategy whereby all drawdowns on the asset portfolio to pay claims will involve the same proportions by asset sector. Specifically, the amount C_k will be realised in the proportions A^F/A from fixed interest assets and A^V/A from volatile assets.

This strategy may sometimes be viewed as over-simplified. For example, it is more natural for a running off operation than for a going concern. For present purposes, however, it does avoid obscuring the simple valuation concepts with structural complexities.

The investment strategy can always be changed, and the reasoning which follows will change correspondingly. The concepts will remain in place, but the complexity may increase.

Under the investment strategy adopted here,

$$L_T = \sum_{k>K} C_k[(\mathrm{A}^F/\mathrm{A})R_{kT}^F + (\mathrm{A}^V/\mathrm{A})R_{kT}^V]. \tag{3.3}$$

This is a stochastic quantity since the C_k, R_{kT}^F and R_{kT}^V are stochastic.

Since it is assumed that paid losses and investment returns are stochastically independent, (3.3) yields:

$$\Lambda_T = \sum_{k>K} \gamma_k[(\mathrm{A}^F/\mathrm{A})\rho_{kT}^F + (\mathrm{A}^V/\mathrm{A})\rho_{kT}^V]. \tag{3.4}$$

3.3 Surplus assets

The definition of surplus in Section 3.1 left open the value to be assigned to accumulated assets. Again, this will depend on investment strategy, specifically the balancing of assets between the fixed interest and volatile sectors from time to time.

Again, this paper will take a rather simple approach by assuming that, apart from the drawdowns discussed in Section 3.2, all assets of a particular sector and all revenue generated by them are retained in that sector.

Then

$$\begin{aligned} A_T &= A^F R_{KT}^F + A^V R_{KT}^V \\ &= A\left[(A^F/A) R_{KT} + (A^V/A) R_{KT}^V\right]. \end{aligned} \tag{3.5}$$

Substitution of (3.3) and (3.5) into the expression for S_T gives:

$$\begin{aligned} S_T = {} & (A^F/A)\left[A R_{KT}^F - \sum_{k>K} C_k R_{kT}^F\right] \\ & + (A^V/A)\left[A R_{KT}^V - \sum_{k>K} C_k R_{kT}^V\right] \end{aligned} \tag{3.6}$$

and, using (3.4),

$$\begin{aligned} \Sigma_T = {} & (A^F/A)\left[A \rho_{KT}^F - \sum_{k>K} \gamma_k \rho_{kT}^F\right] \\ & + (A^V/A)\left[A \rho_{KT}^V - \sum_{k>K} \gamma_k \rho_{kT}^V\right]. \end{aligned} \tag{3.7}$$

It will be reasonable to regard assets and liabilities as of equal value (in expectation) at end of period K if

$$\Sigma_T = 0. \tag{3.8}$$

i.e. $E[\Lambda_T] = \Lambda_T$

$$\text{i.e. } \Lambda = \sum_{k>K} \gamma_k[(A^F/A)\rho^F_{kT} + (A^V/A)\rho^V_{kT}]/[(A^F/A)\rho^F_{KT} + (A^V/A)\rho^V_{KT}]. \tag{3.9}$$

This indicates that the value of liabilities at end of period K should be taken as:

$$\Lambda = (\rho_{KT})^{-1} \sum_{k>K} \gamma_k[(A^F/A)\rho^F_{kT} + (A^V/A)\rho^V_{kT}] \tag{3.10}$$

$$= \sum_{k>K} \gamma_k \rho_{kT} / \rho_{KT} , \tag{3.11}$$

with

$$\rho_{kT} = (A^F/A)\rho^F_{kT} + (A^V/A)\rho^V_{kT}. \tag{3.12}$$

3.4 Asset and liability risks

Consider the accumulated surplus S_T as represented by (3.6), and rewrite this as:

$$S_T = (A^F R^F_{KT} + A^V R^V_{KT}) - \sum_{k>K} C_k[(A^F/A)R^F_{kT} + (A^V/A)R^V_{kT}] .$$

Then substitute (2.2) for A^F:

$$S_T = \sum_{k>K} [\phi_k \rho^F_{kT}(R^F_{KT}/\rho^F_{KT}) - (A^F/A)C_k R^F_{kT}]$$

$$+ [A^V R^V_{KT} - \sum_{k>K} (A^V/A) C_k R^V_{kT}]. \tag{3.13}$$

The two components of surplus are those generated by fixed interest and volatile assets respectively, relative to the respective portions of liabilities covered by these asset sectors.

Note that S_T may be written in the general form:

$S_T = f(R,C)$,

where f is a real-valued function, R is the vector of values R_{kT}^{F} and R_{kT}^{V}, and C is the vector of claim costs C_k.

Similarly,

$$\Sigma_T = f(\rho,\gamma). \tag{3.14}$$

Then

$$\begin{aligned} S_T - \Sigma_T &= f(R,C) - f(\rho,\gamma) \\ &= \left[f(\rho,C) - f(\rho,\gamma)\right] + \left[f(R,\gamma) - f(\rho,\gamma)\right] \\ &\quad + \left[f(R,C) - f(\rho,C) - f(R,\gamma) + f(\rho,\gamma)\right] \\ &= \Delta_L + \Delta_A + \Delta_{AL}, \end{aligned} \tag{3.15}$$

where

$$\Delta_L = f(\rho,C) - f(\rho,\gamma) = \text{liability component}, \tag{3.16}$$

$$\Delta_A = f(R,\gamma) - f(\rho,\gamma) = \text{asset component}, \tag{3.17}$$

$$\begin{aligned} \Delta_{AL} &= f(R,C) - f(\rho,C) - f(R,\gamma) + f(\rho,\gamma) \\ &= \text{asset-liability interaction component.} \end{aligned} \tag{3.18}$$

An explicit expression can be obtained for each of the components (3.16)-(3.18) by writing out the functions f in full. As one example,

$$\Delta_L = \sum_{k>K} \left[(A^F/A)\,\rho_{kT}^{F} + (A^V/A)\,\rho_{kT}^{V}\right](\gamma_k - C_k), \tag{3.19}$$

which is the accumulated sum of deviations of liability cash flows from their expectations. It is the variation of accumulated surplus from its mean when asset performance is known and liability cash flows are the only source of variation.

Similarly, Δ_A is the variation in accumulated surplus when liability cash flows are certain and asset performance is the only source of variation.

Finally, Δ_{AL} is the additional variation of accumulated surplus due to the **joint** uncertainty in asset performance and liability cash flows. Its meaning is best illustrated by example.

Consider the case in which liability cash flows are perfectly matched with default free fixed interest assets, these being the only assets, i.e.

$$A^F = A,\ A^V = 0,$$

$$\phi_k = \gamma_k. \tag{3.20}$$

Substitution of these identities into (3.18) leads to the following result:

$$\Delta_{AL} = \sum_{k>K} (\gamma_k - C_k)(R_{kT}^F - \rho_{kT}^F). \tag{3.21}$$

Each term in this sum is the product of liability uncertainty and asset uncertainty.

3.5 Decomposition of asset risk

The asset risk defined in (3.17) can be decomposed further:

$$\begin{aligned}
\Delta_A &= \sum_{k>K} \phi_k (R_{KT}^F - \rho_{KT}^F)\rho_{kT}^F/\rho_{KT}^F + A^V(R_{KT}^V - \rho_{KT}^V) \\
&\quad - \sum_{k>K} \gamma_k\left[(A^F/A)(R_{kT}^F - \rho_{kT}^F) + (A^V/A)(R_{kT}^V - \rho_{kT}^V)\right] \\
&= \sum_{k>K} (\gamma_k - \phi_k)\rho_{kT}^F + \sum_{k>K}\left[\phi_k \rho_{kT}^F (R_{KT}^F/\rho_{KT}^F) - \gamma_k R_{kT}^F\right] \\
&\quad + A^V(R_{KT}^V - \rho_{KT}^V) - \sum_{k>K} \gamma_k (A^V/A)\left[(R_{kT}^V - \rho_{kT}^V) - (R_{kT}^F - \rho_{kT}^F)\right] \\
&= \Delta_{Am} + \Delta_{ApF} + \Delta_{ApV} + \Delta_{Amp},
\end{aligned} \tag{3.22}$$

with

$$\Delta_{Am} = \sum_{k>K} (\gamma_k - \phi_k)\ \rho_{kT}^F, \tag{3.23}$$

$$\Delta_{ApV} = A^V(R_{KT}^V - \rho_{KT}^V) - (A^V/A)\sum_{k>K} \gamma_k (R_{kT}^V - \rho_{kT}^V), \tag{3.24}$$

$$\Delta_{ApF} = (A^V/A)\sum_{k>K} \gamma_k (R_{kT}^F - \rho_{kT}^F), \tag{3.25}$$

$$\Delta_{Amp} = \sum_{k>K}\left[\phi_k \rho_{kT}^F (R_{KT}^F/\rho_{KT}^F) - \gamma_k R_{kT}^F\right]. \tag{3.26}$$

Now $\Delta_{Am} = 0$ if expected revenue from fixed interest assets matches expected claim costs year by year. Thus, Δ_{Am} may be regarded as an **asset mismatch component** of Δ_A.

Similarly Δ_{ApF} and Δ_{ApV} may be regarded as **fixed interest asset performance** and **volatile asset performance** components. Finally, Δ_{Amp} is a mismatch - performance interaction component.

3.6 Measurement of risk

Let

$$\Delta = S_T - \Sigma_T \tag{3.27}$$

By (3.15) and (3.22),

$$\begin{aligned} \Delta &= \Delta_L + \Delta_A + \Delta_{AL} \\ &= \Delta_L + (\Delta_{Am} + \Delta_{ApF} + \Delta_{ApV} + \Delta_{Amp}) + \Delta_{AL}. \end{aligned} \tag{3.28}$$

By (2.4),

$$Cov[\Delta_L, \Delta_A] = 0, \tag{3.29}$$

and Appendix A shows that

$$Cov[\Delta_L, \Delta_{AL}] = Cov[\Delta_A, \Delta_{AL}] = 0. \tag{3.30}$$

Therefore,

$$V[\Delta] = V[\Delta_L] + V[\Delta_A] + V[\Delta_{AL}]. \tag{3.31}$$

This may be expressed verbally as:

Total risk = liability risk
\+
asset risk
\+
asset-liability interaction risk, (3.32)

where

$$V[\Delta] = \text{total risk},$$

$$V[\Delta_L] = \text{liability risk},$$

$$V[\Delta_A] = \text{asset risk},$$

$$V[\Delta_{AL}] = \text{asset-liability interaction risk}, \tag{3.33}$$

Note that it is not possible to express asset risk as a similar sum of variances on the basis of (3.22). For example, it is unlikely that R_{kT}^F and R_{kT}^V would be stochastically independent, and so neither would Δ_{ApF}^F and Δ_{ApV}.

4. Capital and Risk

4.1 Asset-liability ratio

Consider the case in which Δ defined by (3.27) may be adequately approximated by a normal distribution:

$$\Delta \sim N(0,\sigma^2) \tag{4.1}$$

The insurer's survival probability is given by:

$$Prob[S_T \geq 0] = Prob[\Delta/\sigma \geq -\Sigma_T/\sigma], \tag{4.2}$$

where the variable within the last square bracket is unit normal.

Thus the survival probability is p if:

$$\Sigma_T/\sigma = z_p, \tag{4.3}$$

with $\Phi(z_p) = p$.

By (3.2), (4.3) may be re-expressed as:

$$E[A_T]/\Lambda_T = 1 + z_p\omega \tag{4.4}$$

with

$$\omega = \{V[\Delta]\}^{½}/\Lambda_T. \tag{4.5}$$

If both numerator and denominator on the left of (4.4) are divided by ρ_{KT}, as defined by (3.12), the result is

$$\mathrm{A}/\Lambda = 1 + z_p\omega, \tag{4.6}$$

where use has been made of (3.9) and (3.10).

Note that substitution of (3.31) in (4.5) yields:

$$\omega^2 = \omega_L^2 + \omega_\mathrm{A}^2 + \omega_{\mathrm{A}L}^2, \tag{4.7}$$

where

$$\omega_X^2 = V[\Delta_X]\,/\Lambda_T^2, \text{ for } X = L,\mathrm{A},\mathrm{A}L. \tag{4.8}$$

Call ω_L, ω_A, $\omega_{\mathrm{A}L}$ the liability, asset, and asset-liability interaction **risk measures** respectively.

4.2 Liability risk

Consider how required assets vary with the liability risk measure ω_L. Assume that Λ and ω_A are held constant. Then, from (4.6),

$$\begin{aligned} d\mathrm{A} &= \Lambda z_p d\omega \\ &= \Lambda z_p(\omega_L d\omega_L + \omega_{\mathrm{A}L} d\omega_{\mathrm{A}L})/\omega, \end{aligned} \tag{4.9}$$

by (4.7). Hence the following result.

Proposition 4.1. Assets required to achieve a prescribed probability of survival p increase with increasing liability risk measure, if asset-liability interaction risk measure increases in sympathy. □

In practice, it will be difficult to check the proviso in this statement. However, the example given in (3.21) indicates that it is likely to hold in usual circumstances; and that, in any event, $\omega_{\mathrm{A}L}$ is likely to be small, of order $\omega_\mathrm{A}\ \omega_L$, in which case even failure of the proviso would be unlikely to change the conclusion of Proposition 4.1.

4.3 Asset risk

Now consider how the volume of required assets varies with changes in the composition of the asset portfolio.

Recall (4.5) and (4.6), to obtain

$$\mathrm{A} = \Lambda + z_p\{V[\Delta]\}^{1/2}\Lambda/\Lambda_T$$

$$= \Lambda + z_p\{V[\Delta]\}^{1/2}\rho_{KT}^{-1}, \tag{4.10}$$

by (3.4) and (3.10).

Then

$$d\mathrm{A} = d\Lambda + z_p\{\tfrac{1}{2}\rho_{KT}\, dV[\Delta] - V[\Delta]\, d\rho_{KT}\}/\rho_{KT}^2\,\{V[\Delta]\}^{1/2}, \tag{4.11}$$

for variations in the asset portfolio.

Recall (3.31) and disregard $V[\Delta_{AL}]$ as second order. Then (4.11) becomes:

$$d\mathrm{A} = d\Lambda + z_p\{[\tfrac{1}{2}\rho_{KT}\, dV[\Delta_L] - V[\Delta_L]\, d\rho_{KT}]$$

$$+ [\tfrac{1}{2}\rho_{KT}\, dV[\Delta_{\mathrm{A}}] - V[\Delta_{\mathrm{A}}] d\rho_{KT}]\}/\rho_{KT}^2\{V[\Delta]\}^{1/2}, \tag{4.12}$$

It is difficult to consider variations of this type in generality. Consider therefore the special case of a shift between the fixed interest and volatile asset sectors, but with no change in intra-sector structure.

To define this idea precisely, let $\pi = \mathrm{A}^V/\mathrm{A}$ = proportion of assets in volatile sector.

This notation allows (3.3) - (3.5) to be written as:

$$L_T = \sum_{k>K} C_k[(1-\pi)R_{kT}^F + \pi R_{kT}^V], \tag{4.13}$$

$$\Lambda_T = \sum_{k>K} \gamma_k[(1-\pi)\rho_{kT}^F + \pi\rho_{kT}^V], \tag{4.14}$$

$$\mathrm{A}_T = \mathrm{A}[(1-\pi)R_{kT}^F + \pi R_{kT}^V]. \tag{4.15}$$

The shift between asset sectors described above will consist of a variation $d\pi$ in π. Appendix B evaluates the various contributions to $d\Lambda$ under this variation.

Appendix B.2 shows that, for $d\pi > 0$,

$$d\Lambda < 0, \tag{4.16}$$

provided that

$$\rho_{kT}^{V} / \rho_{kT}^{F} \text{ decreases with increasing } k. \tag{4.17}$$

Appendix B.3 shows that, for $d\pi > 0$,

$$\tfrac{1}{2}\rho_{KT}\, dV[\Delta_L] - V[\Delta_L]\, d\rho_{KT} \leq 0, \tag{4.18}$$

if (4.17) holds and

$$Cov[C_k, C_l] \geq 0 \text{ for all } k,\ l. \tag{4.19}$$

The behaviour of the remaining contribution to $d\Lambda$ in (4.12) is more complex. From Appendix B.1,

$$\begin{aligned} &\tfrac{1}{2}\rho_{KT}\, dV[\Delta_A] - V[\Delta_A]\, d\rho_{KT} \\ &\quad = (d\pi)\ Cov[P + \pi Q,\ \rho_{KT}^{F} Q - (\rho_{KT}^{V} - \rho_{KT}^{F}) P], \end{aligned} \tag{4.20}$$

where

$$P = \mathrm{A} R_{KT}^{F} - \sum_{k>K} \gamma_k R_{kT}^{F}, \tag{4.21}$$

$$Q = \mathrm{A}(R_{KT}^{V} - R_{KT}^{F}) - \sum_{k>K} \gamma_k (R_{kT}^{V} - R_{kT}^{F}). \tag{4.22}$$

The right side of (4.20) is not conveniently quantifiable, but in practice it is likely that volatility of fixed interest returns will be low. In the special case in which it is zero, covariances involving P vanish, and the right side of (4.20) reduces to:

$$(d\pi)\ \pi\ \rho_{KT}^{F}\ V[Q] \tag{4.23}$$

$$> 0, \text{ for } d\pi > 0. \tag{4.24}$$

More generally then, (4.20) is positive for $d\pi > 0$, provided that fixed interest volatility is sufficiently low.

From this and (4.18), it is seen that the multiplier of z_p in (4.12) consists of one negative and one positive quantity when the relevant conditions hold. Which of these quantities dominates depends on the relation between:

(a) the intrinsic liability risk; and
(b) the magnitude of net assets.

This can be seen by examining (B.17) and (B.18) in relation to (a), and (4.23) and (B.6) in relation to (b).

It is seen from (B.17) and (B.18) that, if all terms $Cov[C_k, C_l]$ become small, then (4.18) becomes small, and the multiplier of z_p in (4.12) is dominated by the asset contribution.

Consider the case of large net assets. By (B.6), Q will be large (in probability). Then (4.23) and the associated discussion show that the asset contribution to the multiplier z_p is again dominant. Note that large net assets are associated with large z_p.

All of this discussion is summarised in Propositions 4.2 and 4.3.

Proposition 4.2. When asset risk is increased (in the sense defined above), the discounted value of liabilities decreases (see (4.16)) provided that condition (4.17) holds. □

Condition (4.17) is a weak requirement. For example, in the simple case in which asset returns from disjoint time intervals are stochastically independent,

$$\rho_{kT}^{V} / \rho_{kT}^{F} = \prod_{l=k+1}^{T} (\rho_l^V / \rho_l^F)$$

$$= (const.)^{T-k}, \tag{4.25}$$

in the special case for which ρ_l^V / ρ_l^F = const. such as in model of log normal R_l^V (e.g. Rubinstein, 1976, p.416).

The constant in (4.25) will exceed 1 reflecting the higher volatility of the V asset sector, and so (4.25) will reduce as k increases.

Proposition 4.3 Suppose that (4.17) and (4.19) hold. When asset risk is increased (still in the same sense as above), assets required to provide a defined confidence of adequacy will increase or decrease depending on the level of confidence. For low levels of confidence ≥50% (e.g. $p = 50\%$ in (4.12)), required assets decrease. For high levels of confidence, required assets increase. This last result holds at sufficiently high levels of confidence without the assumption of (4.19).

The value of p at which required assets are unchanged by increased asset risk reduces as all the liability (co)variances $Cov[C_k, C_l]$ diminish toward zero. □

The practical meaning of Propositions 4.2 and 4.3 is that whether the net asset position of an insurer is improved or worsened by an increase in asset risk will depend on the basis used by the insurer to set its loss reserves. If this basis comprises the **median discounted liability** ($p=50\%$, $z_p=0$), then net assets will be improved; if the basis includes a prudential margin providing **high confidence of adequacy** ($p \to 100\%$), then net assets will be worsened.

The same result may be put another way. Suppose the insurer in question reserves on the median discounted liability basis. Then the required capitalisation of that insurer will depend on the degree of confidence which the capital base is to provide. If it is high (low), then an increase in the insurer's asset risk will necessitate an increase (decrease) in the capital base.

5. Capital and Solvency

Taylor (1995) examined the pricing by, and capitalisation of, insurers in equilibrium of a free competitive market. The main result concerning solvency, given on p. 418 of that work, is that a certain risk parameter d is constant across all insurers.

According to equation (45) of the paper referred to, d is defined by:

$$\log(1+\eta+\delta) = d\sigma - \tfrac{1}{2}\sigma^2 - \tfrac{1}{2}\sigma_L^2, \tag{5.1}$$

where, in the present notation,

$1+\eta$ = ratio of discounted premium, net of acquisition costs, to corresponding discounted losses and associated expenses;

δ = ratio of net assets to discounted losses and associated expenses deriving from the next year's premium income;

$$\sigma_L^2 = \log\{1 + V[\Delta_L]/\Lambda^2\} \tag{5.2}$$

$$\sigma_A^2 = \log\{1 + V[\Delta_A]/A^2\}, \tag{5.3}$$

$$\sigma^2 = \sigma_L^2 + \sigma_A^2. \tag{5.4}$$

In fact, Taylor works with a single period model, and defines the quantities appearing in (5.1) accordingly. The present paper simply extrapolates those definitions to the current multi-period model. In these definitions, discounting is carried out at risk free RORs.

To interpret d, consider the case in which the insurer holds only risk free assets, so that the risk free discounting in η, δ corresponds to the real situation.

Consider the case in which losses arising from a year's premium income are log normally distributed with dispersion parameter σ. Then the probability that the premium income plus capital are sufficient to cover discounted claims is p where

$$\Phi(z_p) = p,$$

with

$$z_p = [\log(1+\eta + \delta)]/\sigma. \tag{5.5}$$

By (5.1) and (5.5),

$$d = z_p + \tfrac{1}{2}\sigma + \tfrac{1}{2}\sigma_L^2/\sigma. \tag{5.6}$$

Thus, constancy of d across insurers implies constancy of p except to the extent that σ and σ_L^2, i.e. σ_A^2 and σ_L^2, vary across insurers.

Indeed, (5.6) yields:

$$z_p + \tfrac{1}{2}\sigma \le d < z_p + \sigma. \qquad \text{[by (5.4)]} \tag{5.7}$$

If σ is small relative to z_p, then d is a reasonable approximation to z_p.

These findings may be summarised as follows.

Proposition 5.1 In a competitive free market in equilibrium, insurers with risk free asset portfolios all have the same security, in the sense of probability of adequacy of existing capital plus premiums to meet corresponding discounted claim cost, except to the extent that asset risk and liability risk vary across those insurers.

In fact, the constant probability p is given by $\Phi(d) = p$ with d, the parameter from Taylor (1995), defined by (5.1), provided that asset risk and liability risk are small relative to d. □

It follows therefore, at least to a reasonable degree of approximation, that each insurer's equilibrium capitalisation, as represented by δ, will follow from (5.1), η, σ_L and σ_A, thus:

$$\delta = \exp(d\sigma - \tfrac{1}{2}\sigma^2 - \tfrac{1}{2}\sigma_L^2) - (1+\eta). \tag{5.8}$$

6. Capital and profit margin

Let δ and η be referred to as **capitalisation** and **profit margin** respectively. The present section will consider the equilibrium relation between the two.

Taylor (1995, pp. 420-423) shows that equilibrium premiums consist of discounted expected claim cost, but adjusted to include the insurer's insolvency put. It is seen, however, that the adjustment is virtually negligible for most practical purposes.

Taylor's work excludes tax effects. Elsewhere these are seen to be considerable. The simplest example of this is provided by Fairley (1979), who calculates the "fair" profit margin:

$$-\kappa i^L + N i^0 \tau/(1-\tau), \tag{6.1}$$

where

κ	=	ratio of **undiscounted** technical liabilities to premium;
N	=	ratio of net assets to premium;
i^0	=	risk free ROR;
i^L	=	CAPM risk adjusted ROR applicable to discounting technical liabilities.

Note that Fairley defines profit margin as the addition to **undiscounted** losses and expenses, whereas η relates to **discounted** losses and expenses. In the latter case, annual profit is depressed by κi^L relative to the former, and so (6.1) gives the fair profit margin as:

$$\eta = N i^0 \tau/(1-\tau). \tag{6.2}$$

In this case the profit margin arises entirely from the operation of tax; $\eta = 0$ if $\tau = 0$.

The fact that a fair price consists of discounted expected costs plus a margin dependent on tax on earnings of the capital base reflects the fact that these earnings are subject to a form of double taxation, relative to earnings on investments held directly by shareholders. For the earnings of an insurer's capital base are taxed once in the hands of the insurer, and again in the hands of the shareholder.

The Fairley approach is unduly simple, but it has been shown (Taylor, 1994) that, under assumptions which spell out precisely the steady state conditions to which it applies, it yields the same result as more sophisticated approaches, such as that of Myers and Cohn (1981).

In summary, then,

(1) The equilibrium profit margin consists of tax effects and a virtually negligible insolvency put adjustment.

(2) The Fairley profit margin consists entirely of tax effects.

(3) The Myers-Cohn profit margin is the same as the Fairley under suitable assumptions.

These results yield the following conclusion.

Proposition 6.1 Provided that the insurance industry equilibrium involves a low risk of insolvency (rendering negligible the insolvency put discussed above), the equilibrium profit margin will be approximated by the Myers-Cohn margin calculated on the basis of the equilibrium capitalisation discussed in Section 5. □

This proposition addresses one of the unsolved problems of insurance pricing, namely the capital base which should be allowed for in fair pricing of the Myers-Cohn (or similar) type? The suggestion made here is that it is the base which would emerge in competitive equilibrium, and Section 5 indicates how that capital base would vary across insurers with different distributions of underwriting by line of business.

Conclusions of this sort can be extrapolated to hypothetical single line insurers to determine the capital base, and hence profit margin, associated with a particular line. This is a form of solution to the problem of **allocation of capital by line**.

It would probably be a fully satisfactory solution if the capital bases of different lines were additive, i.e. if the capital required to support underwriting in a number of lines simultaneously were merely the aggregate of the amounts of capital required to support each of those lines in isolation from the others.

The proviso here does not hold, however. Since the underwriting results of different lines are less than fully correlated (considerably less, in fact), their combination will produce risk reduction in the sense that liability risk (as defined in this paper) of the aggregate will be less than the weighted average of individual line liability risks. It then follows from (5.5) that capital is not even approximately additive.

7. Capitalisation of single line insurers

This section pursues the suggestion made in Section 6 that the earlier results can be extrapolated to hypothetical single line insurers.

The capitalisation equation of an insurer is given by (5.8). Let the equation as written there represent the "typical" insurer. Let the same equation with variables δ, η, σ, σ_L replaced by δ^*, η^*, σ^*, σ_L^* apply to another insurer, e.g. a single line insurer.

Note that, according to Section 5, d is a constant applicable to all insurers.

Solve (5.8) (equivalently (5.1)) for d to obtain:

$$\begin{aligned} d &= [\log(1+\eta+\delta)]/\sigma + \tfrac{1}{2}(\sigma^2+\sigma_L^2)/\sigma \\ &= \log\{(1+\eta+\delta)^{1/\sigma} \exp \tfrac{1}{2}(\sigma^2+\sigma_L^2)/\sigma\}. \end{aligned} \qquad (7.1)$$

Then substitute this in the starred version of (5.8):

$$\begin{aligned} \delta^* &= (\exp d)^{\sigma^*} \exp -\tfrac{1}{2}(\sigma^{*2}+\sigma_L^{*2}) - (1+\eta^*) \\ &= (1+\eta+\delta)^{\sigma*/\sigma} \exp \tfrac{1}{2}[(\sigma^2+\sigma_L^2)\sigma^*/\sigma - (\sigma^{*2} + \sigma_L^{*2})] - (1+\eta^*). \end{aligned} \qquad (7.2)$$

Two observations may be made on this result.

First, if δ is substantially larger than σ, σ^*, σ_L, σ_L^*, then the first factor on the right side of (7.2) will be dominant, and a first approximation to (7.2) will be:

$$\delta^* = (1+\eta+\delta)^{\sigma*/\sigma} - (1+\eta^*). \qquad (7.3)$$

This approximation yields the following result.

Proposition 7.1 When approximation (7.3) is valid, the capitalisation of a single line of business increases with the total risk σ^* of that line. □

The second observation concerns the effect of η, η^* on (7.3). Since they act in opposite directions (and in fact are likely to be small relative to δ, δ^*), their net effect on δ^* will be small.

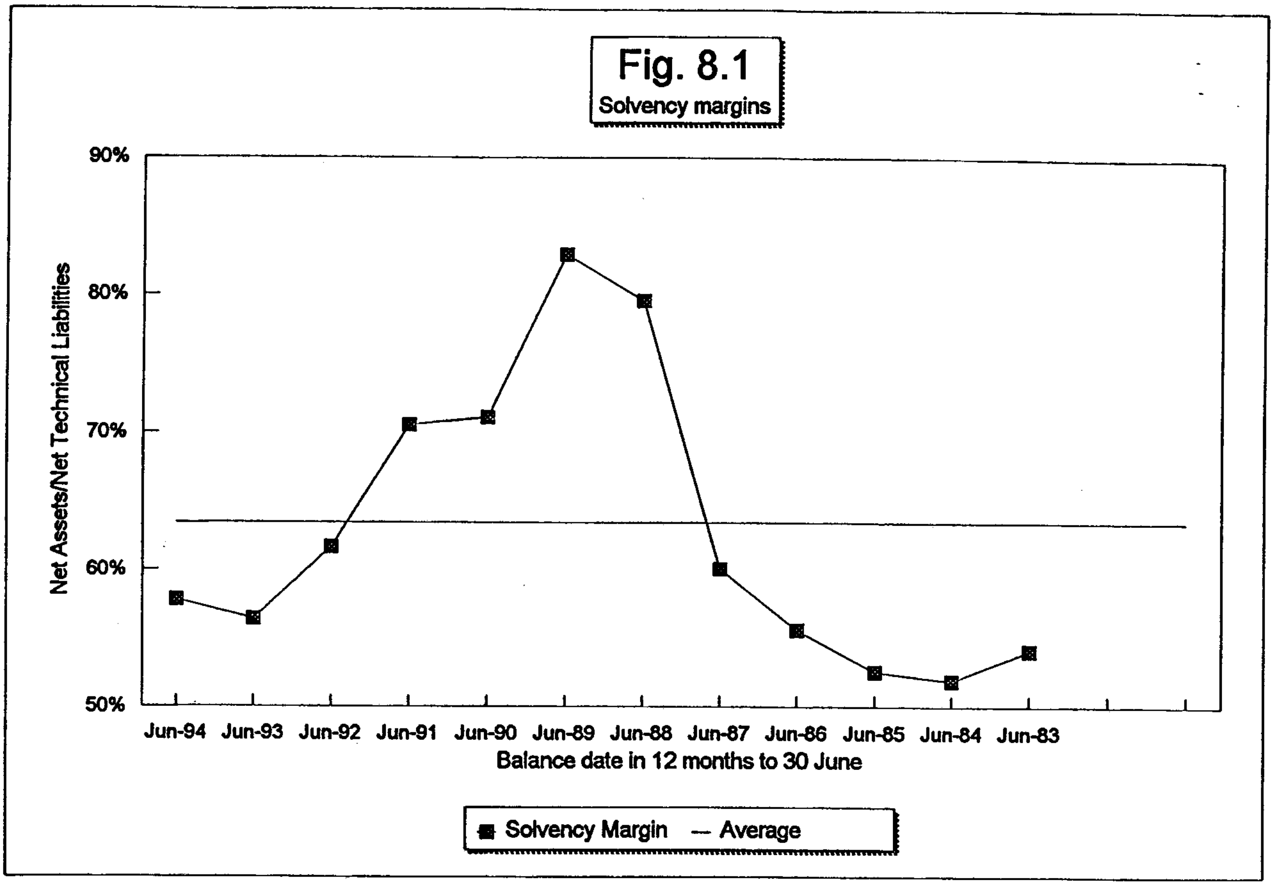
8. Numerical example

8.1 General discussion

The present section provides a numerical example of the results of Sections 4 to 7. The example is based on real data from the whole insurance industry of one particular country.

8.2 Industry capitalisation

Figure 8.1 plots a history of market capitalisation.



It is seen that net assets have stood at about 60% of technical liabilities over the latest three years, slightly below the average over the 14 years. For the present example, it will be assumed that $\delta = 0.6$.

8.3 Asset risk

Table 8.1 is a broad reflection of the industry asset deployment.

Table 8.1
Asset allocation

	%
Cash	5
Fixed Interest	56
Property	6
Local shares	23
International shares	8
Other	2
Total	100

Table 8.2 details the assumptions made in respect of asset volatility.

Table 8.2
Asset volatility

	% p.a.
Cash	2
Fixed Interest	5
Property	18
Local shares	20
International shares	20
Other	10
Total	7.4

The "total" is 5.6% if calculated as a root weighted mean square with weights as in Table 8.1. The figure of 7.4% p.a. includes allowance for sector return correlations. These have been based on an empirical study of the financial markets under consideration, but in the interests of brevity have not been reproduced here.

The extent to which a particular line is subject to asset risk is likely to be related to the duration of its liabilities. This will be related in turn to the ratio of technical liabilities to premium income.

Table 8.3
Duration of Liabilities

Line of business	Proportion of premium income %	Estimated duration of liabilities years
Fire	8	1.88
House	15	1.39
Contractors	1	2.37
Marine	3	1.43
Motor:		
Property	37	0.84
Bodily injury	11	4.76
Worker's compensation:		
Large states	3	4.03
Small states	2	4.03
Public Liability	9	4.02
Other	11	1.68
Total	100	2.05

Note: The "estimated duration of liabilities" is calculated as the ratio of technical liabilities (loss and unearned premium reserves) to net written premium less explicit acquisition cost (e.g. commission, brokerage, etc.).

Industry-wide asset risk is then estimated from Tables 8.2 and 8.3 as follows. If X denotes the (continuous) ROR earned on surplus assets, and D the duration of liabilities, then the average accumulation factor for those assets is:

$$F = e^{DX}. \tag{8.1}$$

If $X \sim N(\mu, \sigma^2)$, then $F \sim \log N(D\mu,\ D\sigma^2)$, and the coefficient of variation of F is $\omega(F)$ with

$$\begin{aligned} \omega^2(F) &= \exp(D\sigma^2) - 1 \\ &= \left[1 + \omega^2(F_1)\right]^D - 1, \end{aligned} \tag{8.2}$$

where F_1 is F in the case $D = 1$.

Now $\omega(F_1)$ is the value of 7.4% found in Table 8.2, and $D = 2.05$ from Table 8.3. Thus, (8.2) gives

$$\omega^2(F) = \left[1 + (0.074)^2\right]^{2.05} - 1$$

whence $\omega(F) = 10.6\%$.

This is an estimate of the coefficient of variation $V^{½}[\Delta_A]/A$. However, to the extent that assets are matched to liabilities, asset variation will cancel corresponding liability variation. This means that the asset coefficient of variation is reduced by a factor of

$$(A - \varepsilon\Lambda)/A = 1 - \varepsilon(\Lambda/A), \tag{8.3}$$

where ε is the effectiveness with which the industry matches its assets to liabilities.

Then the **effective** value of $\omega(F)$ becomes

$$\begin{aligned}\omega(F) &= 10.6\% \times \left[1 - \varepsilon(\Lambda/A)\right] \\ &= 10.6\% \times (1 - \varepsilon/1.6), \qquad \text{[by Section 7.2]}.\end{aligned} \tag{8.4}$$

8.4 Liability risk

Empirical estimates of liability risk for individual lines of business were made by Cumpston (1992). As has been noted elsewhere, liability risk consists of a **systematic** component which is independent of portfolio size and a **non-systematic** component with dies out with increasing portfolio size.

Recall the definition of ω_L in (4.8), but now denote this quantity by $\omega_L(q)$, where q is the market share (of losses) of the insurer under consideration. Then

$$\omega_L^2(q) = \omega_s^2 + w_n^2/q, \tag{8.5}$$

where ω_s^2, ω_n^2 are the systematic and non-systematic parts of ω_L^2 **for the whole market.**

For the present example ω_L^2 needs to be line-specific. Therefore, generalise (8.5) to the following:

$$\omega_l^2(q) = \omega_{sl}^2 + w_{nl}^2/q_l, \tag{8.6}$$

where l denotes the line of business.

Cumpston's estimates of ω_{sl} and $10\,\omega_{nl}$ are set out in Table 8.4. Note that $10\,\omega_{nl}$ is the non-systematic component for an insurer with 1% market share.

Table 8.4
Liability risk by line

Line of business	Liability coefficient of variation: Systematic %	Non-systematic for 1% market share %	Total for 1% market share %
Fire	7	5	9
House	7	2	7
Contractors	7	13	15
Marine	7	8	11
Motor:			
Property	6	2	6
Bodily injury	12	14	18
Worker's compensation:			
Large states	22	7	23
Small states	14	7	16
Public Liability	10	8	13
Other	7	4	8
Total			7.4

The "total" line is subject to the same sort of comment as in Table 8.2. Subjective estimates of correlations between line-specific systematic components of variation have been chosen but, in the interests of brevity, not reproduced here.

Thus, for an insurer with 1% market share in all lines, liability risk is given by:

$$\omega_L = 7.4\%. \tag{8.7}$$

This result does not change much for larger market shares. For example, if the 1% share is replaced by 10%, ω_L changes to 6.7%.

8.5 Total risk

Consider an insurer with 1% market share in all lines. Substitute (8.4) and (8.7) in (5.2)-(5.4) to obtain

$$\sigma_L^2 = \log\left[1 + (0.074)^2\right] = (0.074)^2 \tag{8.8}$$

and

$$\sigma^2 = \log\{[1+(0.074)^2] \times [1+(0.106)^2(1-\varepsilon/1.6)^2]\}. \tag{8.9}$$

The value of ε is unknown. However, most insurers make some broad attempt to match assets to liabilities. It may be reasonable to assume that $\varepsilon = ⅔$, in which case (8.9) gives:

$$\sigma = 9.6\%. \tag{8.10}$$

Strictly, the coefficients of variation reproduced in Table 8.4 apply to **outstanding losses** only, whereas (8.8) and (8.9) treat them as applicable to all technical liabilities. To the extent that they do not reflect the uncertainty in unexpired risk, a distortion will have been introduced in (8.8)-(8.10). Proportionately, any inaccuracy of this type will affect short tail classes more than long tail.

8.6 Industry average security

This is measured by d, given by (5.5) and (5.6). The inputs to these equations are $\delta = 0.6$ (Section 7.2), $\sigma_L = 7.4\%$ (from (8.8)), $\sigma = 9.6\%$ (from (8.10)), and η.

The value of η is unknown. Determination from market data would be difficult, since η is likely to be swamped by the effect on profit of revisions to outstanding losses. However, as pointed out in Section 7, the effect of η on estimated single line capitalisation will be small. A modest allowance of $\eta = 5\%$ is made here.

With these inputs to (5.5) and (5.6),

$$\begin{aligned} d &= (\log 1.65)/0.096 + ½[0.096 + (0.074)^2/0.096] \\ &= 5.26. \end{aligned}$$

Equation (5.6) shows that this is close to z_p, implying that the industry operates at an extremely high level of security. There are several reasons why d might have been falsely inflated.

First, the line specific coefficients of variation appearing in Table 8.4 may have been under-estimated. Second, inaccuracies of the type contemplated at the end of Section 8.5, particularly in relation to property catastrophes, might have lead to under-estimation of the unexpired risk variability implicit in the estimate of d. Third, market frictions in capital raising probably mean that the capital base held by an insurer at any given time supports not only the business underwritten up to that time, but also a certain part of future underwriting.

All of these matters would need to be considered in a practical application of this paper's results. Note, however, that any mis-estimation of d does not necessarily affect an estimated single line capital base materially. Equations (7.2) and (7.3) show that this depends mainly on σ^*/σ, i.e. on the estimated relativities of line-specific total risk parameters rather than their absolute values.

8.7 Single line insurer capitalisation

Now consider a hypothetical single line insurer. Required capitalisation is given by (7.2). With the same parameters as used in Section 8.6, and assuming that $\eta^* = \eta = 5\%$, (7.2) becomes:

$$\delta^* = (1.65)^{\sigma*/\sigma} \exp\left[0.00735\,\sigma^*/\sigma - \tfrac{1}{2}(\sigma^{*2} + \sigma_L^{*2})\right] - 1.05, \qquad (8.11)$$

with σ given by (8.9) and σ^* by its starred counterpart:

$$\sigma^* = \sigma_L^{*2} + \log\{1 + [\omega^*(F)]^2[1 - \varepsilon/(1+\delta^*)]^2\}. \qquad (8.12)$$

Note that, when (8.11) and (8.12) are combined, they yield an implicit equation for δ^*. This needs to be solved numerically.

The value of σ_L^* is based on Table 8.4 but, as was the case with σ_L, it depends on the market share of the single line insurer under consideration. A typical single line insurer will usually have a larger market share of that line than the typical multi-line insurer. Suppose the single line insurer's market share is 10%. Then (8.11) and (8.12) yield the following results, in the case $\varepsilon = 2/3$.

Table 8.5
Single line insurer capitalisation

Line of business	Single line insurer with 10% market share			
	Liability risk %	Asset risk %	Total risk %	Capitalisation %
Fire	7.2	5.8	9.3	57
House	7.0	4.9	8.6	51
Contractors	8.1	6.9	10.7	69
Marine	7.4	5.0	9.0	54
Motor:				
Property	6.0	3.5	7.0	39
Bodily injury	12.8	11.7	17.3	138
Worker's compensation:				
Large states	22.1	12.1	25.2	251
Small states	14.2	10.9	17.9	144
Public Liability	10.3	10.1	14.4	105
Other	7.1	5.5	9.0	54

Note: "Capitalisation" δ^* is , i.e. ratio of net assets of the single line insurer to discounted losses and associated expenses deriving from the next year's premium income.

8.8 Sensitivity Analysis

The results in Table 8.5 are based on several assumptions concerning unknown market parameters, most notably:

- market share of typical multi-line insurer;
- market share of typical single line insurer;
- ε ;
- η .

Table 8.6 indicates the sensitivity of the results to variations in these parameters one at a time.

Table 8.6
Sensitivity analysis

Line of business	Single line insurer capitalisation			
	Market share			
	Typical multi-line insurer = 10%	Typical single line insurer = 30%	ε = 100%	η = 10%
	%	%	%	%
Fire	62	56	57	56
House	55	51	53	50
Contractors	75	63	70	70
Marine	59	52	57	54
Motor:				
Property	42	39	41	37
Bodily injury	153	133	155	149
Worker's compensation:				
Large states	280	250	320	277
Small states	160	143	167	155
Public Liability	116	103	111	111
Other	59	54	55	54

There is some variation between the (widely) different scenarios, but the general shape of capitalisation across the different lines remains much the same.

Appendix A
Independence of total risk components

A.1 Independence of Δ_L and Δ_{AL}

By (3.16) and (3.18),

$$\begin{aligned}\mathrm{Cov}\left[\Delta_L,\Delta_{AL}\right] &= \mathrm{Cov}\left[f(\rho,C)-f(\rho,\gamma),\, f(R,C)-f(R,\gamma)\right]\\ &\quad - V\left[f(\rho,C)-f(\rho,\gamma)\right]. \end{aligned} \tag{A.1}$$

Consider now the first argument of the covariance on the right side of (A.1). By (3.19),

$$f(\rho,C)-f(\rho,\gamma) = -\sum_{k>K}\left[(A^F/A)\,\rho^F_{kT} + (A^V/A)\,\rho^V_{kT}\right]\left[C_k-\gamma_k\right]. \tag{A.2}$$

Similarly, in the case of the second argument,

$$f(R,C)-f(R,\gamma) = -\sum_{k>K}\left[(A^F/A)R^F_{kT} + (A^V/A)R^V_{kT}\right]\left[C_k-\gamma_k\right]. \tag{A.3}$$

Now (A.2) and (A.3) give the covariance on the right side of (A.1) as:

$$\sum_{k,l>K}\left[(A^F/A)\,\rho^F_{kT} + (A^V/A)\,\rho^V_{kT}\right]\left[(A^F/A)\,\rho^F_{lT} + (A^V/A)\,\rho^V_{lT}\right]\mathrm{Cov}\left[C_k,C_l\right], \tag{A.4}$$

when the stochastic independence of assets and liabilities is recognised (see (2.4)).

The variance on the right side of (A.1) may also be evaluated by means of (A.2), and is also found to be equal to (A.4). Thus

$$\mathrm{Cov}\left[\Delta_L,\Delta_{AL}\right] = 0. \tag{A.5}$$

A.2 Independence of Δ_A and Δ_{AL}

The covariance of Δ_A and Δ_{AL} may be approached similarly:

$$\begin{aligned}\mathrm{Cov}\left[\Delta_A,\Delta_{AL}\right] &= \mathrm{Cov}\left[f(R,\gamma)-f(\rho,\gamma),\, f(R,C)-f(\rho,C)\right]\\ &\quad - V\left[f(R,\gamma)-f(\rho,\gamma)\right]. \end{aligned} \tag{A.6}$$

By the same sort of reasoning as in Appendix A.1, it may be shown that both variance and covariance terms appearing on the right side of (A.6) are equal to:

$$V\Big[A^V(R^V_{KT}-\rho^V_{KT}) + \sum_{k>k}\Big\{\phi(\rho^F_{kT}/\rho^F_{KT})(R^F_{KT}-\rho^F_{KT}) - \gamma_k\big[(A^F/A)R^F_{kT} + (A^V/A)R^V_{kT}\big]\Big\}. \tag{A.7}$$

Thus,

$$\mathrm{Cov}\left[\Delta_A,\Delta_{AL}\right] = 0. \tag{A.8}$$

Appendix B
Variation in composition of asset portfolio

B.1 Asset risk

Consider Δ_A as defined by (3.17):

$$\Delta_A = f(R,\gamma) - f(\rho,\gamma), \tag{B.1}$$

with

$$f(R,\gamma) = \sum_{k>K} \phi_k \, \rho^F_{kT}(R^F_{KT}/\rho^F_{KT}) + A^V R^V_{KT} - \sum_{k>K} \gamma_k\left[(A^F/A)R^F_{kT} + (A^V/A)R^V_{kT}\right]. \tag{B.2}$$

By (2.1), this is:

$$\begin{aligned} f(R,\gamma) &= A\left[(1-\pi)R^F_{KT} + \pi R^V_{KT}\right] \\ &\quad - \sum_{k>K} \gamma_k\left[(1-\pi)R^F_{kT} + \pi R^V_{kT}\right] \\ &= \left[AR^F_{KT} - \sum_{k>K} \gamma_k R^F_{kT}\right] \\ &\quad + \pi\left[A(R^V_{KT} - R^F_{KT}) - \sum_{k>K} \gamma_k (R^V_{kT} - R^F_{kT})\right], \end{aligned} \tag{B.3}$$

with π as defined in Section 4.3.

Thus,

$$V[\Delta_A] = V[P + \pi Q] \tag{B.4}$$

with

$$P = AR^F_{KT} - \sum_{k>K} \gamma_k R^F_{kT} \tag{B.5}$$

$$Q = A(R^V_{KT}-R^F_{KT}) - \sum_{k>K} \gamma_k (R^V_{kT}-R^F_{kT}). \tag{B.6}$$

For a variation $d\pi$ in π, (B.4) gives:

$$dV[\Delta_A] = 2(d\pi)\,\mathrm{Cov}[Q,P+\pi Q]. \tag{B.7}$$

Note also that, by (3.12),

$$\rho_{KT} = \rho^F_{KT} + \pi(\rho^V_{KT}-\rho^F_{KT}). \tag{B.8}$$

By (B.4), (B.7) and (B.8),

$$\begin{aligned} &\tfrac{1}{2}\rho\, dV[\Delta_A] - V[\Delta_A]d\rho \\ &\quad = (d\pi)\left\{[\rho^F + \pi(\rho^V-\rho^F)]\,\mathrm{Cov}[Q,P+\pi Q] - (\rho^V-\rho^F)V[P+\pi Q]\right\}, \\ &\quad = (d\pi)\,\mathrm{Cov}[P+\pi Q,\rho^F Q - (\rho^V-\rho^F)P], \end{aligned} \tag{B.9}$$

where it has been convenient to suppress the subscripts on ρ temporarily.

B.2 Discounted liability

Consider the term $d\Lambda$ appearing in (4.11). Recall (3.4) and (3.10):

$$\Lambda = \Lambda_T/\rho_{KT}. \tag{B.10}$$

Therefore,

$$(\rho_{KT})^2\Lambda = \rho_{KT}\, d\Lambda_T - \Lambda_T\, d\rho_{KT}. \tag{B.11}$$

By (4.14),

$$d\Lambda_T = (d\pi) \sum_{k>K} \gamma_k(\rho^V_{kT}-\rho^F_{kT}). \tag{B.12}$$

By (B.8),

$$d\rho_{KT} = (d\pi)\ (\rho^V_{KT} - \rho^F_{KT}). \tag{B.13}$$

Substitution of (B.8), (B.12), (4.14) and (B.13) into (B.11) gives:

$$\begin{aligned}(\rho_{KT})^2 d\Lambda &= (d\pi) \sum_{k>K} \gamma_k \left[\rho^F_{KT}(\rho^V_{kT} - \rho^F_{kT}) - \rho^F_{kT}\ (\rho^V_{KT} - \rho^F_{KT})\right] \\ &= (d\pi)\, \rho^F_{KT} \sum_{k>K} \gamma_k \rho^F_{kT} \left[\rho^V_{kT}/\rho^F_{kT}\ -\ \rho^V_{KT}\ /\rho^F_{KT}\right] \\ &< 0,\ \text{for}\ d\pi > 0,\end{aligned} \tag{B.14}$$

if ρ^V_{kT}/ρ^F_{kT} decreases with increasing k.

B.3 Liability risk

Consider Δ_L as represented in (3.19). This may be written as:

$$\Delta_L = \sum_{k>K} (\gamma_k - C_k)\left[\rho^F_{kT} + \pi\,(\rho^V_{kT} - \rho^F_{kT})\right]. \tag{B.15}$$

Following an argument parallel to that in Appendix B.1, write

$$V[\Delta_L] = V[P + \pi Q], \tag{B.16}$$

with P, Q now defined as follows:

$$P = \sum_{k>K} (C_k - \gamma_k)\,\rho^F_{kT}, \tag{B.17}$$

$$Q = \sum_{k>K} (C_k - \gamma_k)\,(\rho^V_{kT} - \rho^F_{kT}). \tag{B.18}$$

By the same argument as followed (B.4), it is shown that (B.9) holds once again, but this time with Δ_A replaced by Δ_L and with P, Q defined by (B.17) and (B.18).

Consider the second argument of the covariance in (B.9). It is

$$\rho^F Q - (\rho^V - \rho^F) P = \sum_{k>K} (C_k - \gamma_k)\left[\rho^F_{KT}(\rho^V_{kT} - \rho^F_{kT})\ -\ (\rho^V_{KT} - \rho^F_{KT})\,\rho^F_{kT}\right]. \tag{B.19}$$

Note that the square bracketed term is the same as appeared in (B.14), where it was found to be <0, provided that $\rho_{kT}^{V}/\rho_{kT}^{F}$ decreases with increasing k.

Thus, the current version of (B.9) gives $\frac{1}{2}\rho_{KT}\, dV[\Delta_L] - V[\Delta_L]\, d\rho_{KT}$ as $d\pi$ times a covariance of two linear combinations of terms $C_k - \gamma_k$, one linear combination having all coefficients positive, the other all negative. The result is a linear combination of terms $\mathrm{Cov}[C_k, C_l]$ with all coefficients negative.

It then follows that:

$$\tfrac{1}{2}\rho_{KT}\, dV[\Delta_L] - V[\Delta_L]\, d\rho_{KT} \leq 0, \text{ for } d\pi > 0, \tag{B.20}$$

if

$$\mathrm{Cov}[C_k, C_l] \geq 0 \text{ for all } k,l. \tag{B.21}$$

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