A Directional Multiplicative Intensity for Credit Migrations

Ashley Evans*

August 14, 2007

Abstract

Lando & Skødeberg (2002), amongst others, have proposed continuous-time non-Markovian models for credit rating migrations. These studies reject the once-common Markov assumption that the probability of a migration between two particular credit ratings is the same for all issuers at any time. Sparse data, however, have often limited the conclusions that can be drawn from non-Markvoian studies for migrations beyond neighbouring credit ratings, because the probability of issuers migrating long distances is low and so applying a more advanced model fails to reach statistical significance. This paper proposes a novel approach to deal with this limitation, where we first consider the direction of a credit migration, whether upgrade or downgrade, prior to applying a categorical cross-sectional model to the destination credit rating.

This new approach produces statistically significant estimates for the effects of momentum and excitability on credit rating migrations, and thus thoroughly rejects the appropriateness of a simple Markovian matrix for modelling credit rating migrations. This paper also demonstrates that issuers in different industry sectors exhibit different upgrade and downgrade probabilities. Similarly, we show that issuers that do change credit rating classifications do so in ways that vary by industry sector.

Keywords: *credit rating migrations; momentum; excitability; directional intensity; partial likelihood estimation.*

1 introduction

The non-Markovian behaviour of credit rating migrations has attracted extensive research in recent years. This is not surprising, since the Markovian assumption of historical irrelevance—where the probability of a credit rating migration is dependent only on the current credit rating—is intuitively and demonstrably false. Particularly, the dependence of credit rating migrations on the business cycle (Bangia et al. 2002) and issuer-specific information has confirmed a pure Markov assumption as unsuitable. Inference on these more advanced models, however, has been limited due to sparse data for improbable migrations, where low observed frequencies for migrations beyond neighbouring credit ratings cause statistically insignificant results for complex effects.

It has proven difficult to apply these non-Markovian models to credit rating migration data. The reason for this difficulty is that there is a lack of data available on credit rating migrations between, for example, a highly rated category and a category that indicates that an issuer is experiencing financial hardship. It is this lack of data that has inhibited the analyst from drawing *statistically significant* conclusions from such models.

A novel alternative to the traditional approach of directly modelling the migration between two credit ratings is proposed. We model the intensity process of a downgrade or

*any comments or queries can be emailed to a.evans5@pgrad.unimelb.edu.au

upgrade (directional migration) from an issuer's current credit rating, and then the subsequent probability of entering a particular credit rating conditioned on this directional migration. Consider a simplified example, for an issuer with credit rating AA. In previous studies, this issuer would have a migration probability for all credit ratings above and below AA (including default), each one needing to be estimated and tested individually. In the approach proposed here, this issuer has probabilities of an upgrade and a downgrade from AA, which contain all modelled effects, and then simplified categorical probabilities of entering particular credit ratings conditioned on the issuer migrating. Thus, we are able to estimate and test non-Markovian properties of migration to any credit rating, rather than only the most common.

Specific factors relating to issuer heterogeneity for Standard & Poor's credit rating migrations are mostly the focus of this article. The momentum and excitability of an issuer's credit rating were tested and found statistically significant (at least for neighbouring migration) by Lando & Skødeberg (2002). That is, a credit rating downgrade is likely follows another downgrade, and a rating change temporarily increases the probability of another. The equivalent arguments were confirmed (albeit less assured) for rating upgrades. By expanding on Lando & Skødeberg (2002), we test these characteristics of issuer heterogeneity. It is shown that the migration model proposed here is an improvement on the migration model proposed by Lando & Skødeberg (2002) in identification of both momentum and excitability. In addition, we model both momentum and excitability in tandem to gain enhanced results—made possible by the new modelling approach.

A multiplicative intensity process (Andersen & Gill 1982) is used for modelling momentum and excitability because it acknowledges the business cycle and other non-Markovian *common* effects without explicit specification. Also, it is ideal for finding the proposed directional migration intensity by partial likelihood estimation (Cox 1975) because, as we will see, the conditional destination probability can be removed and estimated separately, but the partial likelihood function of migration intensity is still maximised overall.

We adopt a continuous-time model, implied by the estimation of the migration *intensity*, as opposed to a discrete-time model for the following reasons. Firstly, the daily recording of an issuer's credit rating means modelling over annual or even monthly discrete periods would ignore useful information such as exact dates and multiple migrations in a period. Secondly, a discrete-time model assigns zero probability to very rare events, such as default from high credit ratings, whereas our approach does not automatically assign zero probability to such events.

Limitations in data prevented Lando & Skødeberg (2002) from considering the effect of momentum and excitability on migration intensities beyond neighbouring states. For example, momentum and excitability for an issuer in credit rating AA was only tested for migrations to AA+ and AA-. The directional migration approach adopted here alleviates many of these data constraints, with all migrations in the same direction considered in unison when modelling momentum and excitability. Moreover, restricting migration intensities to individual classes exposes the model to a large risk of informative censoring, where we could easily speculate a high positive correlation between that the probability of an issuer being downgraded one class or two classes. The directional migration model proposed reduces this informative censoring[1].

In addition, Markovian models often fail an assumption of industry homogeneity, where, for example, downgrading from a low risk credit rating is more likely for a bank than for an industrial firm (Nickell et al. 2000). Initially, industry sectors are considered all together for comparability with Lando & Skødeberg (2002), but we later distinguish between three

[1]see conclusion for further discussion

of the five industry classifications of Standard and Poor's. We do not explicitly measure industry heterogeneity for the directional migration model, instead allowing the unspecified effects common to all issuers, such as the business cycle, to vary between industry sectors. It is demonstrable that this method of distinction is appropriate, with the relative effect of momentum and excitability equal between industry sectors, and the direct effect of industry sector on the probability of migration nonproportional.

Another advantage of this directional migration intensity approach is that it leaves categorical data for modelling the distribution of the destination credit rating. Testing hypotheses, for which data were previously insufficient, becomes possible because we no longer rely on asymptotic results for our analyses (Zelterman 2006, Section 2.4), thus allowing inference on the entire set of credit rating migrations. Therefore, we can test the validity of credit rating grouping, such as ignoring the positive and negative signs of the credit ratings (Frydman & Schuermann (2005) and Bangia et al. (2002) to name a few). In addition, we explicitly estimate these conditional destination probabilities by industry sector, allowing testing for industry heterogeneity where previous studies could not.

Section 2 gives an overview of the Standard and Poor's data used and outlines the simplifications made in the analyses. Section 3 derives and calibrates the directional migration intensity, where partial likelihood estimation is used to measure the effects of momentum and excitability in both single stratum and industry stratified models. We investigate the consequential conditional destination probabilities in Section 4, testing whether such probabilities differ by credit ratings and between industry sectors. Lastly, we summarise and discuss our results in Section 5.

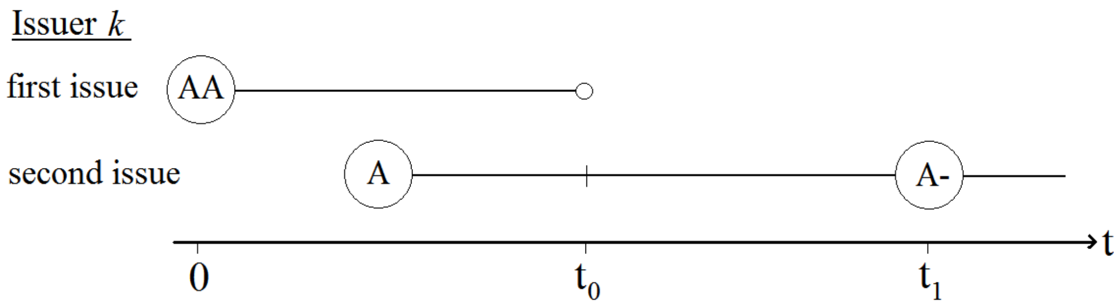
2 the data

We use Standard and Poor's credit rating data from 1 April 1995 to 31 December 2006. The credit ratings, from lowest to highest risk, are labelled AAA, AA+, AA, BBB+, ..., B-, CCC, CCC-, CC and C (see Table 1). We group the final three classes (CCC-, CC and C) into one class labelled CCC-. Furthermore, this paper refers to default as an absorbing credit rating labelled D, and considers it the lowest credit rating. We include the Not Rated class, labelled NR, but consider it neither higher nor lower than any other credit rating. The five industry classifications used by Standard and Poor's are industrial, financial, utilities, government and miscellaneous.

When Standard and Poor's rates multiple issues from a single issuer differently—usually due to issue-specific covenants—this paper uses the credit rating from the earliest issue. Then when this issue expires, we adopt credit rating for the next earliest issue. This possible change in credit rating, however, is not recorded as a migration (unless the credit rating of the second issue changed at that time), but a right-censoring of the first credit rating and a left-truncation of the second credit rating. For example, Figure 1 shows the history of two issues of a hypothetical issuer k. We record the issuer's credit rating as AA up to time t_0. At time t_0, the first issue expires, and is therefore no longer observable—right-censored. We adopt the credit rating of the previously unrecorded second issue, A, as the issuer's credit rating at time t_0—left-truncation. Although the issuer's credit rating has changed, no migration has occurred, since Standard and Poor's have not reclassified the issue. We are simply using a different issue to represent the credit rating of issuer k. The credit rating change that occurs to the second issue at time t_1 is a migration since the issue has been reclassified by Standard and Poor's.

There are 6,433 (32,411) issuers (contributing years of exposure) that qualify for use in the analyses; 3,799 (18,499) in the industrial sector, 1,791 (8,997) in the financial sector, 631

Figure 1: An example of right-censoring and left-truncation using hypothetical issuer k with two issues



(3,650) in the utility sector, 204 (1,254) in the government sector, and 8 (10) categorised as miscellaneous. These issuers contribute 14,772 migrations, of which 1,968 are either to or from NR. If we were to exclude the first migration for each issuer, which is required for an estimate of momentum or excitability, the total migrations would decrease to 9,039[2].

3 the directional migration intensity

This section explores the direction of a rating migration by deriving the downgrade and upgrade intensity processes and their estimators. We then use this modelling framework in two ways. Firstly, we build models for direct comparison with Lando & Skødeberg (2002), measuring the effects of momentum and excitability. Secondly, we build similar models but distinguish between industry sectors. We then assess the benefit from using industry distinction with both Nelson-Aalen estimators and Monte Carlo simulation.

Section 3.1 formulates our migration intensities, with the aim of deriving an estimator for the directional component. Although we encourage an inspection of the multiplicative intensity model used to model directional migration intensity (Equation 3), it is not necessary to understand this derivation to appreciate the results from Section 3.2 onwards.

3.1 partial likelihood estimation

We begin our formulation of the migration intensities with the states representative of the credit ratings as determined by Standard & Poor's; AAA is the highest state (with the lowest risk) and Default is the lowest state. The Not Rated state is treated as neither higher nor lower than other states, with migrations to and from the Not Rated state treated as *lateral* migrations.

The multivariate counting process $\mathbf{N}_{ij,\ell} = (N_{ij,k\ell}\ ; k = 1, \cdots, m_\ell)$ counts each migration from state i to state j of each issuer in stratum $\ell = \{1, \cdots, M\}$, where M is the number of strata and m_ℓ is the number of issuers in stratum ℓ. The *full migration intensity* is the rate at which an issuer migrates between two specific credit ratings; often referred to as a hazard rate or instantaneous probability. Assuming unique migration times from state i within stratum ℓ, we define

$$d\Lambda_{ij,k\ell}(t) = \mathbb{E}[dN_{ij,k\ell}(t)|\mathcal{F}_{t-}]$$

as the full migration intensity (Andersen & Gill 1982) where

$$dN_{ij,k\ell}(t) = \lim_{dt \searrow 0} (N_{ij,k\ell}(t^- + dt) - N_{ij,k\ell}(t^-)) \ .$$

[2]for a comprehensive set of descriptive statistics refer to Addendum A at www.economics.unimelb.edu.au/aevans

$\Lambda_{ij,k\ell}(t)$ is the cumulative intensity process for issuer k migrating to state j from i, and $\{\mathcal{F}_t\}_{t\geq 0}$ is the appropriate filtration generated by the process up to time t. We assume that the applicable filtration is the issuer-specific history (including the current state), giving

$$d\Lambda_{ij,k\ell}(t) = \mathbb{P}[dN_{ij,k\ell}(t) = 1 | N_{gh,k\ell}(s^-), Y_{g,k\ell}(s); \text{for all } g \neq h, 0 < s \leq t]$$

where $Y_{g,k\ell}(s)$ is the *at risk* process equalling one if the issuer k from stratum ℓ is observed in state g at time s and zero otherwise. The *at risk* process is a subtle but necessary tool to ensure that an issuer is subject to the migration intensity from state i only when they are residing in state i. Furthermore, only issuers deemed *at risk* will contribute to an estimation of the migration intensity.

Now consider the migration from i in direction d, where d is the set of credit ratings j either *up*, *down* or *lateral* from the current state i. A new process, $\tilde{N}_{id,k\ell}$, is proposed to count d-migrations, where

$$\tilde{N}_{id,k\ell}(t) = \sum_{j\in d} N_{ij,k\ell}(t)$$

is the total number of migrations in direction d from state i for issuer k up to time t. $d\tilde{N}_{id,k\ell}(t) = 1$ is incidental given $dN_{ij,k\ell}(t) = 1$, because if we know the destination state then we know the direction, and an issuer can only migrate once in any very small time period. So

$$\begin{aligned} d\Lambda_{ij,k\ell}(t) &= \mathbb{P}[dN_{ij,k\ell}(t) = 1, d\tilde{N}_{id,k\ell}(t) = 1 | N_{gh,k\ell}(s^-), Y_{g,k\ell}(s); \text{for all } g \neq h, 0 < s \leq t] \\ &= \mathbb{P}[dN_{ij,k\ell}(t) = 1 | d\tilde{N}_{id,k\ell}(t) = 1, N_{gh,k\ell}(s^-), Y_{g,k\ell}(s); \text{for all } g \neq h, 0 < s \leq t] \\ &\quad \times \mathbb{P}[d\tilde{N}_{id,k\ell}(t) = 1 | N_{gh,k\ell}(s^-), Y_{g,k\ell}(s); \text{for all } g \neq h, 0 < s \leq t] \ . \end{aligned} \tag{1}$$

Firstly, we assume that the probability of a migration to state j at time t, conditioned on a migration in that direction d from state i, is the same for all issuers. We define

$$\begin{aligned} p_{j|id,\ell}(t) &= \mathbb{P}[dN_{ij,k\ell}(t) = 1 | d\tilde{N}_{id,k\ell}(t) = 1, N_{gh,k\ell}(s^-), Y_{g,k\ell}(s); \text{for all } g \neq h, 0 < s \leq t] \\ &= \mathbb{P}[dN_{ij,k\ell}(t) = 1 | d\tilde{N}_{id,k\ell}(t) = 1] \end{aligned} \tag{2}$$

and refer to it as the *conditional destination mass function.* Section 4 explores this distribution in detail.

Secondly, the directional component of Equation 1, the *directional migration intensity*, is modelled using a multiplicative intensity process such that

$$\begin{aligned} d\Lambda_{d|i,k\ell}(t) &= \mathbb{P}[d\tilde{N}_{id,k\ell}(t) = 1 | N_{gh,k\ell}(s^-), Y_{g,k\ell}(s); \text{for all } g \neq h, 0 < s \leq t] \\ &= Y_{i,k\ell}(t) \exp\left\{\boldsymbol{\beta}'_{id}\mathbf{X}_k(t)\right\} d\Lambda_{d|i,0\ell}(t) \ , \end{aligned} \tag{3}$$

where the covariate processes for issuer k are contained in $\mathbf{X}_k$, whose elements are functions of $N_{gh,k\ell}(s^-)$ and $Y_{g,k\ell}(s)$ for $g \neq h \in S, 0 < s \leq t$. The coefficient vector, $\boldsymbol{\beta}_{id}$, is common to all issuers in all strata for a d-migration from state i. All issuers in stratum ℓ and state i are subject to a *baseline* d-migration intensity of $d\Lambda_{d|i,0\ell}(t)$.

The exponential function and *at risk* process components of $d\Lambda_{d|i,k\ell}(t)$ are called a *relative risk process*, where the covariate processes, $\mathbf{X}_k$, have a multiplicative effect on the directional migration intensity. The baseline d-migration intensity, $d\Lambda_{d|i,0\ell}$, can be thought of as a directional migration intensity borne by all issuers in state i and strata ℓ, prior to being scaled for each issuer by their individual relative risk processes.

This multiplicative intensity process has two benefits. Firstly, the business cycle and macroeconomic effects are often cited as affecting the migration intensities, and the multiplicative intensity process allows a baseline intensity process common to all issuers that recognises these effects without explicit specification. This is unlike a fully parametric model, where we would need to specify an explicit functional form to consider these effects. Thus, the multiplicative intensity process allows us to focus our investigation on issuer-specific effects on the migration intensity. Secondly, in estimating the coefficients within a multiplicative intensity process we use the partial likelihood (Cox 1975), which allows us to ignore both the baseline intensity process and the conditional destination mass function. Therefore, we can show that the full migration intensity partial likelihood function is maximised by the directional and categorical components being maximised separately.

By applying partial likelihood estimation to the full migration intensity, $d\Lambda_{ij,k\ell}$, to estimate $\boldsymbol{\beta}_{id}$, we find the score vector and information matrix,

$$\begin{aligned}
\mathcal{U}(\boldsymbol{\beta}_{id}, t) &= \sum_{\ell} \left(\sum_{k=1}^{m_\ell} \int_0^t \mathbf{X}_k(s) d\tilde{N}_{id,d\ell}(s) - \int_0^t \mathcal{E}(\boldsymbol{\beta}_{id}, s)\, d\tilde{N}_{id,\cdot\ell}(s) \right) \quad \text{and} \\
\mathcal{I}(\boldsymbol{\beta}_{id}, t) &= -\sum_{\ell} \int_0^t \mathcal{V}(\boldsymbol{\beta}_{id}, s)\, d\tilde{N}_{id,\cdot\ell}(s) \ ,
\end{aligned}$$

where,

$$\begin{aligned}
\mathcal{E}(\boldsymbol{\beta}_{id}, t) &= \frac{\sum_{k=1}^{m_\ell} \mathbf{X}_k(t) Y_{i,k\ell}(s) \exp\{\boldsymbol{\beta}'_{id} \mathbf{X}_k(t)\}}{\sum_{k=1}^{m_\ell} Y_{i,k\ell}(s) \exp\{\boldsymbol{\beta}'_{id} \mathbf{X}_k(t)\}} \ , \\
\mathcal{V}(\boldsymbol{\beta}_{id}, t) &= \frac{\sum_{k=1}^{m_\ell} \mathbf{X}_k(t)^{\otimes 2} Y_{i,k\ell}(s) \exp\{\boldsymbol{\beta}'_{id} \mathbf{X}_k(t)\}}{\sum_{k=1}^{m_\ell} Y_{i,k\ell}(s) \exp\{\boldsymbol{\beta}'_{id} \mathbf{X}_k(t)\}} - \mathcal{E}(\boldsymbol{\beta}_{id}, t)^{\otimes 2} \ , \ \text{and} \\
\tilde{N}_{id,\cdot\ell}(t) &= \sum_{k=1}^{m_\ell} \tilde{N}_{id,k\ell}(t) \ .
\end{aligned}$$

The estimate for $\boldsymbol{\beta}_{id}$, $\hat{\boldsymbol{\beta}}_{id}$, is found by setting the score vector evaluated over the sample period equal to zero, $\mathcal{U}(\boldsymbol{\beta}_{id}, \tau) = 0$. The standard error and covariance associated with $\hat{\boldsymbol{\beta}}_{id}$ is approximated by the inverse of the observed information matrix, which we use for estimating the statistical significance of the coefficient estimates (Therneau & Grambsch 2000, chap. 3).

3.2 momentum and excitability

The first effect of interest for estimating the relative migration intensities is momentum. We aim to ascertain whether the direction of a credit rating migration influences the direction of the next migration. The covariates for testing momentum indicate whether the migration to the current credit rating was a downgrade or an upgrade. This differs slightly from the definitions used by Lando & Skødeberg (2002), who defined only an upward momentum covariate for both downgrades and upgrades, and it is unclear how migrations to and from NR are categorised. We consider migrations involving NR as neither a downgrade nor an upgrade, and hence we define two indicator covariates related to momentum for issuer k:

$$\begin{aligned}
\mathbb{1}_{k,down}(t) &= \begin{cases} 1 & \text{if the most recent migration at time } t \text{ was down} \\ 0 & \text{otherwise} \end{cases} \\
\mathbb{1}_{k,up}(t) &= \begin{cases} 1 & \text{if the most recent migration at time } t \text{ was up} \\ 0 & \text{otherwise} \end{cases} \ .
\end{aligned}$$

In our analyses, only the downgrade momentum covariate is fit for downgrade intensities ($\mathbf{X}_k(t) = \mathbb{1}_{k,down}(t)$), and likewise for upgrades ($\mathbf{X}_k(t) = \mathbb{1}_{k,up}(t)$), with a positive coefficient estimate signifying that a momentum effect exists.

The second effect of interest is excitability. We aim to deduce whether issuers that have recently migrated have a higher relative migration intensity than those that have maintained their credit rating. To model excitability, we apply a covariate that represents the duration spent in the current credit rating. The duration covariate is defined as the natural logarithm of the years since the issuer's most recent credit rating migration, thus, if t_k^* is the time of the most recent migration if issuer k, $\mathbf{X}_k(t) = \log(t - t_k^*)$. This functional form is chosen because it is well behaved and easily interpreted[3]. A negative coefficient estimate for duration confirms excitability, with a relative migration intensity diminishing as the sojourn in a credit rating increases.

Momentum and duration covariate models are calibrated individually and results are displayed in Table 1 for each credit rating. This is completed for the neighbouring migration intensities proposed by Lando & Skødeberg (2002) (the first and third models) and the directional migrations intensities (the second and fourth models). Thus in the second downgrade model, for example, each credit rating (i) has β_{id} estimated from the following directional migration intensity:

$$d\Lambda_{d|i,k\ell}(t) = Y_{i,k\ell}(t) \exp\left\{\beta_{id}\mathbb{1}_{k,down}(t)\right\} d\Lambda_{d|i,0\ell}(t) \ .$$

Also displayed is the sample size of migrations from credit rating i, and the sample size of migrations to credit rating j or in direction d. The coefficient estimate and its statistical significance are displayed for each model along with the log-likelihood (multiplied by -2) as the Model Fit statistic. We represent a coefficient estimate by the asterisks following the coefficient estimate, with statistical significance from zero at a confidence level above ninety percent indicated by one asterisk; above ninety-five percent indicated by two asterisks; and above ninety-nine percent indicated by three asterisks. A confidence level of at least ninety-five percent is required to accept, statistically, a coefficient estimate as significantly different from zero. We use the Model Fit statistics for comparison between models if the sample sizes are identical, with smaller values meaning less overall deviation of the fitted model from the data. Therefore, using the Model Fit statistics, we compare the momentum effect model to the duration effect model, but cannot compare a neighbouring migration model to a directional migration model. There is no distinction between industry sectors in the models presented in Table 1, with the number of strata, M, set to one.

The downgrade model for the momentum covariate has statistically significant and positive coefficient estimates in most credit ratings, which means a downgraded issuer is relatively more likely to experience a further downgrade. Aside from rare migrations from NA, issuers must migrate downwards into CCC- (not shown) since the only lower credit rating, default, is absorbing. Therefore, the CCC- downgrade covariate is uninformative and the coefficient estimate is statistically insignificant for downgrade intensities. Overall, the directional migration model is more adept at detecting momentum than the neighbouring migration model, particularly in the high risk credit ratings, with it accepting at ninety-nine percent statistical significance the otherwise rejected downgrade coefficient estimate for CCC+. Downgrades from AA+ to AA, however, have statistically insignificant momentum coefficient estimates in both migration models, which is possibly a consequence of the small sample size of AA+. A comparison of coefficient estimates between the neighbouring migration model and the directional migration model is uninformative, since these model estimate different migration intensities.

The downgrade model for the duration covariate also demonstrates statistically significant and negative coefficient estimates in most credit ratings, indicating excitability, with the

[3]for a comparison to other functional forms, refer to Addendum B at www.economics.unimelb.edu.au/aevans

Table 1: Momentum and Duration Coefficient Estimates for the Neighbouring and Directional Migration Models. Coefficient estimates, $\hat{\beta}$, are accompanied with asterisks representing statistical significance: * is ninety percent, ** is ninety-five percent, and *** is ninety-nine percent. The Model Fit statistics are calculated as $-2\times$log-likelihood.

Migration From		Neighbouring Migration Momentum Covariate				Directional Migration Momentum Covariate			Neighbouring Migration Duration Covariate				Directional Migration Duration Covariate		
Rating (i)	#From	Rating (j)	#To	$\hat{\beta}_{ij}^{momentum}$	Model Fit	#To	$\hat{\beta}_{id}^{momentum}$	Model Fit	Rating (j)	#To	$\hat{\beta}_{ij}^{duration}$	Model Fit	#To	$\hat{\beta}_{id}^{duration}$	Model Fit
The Downgrade Migrations															
AA+	41	AA	18	1.0341*	62	26	0.8845*	95	AA	18	−0.2754	64	26	−0.2550	96
AA	129	AA-	66	0.8361***	350	84	0.7282***	450	AA-	66	−0.2957***	348	84	−0.2457***	450
AA-	240	A+	115	0.7989***	803	148	0.6700***	1065	A+	115	0.1655*	817	148	0.2180***	1073
A+	404	A	185	0.4638***	1496	249	0.3182**	2062	A	185	−0.2251***	1488	249	−0.2016***	2050
A	513	A-	202	0.7657***	1810	280	0.6830***	2495	A-	202	−0.1231**	1834	280	−0.0957**	2522
A-	575	BBB+	227	0.7890***	2008	321	0.8494***	2846	BBB+	227	−0.0838	2038	321	−0.0997**	2894
BBB+	657	BBB	284	1.0577***	2530	373	1.0608***	3325	BBB	284	−0.1195***	2589	373	−0.1197***	3402
BBB	684	BBB-	293	0.5952***	2646	373	0.6944***	3372	BBB-	293	−0.1852***	2651	373	−0.1887***	3387
BBB-	671	BB+	201	0.5879***	1788	354	0.9186***	3105	BB+	201	−0.2033***	1786	354	−0.3347***	3077
BB+	523	BB	130	1.5435***	1025	260	1.6249***	2020	BB	130	−0.2164***	1063	260	−0.2886***	2078
BB	405	BB-	103	0.8199***	756	236	1.2184***	1690	BB-	103	−0.2276***	759	236	−0.4120***	1655
BB-	461	B+	154	0.4481***	1195	250	0.8956***	1917	B+	154	−0.0795	1201	250	−0.2139***	1939
B+	565	B	167	1.0351***	1359	300	1.1216***	2420	B	167	−0.3273***	1357	300	−0.3250***	2428
B	630	B-	206	0.8562***	1750	366	1.2035***	3029	B-	206	−0.1130**	1778	366	−0.2657***	3078
B-	698	CCC+	210	0.5914***	1794	422	0.9180***	3548	CCC+	210	−0.1804***	1792	422	−0.3187***	3500
CCC+	626	CCC	179	0.4166	1446	468	0.7653***	3781	CCC	179	−0.1285**	1443	468	−0.2963***	3714
CCC	460	CCC-	286	0.7472**	2001	344	0.8015**	2403	CCC-	286	−0.2867***	1963	344	−0.2638***	2366
The Upgrade Migrations															
AA+	41	AAA	9	17.3817	28	9	17.3817	28	AAA	9	0.6188	32	9	0.6188	32
AA	129	AA+	9	1.2580	52	11	1.4774*	63	AA+	9	−0.4725**	48	11	−0.3905**	63
AA-	240	AA	29	0.2617	192	36	0.3715	244	AA	29	0.2730*	189	36	0.1425	244
A+	404	AA-	80	0.5888***	605	85	0.5758***	653	AA-	80	−0.2731***	599	85	−0.2753***	646
A	513	A+	133	0.5481***	1133	150	0.6448***	1275	A+	133	0.1438*	1139	150	0.1163	1287
A-	575	A	157	0.3148*	1284	182	0.3304**	1514	A	157	0.1720**	1283	182	0.0799	1518
BBB+	657	A-	168	0.5775***	1421	204	0.4987***	1751	A-	168	−0.0159	1434	204	−0.0160	1763
BBB	684	BBB+	158	0.6658***	1411	199	0.5098***	1810	BBB+	158	0.0430	1428	199	0.0681	1821
BBB-	671	BBB	190	0.5425***	1570	246	0.5385***	2077	BBB	190	0.0895	1581	246	0.1097*	2090
BB+	523	BBB-	158	0.5191***	1256	206	0.4466***	1638	BBB-	158	0.0075	1266	206	0.0739	1646
BB	405	BB+	96	0.5217**	649	133	0.6030***	886	BB+	96	0.2587***	647	133	0.1086	894
BB-	461	BB	74	0.6391***	578	142	0.7122***	1082	BB	74	0.2160**	581	142	0.1500**	1094
B+	565	BB-	126	0.3587**	1006	173	0.1546	1388	BB-	126	0.1363*	1007	173	0.0665	1388
B	630	B+	124	0.4752***	978	160	0.4144***	1293	B+	124	0.1151	983	160	0.1052	1296
B-	698	B	124	0.9665***	1012	160	0.9372***	1319	B	124	0.0594	1034	160	0.0877	1346
CCC+	626	B-	65	0.8793***	504	86	0.9661***	670	B-	65	0.3160***	502	86	0.3246***	669
CCC	460	CCC+	35	1.1230**	237	73	1.0137***	496	CCC+	35	0.4627***	234	73	0.3917***	492

directional migration model again improving on the neighbouring migration model in almost all calibrations. We accept the duration coefficient estimates for AA-, A- and BB- under the directional migration model but not the neighbouring migration model, and there is no substantive evidence of excitability for AA+ in either model. AA- contradicts what our expectations and the other credit ratings suggest, with a statistically significant and positive coefficient estimate for duration meaning a longer stay in AA- increases the relative intensity of downgrading. Further investigation into this anomaly does not reveal overt influence from individual issuers on the calibration, and its presence remains a topic for future research[4].

Momentum and excitability effects on the migration intensities are far less convincing in the upgrade models, which we can assume is at least partially because of the smaller sample sizes. Momentum coefficient estimates are statistically significant mostly in the medium to high-risk credit ratings, although it is unclear whether the neighbouring or directional migration model is superior at detect this effect. Both models suggest that it is unlikely there are upgrade momentum effects in the very low risk credit ratings. In addition, we find very little evidence of excitability in the upgrade directional migration intensities, with only two credit ratings, AA and A+, having coefficient estimates that are both negative and statistically significant. We find statistically significant positive duration coefficient estimates for CCC+ and CCC, which suggest that the longer an issuer survives in the high-risk credit ratings, the higher the relative probability of a subsequent upgrade.

We extend our model with no industry distinction to fit two covariates, shown in Table 2, which, given the strong evidence of momentum and excitability, is a natural advancement on previous research. Firstly, we fit a model with both the momentum and duration covariates used in the previous analyses, and secondly, we fit a model with momentum and duration×momentum covariates—implying excitability is present only if the intensities are in the same direction as the previous migration. Thus in the forth downgrade model, for example, each credit rating (i) has an estimate for $\boldsymbol{\beta}_{id}$ from the following directional migration intensity:

$$d\Lambda_{d|i,k\ell}(t) = Y_{i,k\ell}(t) \exp\left\{\boldsymbol{\beta}'_{id}(\mathbb{1}_{k,down}(t), \mathbb{1}_{k,down}(t) \times \log(t - t^*_k))'\right\} d\Lambda_{d|i,0\ell}(t) \ ,$$

where t^*_k is the most recent migration time for issuer k.

We display the sum of all displayed Model Fit statistics, labelled Total Fit statistics, to be compared to models where all credit ratings between AA+ and CCC share identical coefficients ($\boldsymbol{\beta}_{id} \equiv \boldsymbol{\beta}_d$), but still vary in their baseline directional migration intensities. These Total Fit statistics have asterisks indicating whether we can safely reject the common migration coefficients. We also show whether the models with two covariates are a statistically significant improvement over the better fitting model with one covariate, indicated by the Model Fit asterisks. The neighbouring migration model is inferior to the directional migration model in all credit ratings, and is no longer displayed.

The third downgrade model (momentum and duration covariates) and the fourth downgrade model (momentum and duration×momentum covariates) in Table 2 show similar results. The statistical significance of the momentum coefficient estimates are improved when modelled alongside the duration (or duration×momentum) covariate, particularly in the high-risk credit ratings. We accept the momentum coefficient estimates, usually at over ninety-nine percent confidence, with the exception of AA+ and CCC- for the reasons discussed previously.

A slightly diminished level of acceptance applies to the duration coefficient estimates in the third model when compared to the second model. The fourth model, however, accepts the duration related covariate as convincingly as the second model. When comparing the

[4]for details of diagnostics refer to Addendum C at www.economics.unimelb.edu.au/aevans

Table 2: Directional Migrations Model with Momentum Covariate, Duration Covariate, Momentum and Duration Covariates, and Momentum and Momentum×Duration Covariates. Coefficient estimates, $\hat{\beta}$, are accompanied with asterisks representing statistical significance: * is ninety percent, ** is ninety-five percent, and *** is ninety-nine percent. The Model Fit statistics (calculated as −2×log-likelihood) for the third and forth models are accompanied with asterisks representing the improvement over the first or second model. Total Fit statistics are the overall fit with asterisks representing statistical significance over a common coefficient.

Migrations			Momentum Covariate		Duration Covariate		Momentum and Duration Covariatese			Momentum and Momentum×Duration Covariates		
Rating (i)	#From	#To	$\hat{\beta}_{id}^{momentum}$	Model Fit	$\hat{\beta}_{id}^{duration}$	Model Fit	$\hat{\beta}_{id}^{momentum}$	$\hat{\beta}_{id}^{duration}$	Model Fit	$\hat{\beta}_{id}^{momentum}$	$\hat{\beta}_{id}^{mom\times dur}$	Model Fit
The Downgrade Migrations												
AA+	41	26	0.8845*	95	−0.2550	96	0.8926*	−0.2537	93*	0.7547	−0.3436	92*
AA	129	84	0.7282***	450	−0.2457***	450	0.6236***	−0.2128***	442***	0.6265***	−0.2670***	442***
AA-	240	148	0.6700***	1065	0.2180***	1073	0.8082***	0.3171***	1051***	0.5972***	0.4398***	1052***
A+	404	249	0.3182**	2062	−0.2016***	2050	0.2864**	−0.1958***	2045***	0.3325**	−0.1785***	2054***
A	513	280	0.6830***	2495	−0.0957**	2522	0.6685***	−0.0743	2492***	0.6850***	−0.1690***	2486***
A-	575	321	0.8494***	2846	−0.0997**	2894	0.8316***	−0.0653	2844***	0.8223***	−0.1467***	2839***
BBB+	657	373	1.0608***	3325	−0.1197***	3402	1.0370***	−0.0596	3323***	1.0404***	−0.1000**	3320***
BBB	684	373	0.6944***	3372	−0.1887***	3387	0.6607***	−0.1732***	3354***	0.6264***	−0.2899***	3330***
BBB-	671	354	0.9186***	3105	−0.3347***	3077	0.8633***	−0.3209***	3032***	0.7718***	−0.3696***	3022***
BB+	523	260	1.6249***	2020	−0.2886***	2078	1.5594***	−0.2577***	1985***	1.4441***	−0.3117***	1973***
BB	405	236	1.2184***	1690	−0.4120***	1655	1.0778***	−0.3852***	1614***	0.8374***	−0.4483***	1597***
BB-	461	250	0.8956***	1917	−0.2139***	1939	0.8675***	−0.1982***	1899***	0.6977***	−0.3483***	1871***
B+	565	300	1.1216***	2420	−0.3250***	2428	1.0438***	−0.2936***	2363***	0.8492***	−0.3865***	2336***
B	630	366	1.2035***	3029	−0.2657***	3078	1.1530***	−0.2387***	2987***	0.9905***	−0.3104***	2968***
B-	698	422	0.9180***	3548	−0.3187***	3500	0.8724***	−0.3112***	3463***	0.6971***	−0.3313***	3461***
CCC+	626	468	0.7653***	3781	−0.2963***	3714	0.7722***	−0.2963***	3700***	0.5183**	−0.3136***	3692***
CCC	460	344	0.8015**	2403	−0.2638***	2366	0.7934**	−0.2635***	2359***	0.5598*	−0.2757***	2355***
Total Fit	8282	4854		39621***		39710***			39045***			38891***
$\boldsymbol{\beta}_{id} \equiv \boldsymbol{\beta}_d$	8282	4854	0.9044***	39680	−0.2439***	39821	0.8512***	−0.2248***	39228	0.7876***	−0.2922***	39034
The Upgrade Migrations												
AA+	41	9	17.3817	28	0.6188	32	17.2095	0.3218	28**	16.9951	0.3218	28**
AA	129	11	1.4774*	63	−0.3905**	63	1.5662*	−0.4085**	58**	1.4856*	−0.2740	62
AA-	240	36	0.3715	244	0.1425	244	0.3137	0.1199	243	0.2770	0.1959	243
A+	404	85	0.5758***	653	−0.2753***	646	0.5255**	−0.2619***	640***	0.5118**	−0.3427***	638***
A	513	150	0.6448***	1275	0.1163	1287	0.6366***	0.1075	1272***	0.5445***	0.2151**	1270***
A-	575	182	0.3304**	1514	0.0799	1518	0.3204**	0.0707	1513**	0.3221**	0.0259	1514*
BBB+	657	204	0.4987***	1751	−0.0160	1763	0.5027***	−0.0267	1750***	0.5134***	−0.0981	1748***
BBB	684	199	0.5098***	1810	0.0681	1821	0.5068***	0.0631	1809***	0.5174***	−0.0378	1810***
BBB-	671	246	0.5385***	2077	0.1097*	2090	0.5415***	0.1130**	2073***	0.5394***	0.0549	2076***
BB+	523	206	0.4466***	1638	0.0739	1646	0.4313***	0.0555	1637***	0.4467***	0.0027	1638***
BB	405	133	0.6030***	886	0.1086	894	0.5922***	0.0970	884***	0.5824***	−0.0720	885***
BB-	461	142	0.7122***	1082	0.1500**	1094	0.6990***	0.1436*	1078***	0.7128***	0.0055	1082***
B+	565	173	0.1546	1388	0.0665	1388	0.1578	0.0680	1387	0.1797	0.1010	1387
B	630	160	0.4144***	1293	0.1052	1296	0.3934**	0.0921	1291**	0.4246***	0.0694	1292**
B-	698	160	0.9372***	1319	0.0877	1346	0.9468***	0.1011	1317***	0.9525***	0.0453	1319***
CCC+	626	86	0.9661***	670	0.3246***	669	1.0114***	0.3413***	659***	1.0269***	0.1254	670
CCC	460	73	1.0137***	496	0.3917***	492	0.9831***	0.3941***	483***	1.1318***	0.4615	494
Total Fit	8282	2255		18186*		18288***			18123***			18155***
$\boldsymbol{\beta}_{id} \equiv \boldsymbol{\beta}_d$	8282	2255	0.5356***	18212	0.0788***	18336	0.5294***	0.0717***	18196	0.5355***	−0.0055	18212

models with two covariates, we find that the fourth model is equivalent or superior to the third model in almost all credit ratings when we compare the Model Fit statistics, with the third model only better for AA-. It is inconclusive which of the third or fourth model is preferable for AA+, but neither is a statistically significant improvement over the first model (using a χ_1^2 distribution). In all other credit ratings, the better fitting two-covariate model is superior to the better fitting one-covariate model, justifying our advancement of the model to fit two covariates. The Total Fit statistics provide further evidence that the fourth model is preferable overall.

It is not surprising that the third and fourth upgrade models deliver unconvincing results, since there is limited data when fitting even the first and second upgrade models. A comparison of the Model Fit statistics illustrates that the third and fourth models make little-to-no improvement over the first or second models. The exceptions to this are the credit ratings A+ and BBB+, which demonstrate a statistically significant improvement over the one-covariate models. Furthermore, the Total Fit statistic for the first model is a not statistically significant improvement over the Model Fit for a model with the common β_d, so caution must be exercised if inferring upward momentum effects are more apparent in some credit ratings than others.

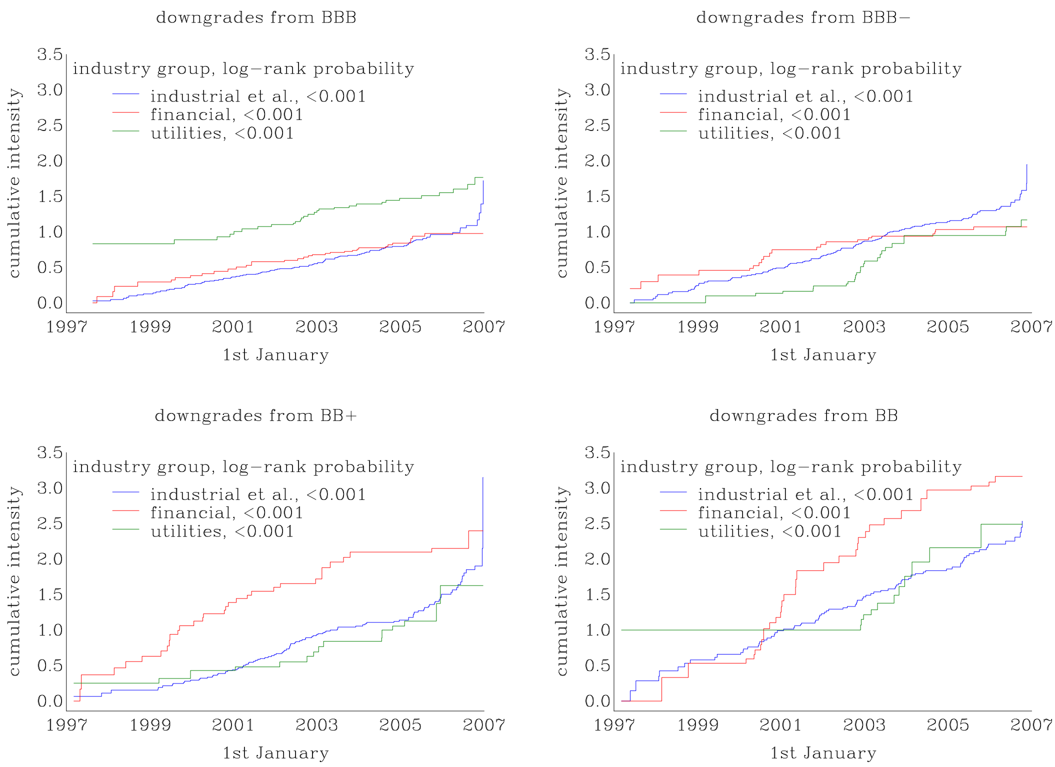
Our final advancement in the modelling of issuer-specific effects on credit rating migrations is to investigate the effect of momentum and excitability when the assumption of industry homogeneity is relaxed. We achieve this by allowing each industry sector to have a unique baseline directional migration intensity. As mentioned previously, Standard and Poor's classify industry sectors as industrial, finance, utility, government and miscellaneous. For partial likelihood estimation, a small stratum sample can easily compromise our estimates, where, by considering the effect on the denominator of the partial likelihood process, small samples make a disproportionately large contribution. Unfortunately, there is insufficient data in the government and miscellaneous sectors to give reasonable results, so these sectors are included in the industrial category, thus leaving three industry strata ($M = 3$).

Initially, we justify the stratification by appealing to the Nelson-Aalen estimates (Aalen 1978), a non-parametric method of estimating the cumulative intensity function from 1 April 1995. A stratified log-rank test is used to judge whether, statistically, the cumulative directional migration intensity estimate of each industry group is significantly different from a non-stratified estimate (Andersen 1982, Section V.3.2). The log-rank probability—the estimated probability that the industry sector Nelson-Aalen estimate is the same as a non-stratified Nelson-Aalen estimate—is presented with each Nelson-Aalen curve in Figure 2. For brevity, only the downward cumulative migration intensities for credit ratings BBB, BBB-, BB+ and BB between 1 January 1997 and 31 December 2006 are displayed, but the null hypotheses of industry homogeneity were rejected with a confidence above 99.9 percent for every industry group in all credit ratings over the full observation period[5]. Visually, the behaviour of the cumulative migration intensities over time confirms the use of stratification rather than introducing indicator covariates for the industry sectors. Using industry sector indicator covariates would assume the baseline directional migration intensities are proportional, which appears unlikely given the cumulative migration intensities in Figure 2 do not diverge at a constant rate. Furthermore, we find support for our use of different baseline directional migration intensities between credit ratings from the differences in the cumulative directional migration intensities of these four adjacent credit ratings.

Whilst useful for validating our assumptions, a comparison of the Nelson-Aalen estimates does not deliver a measure of how our directional migration intensities with industry

[5]for full investigation into the Nelson-Aalen estimates refer to Addendum D at www.economics.unimelb.edu.au/aevans

Figure 2: Nelson-Aalen Estimates for the Cumulative Downgrade Intensities with Stratified Log-Rank Probabilities for credit ratings: BBB, BBB-, BB+ and BB; and between industry sectors: industrial et al., financial and utilities



distinction compare to those without. We cannot be confident that momentum and duration covariate values are not cause of differences in the cumulative migration intensities between industry sectors. This concern is investigated below, where we compliment the simple Nelson-Aalen estimate with a more precise but computationally intensive Monte Carlo simulation.

We propose a Monte Carlo technique for testing if stratifying a model by industry sector is superior to an unstratified model. Since the log-likelihood statistics of stratified and non-stratified partial likelihood estimate are not comparable, we instead work under the equivalent null hypothesis that industry distinction of the initial sample is non-informative[6], that is, any arbitrary stratification would achieve the similar results to industry distinction. The procedure for testing is as follows:

1. Find the empirical distribution for the industry sector classification between issuers in each credit rating, that is, the proportion of issuers residing in each industry sector.

2. Allocate each issuer to a category randomly based on the industry sector empirical distribution.

3. Calibrate the multiplicative intensity model (Equation 3) while stratifying by the randomly allocated categories, recording the Model Fit ($-2\times$log-likelihood).

4. Repeat steps 2 and 3 until a representative distribution of the Model Fit statistic has been achieved. (We complete ten thousand iterations for each model because the

[6]The author is unaware of previous attempts at such a test—likely due the computational power required. We welcome any references or suggestions

process is computationally intensive, but the Model Fit distributions have converged adequately to test our hypotheses.)

5. The simulated probability is the proportion of the simulated Model Fit statistics that equal or exceed the observed Model Fit statistics when stratified by industry sector.

We support industry distinction if the simulated probability is less than five percent, and claim that, statistically, the industry distinction model is significantly different from a non-informative stratification.

Table 3 displays the coefficient estimates and significance under industry distinction for all previously presented directional migration models. Before we compare the new coefficient estimates under the models with industry distinction, we focus on the results of the Monte Carlo simulation to assess the validity of the stratification by industry sector. The Model Fit asterisks represent whether, by the Monte Carlo method outline above, a stratification of the sample by industry sector is a statistically significant improvement from a random stratification of similar dimensions, and thus informative (except the Total Fit, where asterisks are already defined). We stress that the different definitions for the asterisks following the Model Fit statistics for the third and fourth models than in Table 2. It may reassure the more sceptical that this stratification is required to then later stratify the conditional destination mass functions (Section 4), which, as we will see, do show measurable statistically significant industry heterogeneity.

As suspected from the Nelson-Aalen estimates for downgrade intensities, distinction between industrial (with government and miscellaneous), financial and utilities industry sectors is a statistically significant improvement over a non-informative stratification in the majority of credit ratings. We haven't sufficient evidence, however, to conclude that industry distinction is informative for AA+, BB+, BB, B+ and, for all except the first model, CCC+. We observe in Figure 2 that the financial sectors depart from the industrial sectors in downgrade intensity for both BB+ and BB, and this observation is supported by very small stratified log-rank test statistics. This discrepancy justifies use of the Monte Carlo method in testing the suitability of industry distinction, since this method captures the effects of momentum and excitability. The divergence between the financial and industrial sectors in Figure 2 is caused by one sector, on average, having covariate values that would increase the downgrade intensity relative to the other sector.

For example, between 1 January 1997 and 1 January 2001, the proportion of issuers in BB+ that entered by a downgrade is between ten and twenty percent higher for the financial sector than the industrial sector[7]. Thus, momentum effects rather than industry heterogeneity can explain the higher downgrade intensities over this period. Using a similar argument, we explain the high (relative) downgrade intensity for industrial sector issuers in BB+ from 1 January 2004 by identifying that the average time spent in BB+ is a year shorter than for the financial sector. Thus, under a model demonstrating excitability, we would expect the industrial sector, on average, to suffer a higher downgrade intensity over this period, even under industry homogeneity. For BB, The Nelson-Aalen estimates for the downgrade intensities differ greatest between industry sectors in the period surrounding 1 January 2001. The proportion of issuers in BB that entered by a downgrade is approximately twenty percent higher for the financial sector over this period when compared to the industrial sector. Again, we can assert that a momentum effect could explain the divergence in the Nelson-Aalen estimates for the cumulative downgrade intensities. Therefore, we cannot reject industry homogeneity in these specific credit ratings. It is prudent to remember,

[7] for detailed descriptive statistics refer to Addendum A at www.economics.unimelb.edu.au/aevans

Table 3: Directional Migrations Model under Industry Distinction with the Momentum Covariate, Duration Covariate, Momentum and Duration Covariates, and Momentum and Momentum×Duration Covariates. Coefficient estimates, $\hat{\beta}$, are accompanied with asterisks representing statistical significance: * is ninety percent, ** is ninety-five percent, and *** is ninety-nine percent. Model Fit statistics (calculated as −2×log-likelihood) are accompanied with asterisks representing the simulated probability of being an uninformative stratification. Total Fit statistics are the overall fit with asterisks representing statistical significance over a common coefficient.

Migrations			Momentum Covariate		Duration Covariate		Momentum and Duration Covariates			Momentum and Momentum×Duration Covariates		
Rating (i)	#From	#To	$\hat{\beta}_{ij}^{momentum}$	Model Fit	$\hat{\beta}_{ij}^{duration}$	Model Fit	$\hat{\beta}_{ij}^{momentum}$	$\hat{\beta}_{ij}^{duration}$	Model Fit	$\hat{\beta}_{ij}^{momentum}$	$\hat{\beta}_{id}^{mom\times dur}$	Model Fit
The Downgrade Migrations												
AA+	41	26	0.9778*	57	−0.4463*	56*	0.9912	−0.4803*	53*	0.7943	−0.5677	54
AA	129	84	0.7184***	325**	−0.2021***	327**	0.6370**	−0.1695**	321**	0.6166**	−0.2132**	320**
AA-	240	148	0.6387***	763***	0.2094**	769***	0.7895***	0.3057***	751***	0.5635***	0.5354***	746***
A+	404	249	0.2503*	1568***	−0.1819***	1557***	0.2200	−0.1771***	1554***	0.2611*	−0.1671***	1560***
A	513	280	0.6356***	1943***	−0.1293***	1963***	0.6112***	−0.1089**	1938***	0.6400***	−0.1979***	1932***
A-	575	321	0.8274***	2234***	−0.0860*	2280***	0.8131***	−0.0477	2233***	0.8017***	−0.1265**	2229***
BBB+	657	373	1.0226***	2619***	−0.1301***	2687***	0.9943***	−0.0771*	2615***	0.9999***	−0.1244**	2612***
BBB	684	373	0.6794***	2747***	−0.2021***	2756***	0.6451***	−0.1889***	2726***	0.6164***	−0.3239***	2697***
BBB-	671	354	0.9049***	2488***	−0.3155***	2468***	0.8388***	−0.2997***	2427***	0.7666***	−0.3430***	2420***
BB+	523	260	1.5601***	1653	−0.2784***	1705	1.4936***	−0.2433***	1624	1.3988***	−0.3008***	1613
BB	405	236	1.1619***	1335	−0.3950***	1303	1.0387***	−0.3711***	1267	0.8233***	−0.4421***	1251
BB-	461	250	0.8314***	1566**	−0.2016***	1584**	0.8075***	−0.1876***	1552*	0.6256***	−0.3708***	1523**
B+	565	300	1.0834***	2105	−0.2983***	2117	1.0113***	−0.2692***	2059	0.8294***	−0.3651***	2033
B	630	366	1.1292***	2611***	−0.2523***	2651***	1.0884***	−0.2310***	2574***	0.9422***	−0.3019***	2557***
B-	698	422	0.8857***	3233**	−0.3017***	3195**	0.8236***	−0.2911***	3163**	0.6841***	−0.3062***	3163**
CCC+	626	468	0.7279***	3464**	−0.2905***	3401*	0.7284***	−0.2899***	3389*	0.4936**	−0.3069***	3382*
CCC	460	344	0.8146**	2110***	−0.2472***	2082***	0.7518**	−0.2435***	2076***	0.5744*	−0.2527***	2074***
Total Fit	8282	4854		32821***		32900***			32322***			32166***
$\beta_{id} \equiv \beta_d$	8282	4854	0.8655***	32876	−0.2355***	32989	0.8121***	−0.2170***	32475	0.7567***	−0.2844***	32300
The Upgrade Migrations												
AA+	41	9	17.9507	19	0.2650	25	18.0592	−0.0620	19	18.0529	−0.0620	19
AA	129	11	1.9243**	46	−0.3336*	49	2.1924**	−0.3739**	42	2.0251**	−0.2907	44
AA-	240	36	0.6377*	184	0.1630	186	0.5749	0.1086	184	0.5512	0.1638	184
A+	404	85	0.4347*	512	−0.2228***	507	0.3944*	−0.2120***	504	0.3955*	−0.2765***	502
A	513	150	0.4871***	1041	0.1541*	1045	0.4757***	0.1483*	1038	0.3553*	0.2795**	1034
A-	575	182	0.2827*	1175***	0.0540	1178***	0.2777*	0.0482	1174***	0.2709*	0.0327	1175***
BBB+	657	204	0.4744***	1378***	−0.0303	1388***	0.4786***	−0.0372	1377***	0.4906***	−0.0960	1376***
BBB	684	199	0.4135***	1454***	0.0779	1460***	0.4130***	0.0771	1452***	0.4170***	−0.0213	1454***
BBB-	671	246	0.5359***	1665	0.1195**	1676*	0.5360***	0.1193**	1660	0.5353***	0.0713	1664
BB+	523	206	0.4074***	1278***	0.0595	1284***	0.3964***	0.0448	1277***	0.4074***	−0.0008	1278***
BB	405	133	0.6043***	646***	0.1213	653***	0.5866***	0.1036	644***	0.5753***	−0.1112	644***
BB-	461	142	0.7381***	873**	0.1445*	885**	0.7263***	0.1418*	869*	0.7370***	−0.0127	873*
B+	565	173	0.1887	1169***	0.0592	1169***	0.1946	0.0626	1168***	0.2151	0.0991	1168***
B	630	160	0.4303**	1046***	0.0939	1051***	0.4165**	0.0838	1045***	0.4384**	0.0548	1046***
B-	698	160	0.9102***	1124***	0.0928	1147***	0.9202***	0.1054	1121***	0.9262***	0.0483	1124***
CCC+	626	86	0.9288***	557***	0.2769***	556***	0.9924***	0.3008***	548***	1.0184***	0.1949	556***
CCC	460	73	0.9915***	427**	0.3853***	423**	0.9913***	0.3986***	415**	1.1478***	0.4316	425**
Total Fit	8282	2255		14592*		14683***			14538***			14565***
$\beta_{id} \equiv \beta_d$	8282	2255	0.5066***	14618	0.0772***	14721	0.5015***	0.0718***	14603	0.5066***	−0.0047	14618

however, that this does not imply we accept industry homogeneity; simply that we have insufficient evidence to disprove it.

BBB and BBB- are examples where the Monte Carlo simulated probabilities confirm the conclusions of the Nelson-Aalen stratified log-rank tests. There is a very small group *at risk* early in 1997 for the utilities sector issuers rated BBB, which is why the cumulative downgrade intensity is initially higher than in the industrial or financial sectors. Otherwise, the Nelson-Aalen estimates for the cumulative downgrade intensities for each industry sector appear approximately parallel, which supports industry homogeneity. The proportion of the financial sector issuers in BBB that entered by a downgrade over the period 1 January 1977 to 1 January 1999, however, is approximately twice that of the industrial sector issuers. Thus, after allowing for momentum effects, we would expect that, under industry homogeneity, the financial sector downgrade intensity over this period to exceed the industrial sector downgrade intensity. The Nelson-Aalen estimates for the issuers in BBB- suggests the financial sector downgrade intensity is less than the industrial sector downgrade intensity between 1 January 2002 and 31 December 2006. The proportions of issuers that entered by a downgrade and the average times spent in BBB- are similar between industry sectors, and therefore do not explain the different downgrade intensities; thus industry heterogeneity is a valid concern.

Table 3 shows that the coefficient estimates under industry distinction are similar to when industry homogeneity was assumed, although this likeness is not to be mistaken for proof of industry homogeneity in the baseline directional migration intensity. In fact, the similarity inspires confidence in the assumption that the coefficients are equal between industry sectors, since we obtain these estimates despite the different effective weighting of observations.

The level of coefficient estimate acceptance for the downgrade models is similar to when there was no industry distinction. Most notable, however, is that the momentum coefficient estimates for A+ are now rejected in the first, third and fourth models. After introducing industry distinction, we see the third and fourth models remain statistically significant improvements over directional migration intensities when we model momentum or duration alone. The superior model for each credit rating is the same as when we assume industry homogeneity. In most cases, this is the model containing the momentum and duration$\times$momentum covariates. The Total Fit statistics again show that we must discriminate in coefficient estimates between credit ratings, and that the overall performance of the fourth model is superior to the third model.

Coefficient estimates and significance in the upgrade models in Table 3 are also very similar to those in Table 2. The momentum coefficient estimates are no longer statistically significant for A+ and A- when modelled alone, but now qualify for AA, although we must remember that we do not have enough evidence to reject that all momentum upgrade intensities share the same coefficient values. The duration covariate model varies little, with the confidence in coefficient estimates slightly decreasing for AA and BB-. These observations also apply to changes in the momentum and duration coefficient estimates in the third and fourth models. Just as with no industry distinction, the presence of momentum and excitability in the upgrade models is underwhelming in comparison to the downgrade models. Despite concerns regarding the usefulness of these upgrade models, they confirm that industry distinction is a significant improvement over industry homogeneity in medium to high-risk credit ratings.

These models were extended to allow the momentum and duration coefficient estimates to also vary between industry sectors, but the results are not shown because the difference in the momentum and duration coefficient estimates are not statistically significant between industry sectors for any credit rating. Thus we conclude that, on the data sample used, industry distinction between momentum and excitability effects is not profound enough to warrant separate determination, despite industry distinction being compelling for the direction migration intensity in general.

We leave it to future research to propose suitable forms of the baseline directional migration intensity processes.

4 conditional destination probability

Let us consider now the conditional destination mass function defined in Equation 2. Here, we adopt a unique approach of assuming that given that a directional migration occurs, we have a categorical distribution of the destination credit rating. In other words, given an issuer has downgraded (or upgraded), with what probability does the issuer enter each of the below (or above) credit ratings? We derive conditional destination mass function before initiating our investigation. We assume that this function is equal between industry sectors and does not change over time. For brevity, this section presents cross-sectional analyses of downgrade destinations only[8].

After proposing a simple model for the categorisation of rating migrations, we test common homogeneity hypotheses. For these hypotheses, we often consider the distance downgraded, j^*, (for example, a migration from AA+ to AA- is a downgrade of two) instead of the destination credit rating, j. A comparison of the mass function between credit ratings, i, of the distance downgraded is intuitively more reasonable than a similar comparison of the downgrade destination. For example, we'd expect the probability of migrating from BB- to B+ to be comparable to the probability of migrating from BB to BB-, and not to B+. The initial hypothesis assumes that the different credit ratings share the same conditional mass function for the distance downgraded. In addition, we test special cases of this by discarding the positive and negative signs from the credit ratings and thus creating larger credit rating groups. The later hypotheses assume that the conditional destination mass functions are homogeneous between industry sectors.

4.1 categorical model estimation

In this section we specify the conditional destination mass function from equation 2, before finding appropriate estimators. A basic understanding of categorical distributions—particularly the multinomial distribution—is all that is required to appreciate the results in Section 4.2, and the remainder of this section may be ignored by those unfamiliar with the specifics of the multinomial distribution.

Recall, the conditional destination mass function, $p_{j|id,\ell}(t)$, determines the probability of an issuer in stratum ℓ entering credit rating j given that it is undergoing a d-directional migration from credit rating i at time t. An important constraint of this function is that we cannot stratify the sample unless the same stratification occurred in the estimation of the directional migration intensity, because of the assumptions underlying excluding $p_{j|id,\ell}(t)$ from the partial likelihood estimation of Equation 1. Note, however, that the multiplicative intensity model estimated using the partial likelihood is naturally stratified by time.

Although theoretically possible, it would be foolish to have the conditional destination mass function be different for each event time because the partial likelihood estimation in Section 3.1 allows a maximum of one migration to occur in state i and stratum ℓ at any time t—the regression would be performed on a single observation! Instead, we define the

[8]The equivalent upgrade destination analyses can be found in Addendum E.1 at www.economics.unimelb.edu.au/aevans

conditional destination mass function over a range of times, possibly the entire range, to gain results that are more meaningful. We denote the time range (s,t) and hence define the conditional distance mass function, $p_{j|id,\ell}(s,t)$, as the probability of migrating to credit rating j between the times s and t given a d-directional migration from credit rating i in stratum ℓ.

For our analyses, we assume the conditional distribution of the destination credit rating j follows a product multinomial distribution. The choice of the conditional destination mass function, however, is not limited to the product multinomial distribution. This choice involves few constraints and renders hypothesis testing convenient. One such constraint is that the observations are independent; an imposition that may not hold if an issuer downgrade from the same credit rating on more than one occasion, although such events are rare and thus of little consequence if the observation independence of an issuer is a concern. Future research could assume the credit ratings are ordinal—although such an assumption seems viable, it is dismissed here as unnecessary and may cause problems in high risk credit ratings, where declaration of default is partially a decision of the issuer rather than Standard & Poor's.

Finally, to express our product multinomial distribution we must define:

$$\begin{aligned} N_{ij,k\ell}(s,t) &= N_{ij,k\ell}(t) - N_{ij,k\ell}(s) \ , \text{ and} \\ N_{ij,\cdot\ell}(s,t) &= \sum_{k=1}^{m_\ell} N_{ij,k\ell}(s,t) \end{aligned}$$

with observed values $n_{ij,\cdot\ell}(s,t)$, and recall that

$$\tilde{N}_{id,\cdot\ell}(s,t) = \sum_{j\in d}\sum_{k=1}^{m_\ell} N_{ij,k\ell}(s,t)$$

with observed values $\tilde{n}_{id,\cdot\ell}(s,t)$. Thus,

$$\begin{aligned} &\mathbb{P}[N_{ij,\cdot\ell}(s,t) = n_{ij,\cdot\ell}(s,t), \text{ for all } j\in d|\tilde{N}_{id,\cdot\ell}(s,t) = \tilde{n}_{id,\cdot\ell}(s,t); 0\le s<t] \\ &= \begin{cases} \tilde{n}_{id,\cdot\ell}(s,t)!\prod_{j\in d}\frac{p_{j|id,\ell}(s,t)^{n_{ij,\cdot\ell}(s,t)}}{n_{ij,\cdot\ell}(s,t)!} & \text{if } \sum_j n_{ij,\cdot\ell}(s,t) = \tilde{n}_{id,\cdot\ell}(s,t) \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{4}$$

under the constraint

$$\sum_{j\in d} p_{j|id,\ell}(s,t) = 1 \ .$$

It can be shown that we find the unbiased and consistent estimator for $p_{j|id,\ell}(s,t)$ by applying the method of Lagrage multipliers to the natural logarithm of the likelihood equation (Lawal 2003, Section 2.4.4), and thus an estimate of

$$\hat{p}_{j|id,\ell}(s,t) = \frac{n_{ij,\cdot\ell}(s,t)}{\tilde{n}_{id,\cdot\ell}(s,t)} \ .$$

The standard error of this estimate is

$$s.e.[\hat{p}_{j|id,\ell}(s,t)] = \sqrt{\frac{\hat{p}_{j|id,\ell}(s,t)(1-\hat{p}_{j|id,\ell}(s,t))}{\tilde{n}_{id,\cdot\ell}(s,t)}} \ .$$

The sample for improbable events is small, which, in the past, has been problematic for inference on credit migrations. Recall, the motivation for splitting the full migration intensity into a directional intensity and a conditional probability was to maximise the inference possible on the available data. Although we increased data sample sizes for the directional components, we encounter the same data restrictions on the conditional probability component that has burdened previous research—low observation counts relating to unlikely events. Inference via categorical analysis, however, is better suited to deal with these small sample issues, with hypotheses tested under an exact method—using a multivariate hypergeometric distribution.

We calculate Fisher's p-value (Zelterman 2006, Chapter 2) to test the hypothesis that the conditional mass functions for the distance downgraded are equal between credit ratings. This Fisher's p-value uses the multivariate hypergeometric distribution to find the probability that a distribution equally or less likely than the observed distribution occurs. Since this method does not rely on asymptotic results, we can compare estimated mass functions of samples with very low residency. When comparing many samples, such as every credit rating, a Monte Carlo method is used to estimate Fisher's p-value, since doing so directly is computationally intensive. For this estimation, we simulate until the estimate of Fisher's p-value is within 0.0001 of its true value with a probability of ninety-five percent (this never requires more than 100,000 simulations). We consider the relevant null hypothesis as rejected if the Fisher's p-value is less than or equal to five percent, that is, if the conditional distributions of distance downgraded are equal between strata, more than ninety-five percent of possible observation distributions are more probable than the one observed.

4.2 testing hypotheses

The estimates, $\hat{p}_{j|id,\ell}(s,t)$, and their standard errors, $s.e.[\hat{p}_{j|id,\ell}(s,t)]$, are shown in the top panel of Table 4. For the reasons previously mentioned, we have exchanged the destination credit rating j with the number of classes downgraded j^*. Changing the focus from j to j^* does not affect the model whatsoever. j^* is truncated to equal a maximum of ten, with any higher observation (the highest observed is eleven) included in this group. The maximum distance downgraded decreases as we approach the high-risk credit ratings because an issuer cannot be downgraded any further than default.

Confirming expectations, as j^* increases, $p_{j|id,\ell}(s,t)$ decreases. There are exceptions to this observation for very high values of j^*, however, this is not surprising given the infrequent observation of migrations to these categories. The standard errors presented demonstrate the strong confidence in the probability estimates for close migrations, although estimates become dubious for further migrations, which is a common complaint of low probability categories in the multinomial distribution. A downgrade from CCC+ is notable because it is more likely to downgrade two credit ratings than one. As CCC- is the lowest transient credit rating, it has only one possible downward migration—default.

A lack of observations for some rare events is an obvious deficiency of the estimates, shown in the upper right-hand side of Table 4. Since no migrations to these categories have occurred, they are not included in our estimate of the product multinomial distribution, and so carry no probability estimate. This poses no problem for application purposes, since when using the model over any discrete period, such migrations are possible, albeit in two steps.

The middle panel of Table 4 fits the multinomial model under the null hypothesis (H_0^{all}) that the conditional mass function for the distance downgraded is equal for all credit ratings. We truncate the possible distance downgraded to a maximum of five so each credit rating considered has the same range of migration distances. Also for this reason, we exclude B-, CCC+, CCC and CCC-. Fisher's p-value is used to test the null hypothesis that all included classes share the same conditional mass function, giving a probability of 0.01 percent that the distribution of the distance downgraded is identical for the credit ratings AAA to B; a rejection of the null hypothesis. We test this hypothesis with a maximum distance

Table 4: Conditional Mass Functions for Distance Downgraded—Estimations and Testing Class Homogeneity. $\hat{p}_{j|id,\ell}(s,t)$ $(s.e.\hat{p}_{j|id,\ell}(s,t)])$. The hypotheses test whether the ungrouped distributions are a statistically significant improvement over the grouped distributions.

Conditional Destination Mass Function Estimate for the Number of States Downgraded, j^*, for Each Rating Class, i

i/j^*	1	2	3	4	5	6	7	8	9	10+
AAA	0.5408 (0.0503)	0.1939 (0.0399)	0.1122 (0.0319)	0.0612 (0.0242)	0.0306 (0.0174)	0.0102 (0.0102)	0.0306 (0.0174)	0.0204 (0.0143)	-	-
AA+	0.6974 (0.0527)	0.1711 (0.0432)	0.0526 (0.0256)	0.0395 (0.0223)	-	0.0132 (0.0131)	-	0.0132 (0.0131)	-	0.0132 (0.0131)
AA	0.7232 (0.0336)	0.1525 (0.0270)	0.0791 (0.0203)	0.0113 (0.0079)	0.0056 (0.0056)	0.0226 (0.0112)	-	-	-	0.0056 (0.0056)
AA-	0.8026 (0.0226)	0.1003 (0.0171)	0.0485 (0.0122)	0.0259 (0.0090)	0.0065 (0.0046)	0.0065 (0.0046)	-	-	-	0.0097 (0.0056)
A+	0.7461 (0.0206)	0.1730 (0.0179)	0.0449 (0.0098)	0.0202 (0.0067)	0.0090 (0.0045)	0.0022 (0.0022)	0.0022 (0.0022)	-	0.0022 (0.0022)	-
A	0.7300 (0.0194)	0.1730 (0.0165)	0.0627 (0.0106)	0.0228 (0.0065)	0.0038 (0.0027)	0.0038 (0.0027)	-	0.0019 (0.0019)	0.0019 (0.0019)	-
A-	0.7344 (0.0189)	0.1813 (0.0165)	0.0513 (0.0094)	0.0110 (0.0045)	0.0073 (0.0036)	0.0037 (0.0026)	0.0055 (0.0032)	0.0018 (0.0018)	-	0.0037 (0.0026)
BBB+	0.7591 (0.0174)	0.1678 (0.0152)	0.0249 (0.0064)	0.0233 (0.0061)	0.0100 (0.0040)	0.0033 (0.0023)	0.0050 (0.0029)	0.0066 (0.0033)	-	-
BBB	0.7818 (0.0173)	0.1326 (0.0142)	0.0436 (0.0085)	0.0122 (0.0046)	0.0140 (0.0049)	0.0052 (0.0030)	0.0070 (0.0035)	-	0.0017 (0.0017)	0.0017 (0.0017)
BBB-	0.6108 (0.0207)	0.1856 (0.0165)	0.0811 (0.0116)	0.0631 (0.0103)	0.0198 (0.0059)	0.0180 (0.0056)	0.0036 (0.0025)	0.0054 (0.0031)	0.0072 (0.0036)	0.0054 (0.0031)
BB+	0.5059 (0.0272)	0.2692 (0.0241)	0.1065 (0.0168)	0.0444 (0.0112)	0.0473 (0.0116)	0.0148 (0.0066)	0.0030 (0.0030)	0.0030 (0.0030)	0.0059 (0.0042)	
BB	0.4505 (0.0281)	0.2588 (0.0248)	0.1534 (0.0204)	0.0799 (0.0153)	0.0192 (0.0078)	0.0128 (0.0063)	0.0256 (0.0089)	-		
BB-	0.6474 (0.0240)	0.1990 (0.0200)	0.0655 (0.0124)	0.0277 (0.0082)	0.0227 (0.0075)	0.0202 (0.0071)	0.0176 (0.0066)			
B+	0.6188 (0.0208)	0.2228 (0.0179)	0.0718 (0.0111)	0.0442 (0.0088)	0.0313 (0.0075)	0.0110 (0.0045)				
B	0.6088 (0.0183)	0.2246 (0.0157)	0.0734 (0.0098)	0.0607 (0.0090)	0.0325 (0.0067)					
B-	0.5692 (0.0173)	0.2209 (0.0145)	0.1748 (0.0132)	0.0352 (0.0064)						
CCC+	0.4097 (0.0212)	0.4637 (0.0215)	0.1266 (0.0144)							
CCC	0.8391 (0.0190)	0.1609 (0.0190)								
CCC-	1.0000 (0.0000)									

Testing whether all rating classes share the same conditional mass functions

all	1	2	3	4	5+
	0.6744 (0.0051)	0.1958 (0.0043)	0.0735 (0.0028)	0.0294 (0.0018)	0.0269 (0.0018)
	$H_0^{(all)} \rightarrow$	Fisher p-value < 0.001			

Testing whether conditional destination mass functions are common to *letter only* groups

AA	1	2	3	4	5	6	7	8	9	10+
	0.7633 (0.0179)	0.1263 (0.0140)	0.0587 (0.0099)	0.0231 (0.0063)	0.0053 (0.0031)	0.0125 (0.0047)	-	0.0018 (0.0018)	-	0.0089 (0.0040)
	$H_0^{(AA)} \rightarrow$	Fisher p-value $= 0.192$								
A	1	2	3	4	5	6	7	8	9	10+
	0.7363 (0.0113)	0.1760 (0.0098)	0.0534 (0.0058)	0.0178 (0.0034)	0.0066 (0.0021)	0.0033 (0.0015)	0.0026 (0.0013)	0.0013 (0.0009)	0.0013 (0.0009)	0.0013 (0.0009)
	$H_0^{(A)} \rightarrow$	Fisher p-value $= 0.788$								
BBB	1	2	3	4	5	6	7	8	9	10+
	0.7191 (0.0108)	0.1618 (0.0089)	0.0491 (0.0052)	0.0324 (0.0043)	0.0145 (0.0029)	0.0087 (0.0022)	0.0052 (0.0017)	0.0040 (0.0015)	0.0029 (0.0013)	0.0023 (0.0012)
	$H_0^{(BBB)} \rightarrow$	Fisher p-value < 0.001								
BB	1	2	3	4	5	6	7+			
	0.5429 (0.0154)	0.2395 (0.0132)	0.1050 (0.0095)	0.0487 (0.0066)	0.0296 (0.0052)	0.0162 (0.0039)	0.0181 (0.0041)	-	-	-
	$H_0^{(BB)} \rightarrow$	Fisher p-value < 0.001								
B	1	2	3	4+						
	0.5957 (0.0108)	0.2227 (0.0091)	0.1133 (0.0070)	0.0684 (0.0055)	-	-	-	-	-	-
	$H_0^{(B)} \rightarrow$	Fisher p-value < 0.001								

downgraded of three and four, with the null hypothesis still comprehensively rejected.

The bottom panel of Table 4 tests a series of hypotheses based on aggregating the credit ratings into simplified groups by discarding any positive or negative sign from the credit rating i. Combining credit ratings like this is very common in migration analysis to increase observations in the credit ratings, although here, we retain the true credit ratings for measuring the distance downgraded. We perform the analysis on AA to B, with BB and B truncated to a maximum distance downgraded of their shortest member (BB- and B- respectively), and CCC excluded because we would need to restrict it to one category.

We fail to reject the null hypotheses (H_0^{AA} and H_0^{A}) for credit rating groups AA and A, thus implying that we have insufficient evidence to warrant discriminating between these credit ratings in these groups when applying the conditional mass function. We do, however, strongly reject the null hypotheses for credit rating groups BBB, BB and B, with Fisher's p-values less than 0.01 percent. The distance downgraded truncation of BB and B may vary the accuracy of this result, although such extreme Fisher's p-values are compelling.

We test null hypotheses of industry homogeneity applied to the results from the top panel of Table 4, and find the Fisher's p-values shown in the right-hand column of Table 5. As with Section 3, we aggregate the industrial, government and miscellaneous sectors into one sector (industrial), because the latter two have very few migration observations and do not demonstrate a statistically significant departure in distribution from the industrial sector. Regardless, this aggregation is necessary because we grouped these classes for the directional migration intensity. The results, however, are similar if this aggregation does not occur. We display a comparison of industrial, financial and utility sectors abridged (not truncated) at a distance downgraded of five. We do not display the extreme estimates for each of the conditional destination mass functions because they are burdensome and often uninformative due to high variability.

The assumption of industry homogeneity (under H_0^{ind}) is rejected at a confidence level above ninety-five percent in half of the credit ratings. Although we show the estimates for the utilities sector, the conclusions are less robust since the sector has fewer observations than the other sectors. The utilities sector does, however, deliver very low Fisher's p-values when compared against either the industrial or financial sectors (not shown), and so must be considered separately. The differences in the probability estimates between the industrial and financial sectors are striking, with the downgrade distance in the low-risk credit ratings likely to be further for an industrial sector issuer than for a financial sector issuer, while the opposite is true in the high-risk credit ratings. It would pose an interesting question for future research as to the cause of such blatant discrimination between industry sectors when being downgraded.

Despite concluding our analyses here, industry distinction is not the only possible strata tested. Particularly, future research could investigate time homogeneity, since the sample is naturally stratified by time. This may require, however, a more complex model than the multinomial distribution, since early experiments used a simple time partition but results were very sensitive to the partition position.

5 conclusion

This paper has proposed a new model for credit rating migrations in continuous-time, where we split the traditional migration intensity into a directional and a categorical component. This new model makes full use of all available data, and allows the measurement of issuer-specific effects for migrations beyond the neighbouring credit ratings. The higher concentration of observations allows the credit ratings to be analysed as categorised by Standard

Table 5: Conditional Mass Functions for Distance Downgraded—Testing Industry Homogeneity Between the Industrial, Financial and Utilities Sectors. $\hat{p}_{j|id,\ell}(s,t)$ $(s.e.\hat{p}_{j|id,\ell}(s,t)])$. Fisher's p-values indicate whether industry homogeneity is rejected.

i/j^*	1	2	3	4	5	H_0^{ind}
Industrial, Governmental and Miscellaeous						Fisher's p-value
AAA	0.3333 (0.0727)	0.3095 (0.0713)	0.2381 (0.0657)	0.0952 (0.0453)	–	< 0.001
AA+	0.6667 (0.0861)	0.1000 (0.0548)	0.1000 (0.0548)	0.0667 (0.0455)	–	0.014
AA	0.7097 (0.0471)	0.1505 (0.0371)	0.0645 (0.0255)	0.0215 (0.0150)	0.0108 (0.0107)	0.013
AA-	0.7724 (0.0378)	0.1545 (0.0326)	0.0244 (0.0139)	0.0244 (0.0139)	0.0081 (0.0081)	0.005
A+	0.7206 (0.0314)	0.1863 (0.0273)	0.0441 (0.0144)	0.0245 (0.0108)	0.0098 (0.0069)	0.057
A	0.7184 (0.0287)	0.1755 (0.0243)	0.0776 (0.0171)	0.0163 (0.0081)	0.0041 (0.0041)	0.055
A-	0.7094 (0.0279)	0.2075 (0.0249)	0.0453 (0.0128)	0.0075 (0.0053)	0.0075 (0.0053)	0.082
BBB+	0.7459 (0.0250)	0.1815 (0.0221)	0.0330 (0.0103)	0.0198 (0.0080)	0.0099 (0.0057)	0.037
BBB	0.7778 (0.0228)	0.1351 (0.0187)	0.0360 (0.0102)	0.0150 (0.0067)	0.0180 (0.0073)	0.507
BBB-	0.6688 (0.0264)	0.1672 (0.0210)	0.0631 (0.0137)	0.0631 (0.0137)	0.0158 (0.0070)	0.002
BB+	0.5371 (0.0329)	0.2533 (0.0287)	0.1048 (0.0202)	0.0437 (0.0135)	0.0524 (0.0147)	0.069
BB	0.5369 (0.0350)	0.2562 (0.0306)	0.0985 (0.0209)	0.0542 (0.0159)	0.0148 (0.0085)	< 0.001
BB-	0.6844 (0.0268)	0.1694 (0.0216)	0.0731 (0.0150)	0.0233 (0.0087)	0.0100 (0.0057)	0.006
B+	0.6173 (0.0232)	0.2255 (0.0199)	0.0706 (0.0122)	0.0387 (0.0092)	0.0364 (0.0089)	0.299
B	0.6279 (0.0196)	0.2115 (0.0165)	0.0689 (0.0103)	0.0557 (0.0093)	0.0361 (0.0075)	0.095
B-	0.5746 (0.0180)	0.2246 (0.0152)	0.1691 (0.0136)	0.0317 (0.0064)		0.010
CCC+	0.4132 (0.0224)	0.4587 (0.0226)	0.1281 (0.0152)			0.355
CCC	0.8308 (0.0208)	0.1692 (0.0208)				0.511
CCC-	1.0000 (0.0000)					1.000
Financial						
AAA	0.7255 (0.0625)	0.1176 (0.0451)	0.0196 (0.0194)	0.0392 (0.0272)	0.0392 (0.0272)	
AA+	0.7949 (0.0647)	0.1282 (0.0535)	0.0256 (0.0253)	0.0256 (0.0253)	–	
AA	0.8235 (0.0462)	0.1176 (0.0391)	0.0441 (0.0249)	–	–	
AA-	0.8855 (0.0278)	0.0305 (0.0150)	0.0458 (0.0183)	0.0229 (0.0131)	–	
A+	0.8324 (0.0284)	0.1272 (0.0253)	0.0231 (0.0114)	0.0116 (0.0081)	0.0058 (0.0058)	
A	0.7566 (0.0312)	0.1746 (0.0276)	0.0370 (0.0137)	0.0212 (0.0105)	–	
A-	0.8299 (0.0310)	0.1156 (0.0264)	0.0408 (0.0163)	–	0.0068 (0.0068)	
BBB+	0.8447 (0.0285)	0.0994 (0.0236)	0.0062 (0.0062)	0.0248 (0.0123)	–	
BBB	0.7559 (0.0381)	0.1417 (0.0309)	0.0630 (0.0216)	0.0079 (0.0078)	0.0157 (0.0110)	
BBB-	0.5462 (0.0456)	0.2857 (0.0414)	0.0756 (0.0242)	0.0504 (0.0201)	0.0084 (0.0084)	
BB+	0.4412 (0.0602)	0.3529 (0.0580)	0.0882 (0.0344)	0.0441 (0.0249)	0.0441 (0.0249)	
BB	0.3125 (0.0579)	0.3125 (0.0579)	0.1250 (0.0413)	0.2031 (0.0503)	0.0156 (0.0155)	
BB-	0.6154 (0.0603)	0.2462 (0.0534)	0.0462 (0.0260)	0.0154 (0.0153)	0.0308 (0.0214)	
B+	0.6667 (0.0556)	0.2222 (0.0490)	0.0278 (0.0194)	0.0556 (0.0270)	0.0139 (0.0138)	
B	0.5303 (0.0614)	0.3030 (0.0566)	0.0606 (0.0294)	0.1061 (0.0379)	–	
B-	0.5000 (0.0680)	0.1852 (0.0529)	0.2407 (0.0582)	0.0741 (0.0356)		
CCC+	0.3684 (0.0783)	0.4737 (0.0810)	0.1579 (0.0592)			
CCC	0.8929 (0.0585)	0.1071 (0.0585)				
CCC-	1.0000 (0.0000)					
Utilities						
AAA	0.4000 (0.2191)	–	–	–	0.2000 (0.1789)	
AA+	0.2857 (0.1707)	0.7143 (0.1707)	–	–	–	
AA	0.3750 (0.1210)	0.3125 (0.1159)	0.3125 (0.1159)	–	–	
AA-	0.6727 (0.0633)	0.1455 (0.0475)	0.1091 (0.0420)	0.0364 (0.0252)	0.0182 (0.0180)	
A+	0.6029 (0.0593)	0.2500 (0.0525)	0.1029 (0.0369)	0.0294 (0.0205)	0.0147 (0.0146)	
A	0.7033 (0.0479)	0.1648 (0.0389)	0.0769 (0.0279)	0.0440 (0.0215)	0.0110 (0.0109)	
A-	0.6791 (0.0403)	0.2015 (0.0347)	0.0746 (0.0227)	0.0299 (0.0147)	0.0075 (0.0074)	
BBB+	0.6884 (0.0394)	0.2174 (0.0351)	0.0290 (0.0143)	0.0290 (0.0143)	0.0217 (0.0124)	
BBB	0.8214 (0.0362)	0.1161 (0.0303)	0.0446 (0.0195)	0.0089 (0.0089)	–	
BBB-	0.5210 (0.0458)	0.1345 (0.0313)	0.1345 (0.0313)	0.0756 (0.0242)	0.0420 (0.0184)	
BB+	0.4390 (0.0775)	0.2195 (0.0646)	0.1463 (0.0552)	0.0488 (0.0336)	0.0244 (0.0241)	
BB	0.2609 (0.0647)	0.1957 (0.0585)	0.4348 (0.0731)	0.0217 (0.0215)	0.0435 (0.0301)	
BB-	0.3548 (0.0859)	0.3871 (0.0875)	0.0323 (0.0317)	0.0968 (0.0531)	0.1290 (0.0602)	
B+	0.5161 (0.0898)	0.1935 (0.0710)	0.1935 (0.0710)	0.0968 (0.0531)	–	
B	0.4194 (0.0886)	0.2903 (0.0815)	0.1935 (0.0710)	0.0645 (0.0441)	0.0323 (0.0317)	
B-	0.5000 (0.1443)	0.1667 (0.1076)	0.2500 (0.1250)	0.0833 (0.0798)		
CCC+	0.4000 (0.1265)	0.6000 (0.1265)	–			
CCC	0.9000 (0.0671)	0.1000 (0.0671)				
CCC-	1.0000 (0.0000)					

and Poor's without removing the positive and negative signs as done by most research in the field.

Grouping all migrations in a similar direction together also reduces informative censoring, since, for example, a positive correlation is likely between the probability of being downgraded one credit rating and two credit ratings. Informative censoring still exists, however, because of negative correlations between migration probabilities in opposite directions, whereas we consider downgrades as right-censored when modelling upgrades and visa versa.

We fit covariates representing momentum and excitability to a multiplicative intensity process, with the directional migration model proving superior to the neighbouring migration model proposed by Lando & Skødeberg (2002). Both momentum and excitability are present in the downgrade intensities for all but the outermost credit ratings. The upgrade intensities deliver less convincing results, with only the medium to low risk credit ratings displaying momentum effects, and AA and A+ demonstrating excitability. Fitting models with both momentum and excitability effects together usually have a statistically significant improvement over fitting them alone, although excitability should be applied only to a migration in the same direction as the previous migration. In addition, we find it is preferable to model the credit ratings individually than to treat all equally. We tested these covariates under the relaxed condition of industry distinction, with both Nelson-Aalen estimates and Monte Carlo simulation confirming that the stratification of the sample by industry sector usually improves the overall fit of the model.

We adopted a multinomial distribution for modelling the categorisation of the migrations into destination credit ratings. Probability estimates for this conditional destination mass function behave as we would expect, with decreasing likelihood as the distance downgraded extends. We find it unreasonable that the distribution of the distance downgraded be considered equal for all credit ratings, although we fail to reject grouping AA+, AA and AA- into a single group, and likewise for A+, A and A-. We find some evidence that we should discriminate between the Standard and Poor's industry sector classifications, where the industrial sector (combined with government and miscellaneous) shows a significantly different conditional destination mass functions from financial and utilities sectors.

The primary purpose of this article was to introduce a new model into the discussion on the appropriate technique for investigating credit migrations. The model's proficiency at isolating relevant effects and performing test on the directional and categorical components separately confirms it as an elegant and useful tool in predicting credit migrations. Although we have completed various analyses, there is scope for extensive further research into the directional migration model. Particularly, other issuer-specific covariates, not necessarily derived from the credit rating counting processes, could be tested; an investigation in the functional form and necessary stratifications of the conditional destination mass functions; and the modelling, whether parametric or non-parametric, of the baseline directional migration intensity.

References

Aalen, O. O. (1978), 'Non parametric inference for a family of counting processes', *Annals of Statistics* **6**, 701–726.

Andersen, P. K. (1982), 'Testing goodness of fit of cox's regression and life model', *Biometrics* **38**(1), 67–77.

Andersen, P. K. & Gill, R. D. (1982), 'Cox's regression model for counting processes: A large sample study', *The Annals of Statistics* **10**(4), 1100–1120.

Bangia, A., Diebold, F. X., Kronimus, A., Schagen, C. & Schuermann, T. (2002), 'Ratings migration and the business cycle, with application to credit portfolio stress testing', *Journal of Banking & Finance* **26**, 445–474.

Barlow, W. E. & Prentice, R. L. (1988), 'Residuals for relative risk regression', *Biometrika* **75**(1), 65–74.

Breslow, N. (1974), 'Covariance analysis of censored survival data', *Biometrics* **30**(1), 89–99.

Cox, D. R. (1972), 'Regression models and life-tables', *Journal of the Royal Statistical Society, Series B (Methodological)* **34**(2), 187–220.

Cox, D. R. (1975), 'Partial likelihood', *Biometrika* **62**(2), 269–276.

Fisher, L. D. & Lin, D. Y. (1999), 'Time-dependent covariates in the cox proportional hazards regression model', *Annual Review of Public Health* **20**, 145–157.

Frydman, H. & Schuermann, T. (2005), 'Credit rating dynamics and markov mixture model', *Working Paper* .

Gadliardini, P. & Gourriéroux, C. (2005), 'Migration correlation: Definition and efficient estimation', *Journal of Banking & Finance* **29**, 865–894.

Hu, Y.-T., Kiesel, R. & Perraudin, W. (2002), 'The estimation of transition matrices for sovereign credit ratings', *Journal of Banking & Finance* **26**, 1383–1406.

Kalbfleisch, J. D. & Prentice, R. L. (2002), *The Statistical Analysis of Failure Time Data*, 2nd edn, Wiley.

Kooman, S. J., Lucas, A. & Klaassen, P. (2005), 'Empirical credit cycles and capital buffer formation', *Journal of Banking & Finance* **29**, 3159–3179.

Lando, D. & Skødeberg, T. M. (2002), 'Analysing rating transitions and rating drift with continuous observations', *Journal of Banking & Finance* **26**, 423–444.

Lane, W. R., Looney, S. W. & Wansley, J. W. (1986), 'An application of the cox proportional hazards model to bank failure', *Journal of Banking & Finance* **10**(4), 511–532.

Lawal, B. (2003), *Categorical Data Analysis with SAS® and SPSS Applications*, Lawrence Erlbaum Associates, Publishers.

LeClere, M. J. (2000), 'The occurrence and timing of events: Survival analysis applied to the study of financial distress', *Journal of Accounting Literature* **19**, 158–189.

LeClere, M. J. (2005), 'Time-dependent and time-invariant covariates within a proportional hazards model: A financial distress application', *Review of Accounting & Finance* **4**(2), 88–106.

Lin, D. Y., Wei, L. J. & Ying, Z. (1993), 'Checking the cox model with cumulative sums of martingale-based residuals', *Biometrika* **80**(3), 557–572.

Lui, K.-J. & Jewell, N. P. (1989), 'A note on the estimate of the relative risk when sample size are small', *Biometrics* **45**(3), 1030–1032.

Marzec, L. & Marzec, P. (1997), 'On fitting cox's regression model with time-dependent coefficients', *Biometrika* **84**(4), 901–908.

Ng'andu, N. H. (1997), 'An empirical comparison of statistical tests for assessing the proportional hazards assumption of cox's model', *Statistics in Medicine* **16**, 611–626.

Nickell, P., Perraudin, W. & Varotto, S. (2000), 'Stability of rating transitions', *Journal of Banking & Finance* **24**, 203–227.

Prentice, R. L. & Self, S. G. (1983), 'Asymptotic distribution theory for cox-type regression models with general relative risk form', *The Annals of Statistics* **11**(3), 804–813.

Satten, G. A., Datta, S. & Robins, J. (2001), 'Estimating the marginal survival function in the presence of time dependent covariates', *Statistics & Probability Letters* **54**, 397–403.

Schoenfeld, D. (1982), 'Partial residuals for the proportional hazards regression model', *Biometrika* **69**(1), 239–241.

Shumway, T. (2001), 'Forecasting bankruptcy more accurately: A simple hazard model', *Journal of Business* **74**(1), 101–124.

Therneau, T. M. & Grambsch, P. M. (2000), *Modeling Survival Data: Extending the Cox Model*, Springer-Verlag.

Therneau, T. M., Grambsch, P. M. & Fleming, T. R. (1990), 'Martingale-based residuals for survival models', *Biometrika* **77**(1), 147–160.

Zelterman, D. (2006), *Models for Discrete Data: Revised Edition*, Oxford University Press.