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Maximally Informative Decision Rules in a Two-Person Decision Problem*

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Abstract

This paper studies how much information can be revealed when agents with private information lack commitment to actions in a given mechanism as well as to the mechanism itself. In a two-person decision problem, agents are allowed to hold on to an outcome in one mechanism while they play another mechanism and learn new information. Formally, decision rule is maximally informative if it is (i) posterior implementable and (ii) robust to a *posterior* proposal of another posterior implementable decision rule. Focusing on a two-person problem, we identify environments where maximally informative decision rules exist. We also show that a maximally informative decision rule must be implemented by a mechanism with a small number of actions (at most 5 for two agents). The result indicates that lack of commitment to a mechanism significantly reduces the amount of information revelation in equilibrium.

Keywords: Information aggregation, Limited commitment, Posterior efficiency, Posterior implementation, Renegotiation-proofness.

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1 Introduction

To make collective decisions using information privately held by individuals, information must be voluntarily revealed. How much information will be revealed if individuals behave strategically? A simple answer, known as the Revelation Principle, is "If the decisions are achieved in a Bayes-Nash equilibrium, all information can be revealed". Behind this claim, however, two kinds of commitment are assumed: (i) agents commit to their actions, and (ii) agents commit to a mechanism. Without these two assumptions, there are few known results about how much information can be revealed. Green and Laffont (1987) relaxed the first assumption by proposing the notion of *posterior implementation*. Posterior implementation is stronger than Bayesian implementation, because agents' strategies must remain optimal against one another after any realization of an equilibrium action profile. Green and Laffont interpret posterior implementable decision rules as representing the outcomes of a cooperative process, where communication takes place without binding commitment to actions. Alternatively, posterior implementable decision rules can also be viewed as representing the outcomes of strategic information revelation, where everyone sends a public message. Therefore, a posterior implementable decision rule results in "an incentive compatible information structure", where it is common knowledge that no agent has an incentive to reveal new information by modifying his message *in a given mechanism*. In this paper, we take this idea one step further by relaxing the second assumption of commitment to a mechanism. Intuitively, *a maximally informative decision rule* results in "a renegotiation-proof information structure", where it is common knowledge that there is no mechanism which leads to a new incentive compatible information structure in which everyone is better off. To put it another way, suppose agents played a posterior equilibrium which implements a particular decision rule. Taking the outcome as a status quo, can a third party propose a new mechanism such that agents unanimously prefer a posterior equilibrium outcome of the new mechanism to the status quo? Which decision rules are robust to such *posterior renegotiation*? How much information can be revealed by such decision rules?

More formally, a decision rule is maximally informative (henceforth MI) if it is posterior implementable and in every revealed information state in equilibrium, no posterior implementable decision rule can reveal further information and make everyone better off. In this concept, an alternative mechanism that can challenge a given mechanism must satisfy: (i) it is exogenously proposed *after* information is endogenously revealed in the play of the first mechanism, (ii) it has a posterior equilibrium, and (iii) it's outcome is compared with that of the first mechanism *after* information is endogenously revealed in the play of the new mechanism. While agents are assumed to be passive in the selection of mechanisms (hence do not learn from the proposal of new mechanisms), they are assumed to actively seek new information by playing a proposed mechanism. Without commitment to any mechanism, it is natural that acceptance/rejection of the challenging mechanism be based on all the information revealed in the play of two mechanisms.[1]

We characterize MI decision rules in a two-person problem of Green and Laffont (1987). We identify conditions where MI decision rules exist. We also show that posterior renegotiation-proofness imposes a significant constraint on a set of implementable decision rules and limits the information aggregation. In particular, any MI decision rule must be implemented with at most five actions. Given a continuum of private information states, this means that information aggregation must be very limited. This result indicates the difficulty of information aggregation in a negotiation process where commitment to a mechanism is hard to achieve.

We provide two economic motivations for our solution concept. First, renegotiation typically changes the incentive compatibility of the original decision rule, once the renegotiation is rationally anticipated. Therefore, without commitment to a mechanism, a concept of incentive compatibility may well be vacuous unless the possibility of renegotiation is properly taken into account. This problem has been well known in the literature (Holmstrom and Myerson 1983 and Forges 1994), but has proven to be difficult to analyze. We add to this literature by providing a complete characterization of MI decision rules in a two-person deci-

[1] For a related idea, see Cramton and Palfrey (1995).

sion problem. Second, the form of posterior renegotiation-proofness considered in this paper has some practical relevance in the context of market competition. Consider a financial service provider who offers an intermediation service for two investors (offers a mechanism in which investors can interact). Suppose that the market is regulated such that (i) a service provider cannot force the payment for its service until two parties reach a voluntary agreement (investor protection) and (ii) a communication process must be made public (disclosure regulation). In this environment, a competing financial service provider may propose a new mechanism to the investors based on the information revealed in the first service. Therefore, without commitment to a mechanism, the possibility of posterior counter-proposals affects the ability of the intermediaries to aggregate information. Our notion of MI decision rules is relevant in this situation. Our analysis indicates that service providers may be able to offer only limited varieties of mechanisms, which do not reveal much information. Therefore, to the contrary of the intension of the regulation, the market may neither be investor-friendly nor aggregate information.

The rest of the paper is organized as follows. The next subsection discusses the related literature. Section 2 describes the model environment. Section 3 defines MI decision rules and characterizes them. Section 4 concludes. Section 5 contains omitted proofs.

1.1 Related literature

One interpretation of MI decision rules is that it is an *endogenous* incentive efficiency concept. Too see this, comparison to the two standard incentive efficiency concepts is helpful. First, consider a set of *ex post* incentive compatible decision rules. If a decision rule is not dominated by any alternative decision rules in this set for *arbitrary coalitions of types*, it is ex post incentive efficient. This criterion is sufficient for posterior renegotiation-proofness. However, the set of ex post incentive compatible decision rules typically contains only constant decision rules (Jehiel et al, 2006). Therefore, the ex post incentive efficiency asks too much for many applications. Second, consider a set of *interim* incentive compatible decision rules.

If a decision rule is not dominated by any alternative decision rules in this set for *coalitions of types that* are *common knowledge events at the interim stage*, it is interim incentive efficient.[2] The interim incentive efficiency may not be sufficient for posterior renegotiation-proofness, because renegotiation (and incentive constraints) after the agents learned new information in the implementation process are ignored. This gap between the interim incentive efficiency and the ex post incentive efficiency arises because both concepts *exogenously* specify a relevant set of coalitions of types over which domination is considered. In contrast, a relevant set of coalitions of types for the MI decision rule is determined *in equilibrium*. Any outcome chosen by MI decision rule must not be dominated by alternative decision rules defined for any *coalitions of types that are common knowledge events created by the public play of the two mechanisms*. By construction, an endogenous information structure is no coarser (a set of coalitions of types is no smaller) than the interim information structure, and no finer (a set of coalitions of types is no larger) than the ex post information structure.[3] Therefore, the notion of MI decision rules lies between the two standard incentive efficiencies.

To study posterior renegotiation-proofness, we build on Green and Laffont (1987). An advantage of this approach is that posterior implementation makes it explicit that a different *indirect* mechanism creates a different information structure, with respect to which a decision rule must remain incentive compatible. We apply the same idea for posterior renegotiation-proofness. Our solution concept naturally captures an open-ended negotiation process between two agents in which new information can be endogenously revealed. Hence, it provides an insight into what the final agreement should look like in such a negotiation process. Forges' (1994) *posterior efficiency* is based on a similar idea, but she focuses on interim incentive compatibility and ignores individual rationality. Therefore, commitment is still assumed. Neeman and Pavlov (2013) study posterior individual rationality, but they assume that agents learn only from outcome.[4] Our solution concept takes into account both

[2]This is a standard incentive efficiency studied by Holmstrom and Myerson (1983).

[3]If no information is revealed, the information structure coincides with that of interim. If information is perfectly revealed, the information structure becomes that of ex post.

[4]Also they focus on a case with private values.

posterior implementability and posterior individual rationality. To our knowledge, our work is the first attempt to do away with commitment to both actions and a mechanism. A disadvantage of our approach is that the characterization of MI decision rules depends on the characterization of posterior implementation, which is still an open question for a general environment. In this paper, we remain in the environment of Green and Laffont (1987).[5] While we present new results on posterior implementation, our main contribution is in the development of the new concept of MI decision rules.

2 Model Environment

Two agents i, j with type $\theta = (\theta_i, \theta_j) \in \Theta_i \times \Theta_j = \Theta$ have utility over two possible decisions $d \in \{d_0, d_1\}$. A decision d_0 yields payoff zero for any types. The payoff from a decision d_1 depends on both types θ. Hence, the payoff of agent i is $u_i(d, \theta) = v_i(\theta)\mathbf{1}\{d = d_1\}$. Note that $\{\theta \in \Theta | v_i(\theta) = 0\}$ is the set of types of both agents for which agent i is indifferent between d_0 and d_1. A *decision rule* $\phi : \Theta \to [0, 1]$ associates any type θ with *an outcome* $\phi(\theta)$, which is the probability of d_1. The joint distribution function $F(\theta)$ and its density function $f(\theta)$ are common knowledge. We assume $\Theta_i \times \Theta_j = [\underline{\theta}_i, \overline{\theta}_i] \times [\underline{\theta}_j, \overline{\theta}_j] \subset \mathbb{R}^2$.

Assumption 1: (a) *$v_i(\theta)$ and $v_j(\theta)$ are continuous and strictly increasing in both arguments.* (b) *The set $\{\theta \in \Theta | v_i(\theta) = v_j(\theta) = 0\}$ has at most finite number of elements.*

Assumption 2: (a) *$f(\theta)$ is continuous and strictly positive on Θ. The conditional density $f_i(\theta_j|\theta_i)$ ($f_j(\theta_i|\theta_j)$ respectively) is strictly positive on Θ_j (on Θ_i).* (b) *For any subinterval $\widehat{\Theta}_j \subset \Theta_j$, the conditional distribution $F_i(\theta_j|\theta_i, \theta_j \in \widehat{\Theta}_j)$ is increasing in θ_i in the sense of first order stochastic dominance. The same condition applies for $F_j(\theta_i|\theta_j, \theta_i \in \widehat{\Theta}_i)$ with $\widehat{\Theta}_i \subset \Theta_i$.*

Assumption 1 is illustrated in **Figure 1**.

[5] Jehiel et al (2007) presents another example.

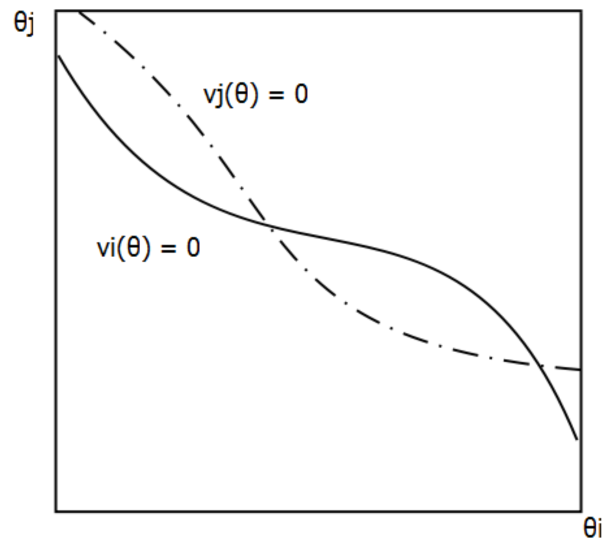

Figure 1. Indifference curves.

Throughout the paper, we measure θ_i in the horizontal direction and θ_j in the vertical direction so that "a vertical segment" means $\{\theta \in \Theta | \theta_i = a, \theta_j \in [b, c]\}$, while "a horizontal segment" means $\{\theta \in \Theta | \theta_i \in [a, b], \theta_j = c\}$. By Assumption 1, two indifference curves $v_i(\theta) = 0$ and $v_j(\theta) = 0$ are strictly decreasing and intersect at most finite number of times. Note that the efficient decision rule under complete information chooses d_1 in the area above two curves, d_0 in the area below two curves, and any random mixture of two decisions in the area between two curves. Assumption 2 is an affiliation property of f, which, together with Assumption 1, makes $\int_{\widehat{\Theta}_j} v_i(\theta_i, \theta_j) f_i(\theta_j|\theta_i) d\theta_j$ monotonic in θ_i for any subinterval $\widehat{\Theta}_j \subset \Theta_j$. This is the expected value of d_1 for agent i of type θ_i, given that agent j's type lies in $\widehat{\Theta}_j$. A symmetric condition holds for $\int_{\widehat{\Theta}_i} v_j(\theta_i, \theta_j) f_j(\theta_i|\theta_j) d\theta_i$.

A *mechanism* (M, g) is a pair of action space $M = M_i \times M_j$ and a measurable function $g : M \to [0, 1]$, where $g(m)$ is a probability of d_1 when $m = (m_i, m_j) \in M$ is chosen by agents. A *strategy* of agent i is a collection of conditional distributions $s_i(m_i|\theta_i)$, $\theta_i \in \Theta_i$. A pair of strategies $s = (s_i, s_j)$ in the mechanism (M, g) results in the decision rule $\phi(\theta) = \int_M g(m) ds_i(m_i|\theta_i) ds_j(m_j|\theta_j)$.

Definition 1: *A pair of strategies* $s = (s_i, s_j)$ *is a Bayes-Nash equilibrium of* (M, g) *if*

$$m_i^* \in \arg\max_{m_i \in M_i} \int_{\Theta_j} \int_{M_j} v_i(\theta_i, \theta_j) g(m) ds_j(m_j|\theta_j) dF_i(\theta_j|\theta_i)$$

for all m_i^* *in the support of* $s_i(m_i|\theta_i)$ *for almost every* θ_i, *and*

$$m_j^* \in \arg\max_{m_j \in M_j} \int_{\Theta_i} \int_{M_i} v_j(\theta_i, \theta_j) g(m) ds_i(m_i|\theta_i) dF_j(\theta_i|\theta_j)$$

for all m_j^* *in the support of* $s_j(m_j|\theta_j)$ *for almost every* θ_j.

Let $\mu^s(m, \theta)$ be the joint distribution over $M \times \Theta$ generated in a Bayes-Nash equilibrium s. Let $\mu_i^s(m_j|\theta_i)$ be the marginal distribution of m_j given θ_i. For every θ_i and $\mu_i^s(m_j|\theta_i)$-almost every m_j, define $F_i(\theta_j|\theta_i, m_j)$ to be the conditional distribution that i of type θ_i would hold about θ_j given m_j. Define $\mu_j^s(m_i|\theta_j)$ and $F_j(\theta_i|\theta_j, m_i)$ in a symmetric manner.

Definition 2: *A Bayes-Nash equilibrium* s *is a posterior equilibrium if*

$$\begin{aligned} m_i &\in \arg\max_{m_i' \in M_i} g(m_i', m_j) \int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i(\theta_j|\theta_i, m_j), \\ m_j &\in \arg\max_{m_j' \in M_j} g(m_i, m_j') \int_{\Theta_i} v_j(\theta_i, \theta_j) dF_j(\theta_i|\theta_j, m_i), \end{aligned}$$

for $\mu^s(m, \theta)$*-almost every* (m, θ).

Definition 3: *A decision rule* ϕ *is posterior implementable if there is a mechanism* (M, g) *with a posterior equilibrium* s *which results in* ϕ.

Green and Laffont (1987) provide a complete characterization of a set of posterior implementable decision rules. Let H be a set of decreasing step functions ξ on $\Theta = \Theta_i \times \Theta_j$ with the following properties:

(i) any vertical segment $(\theta_i, (a, b))$ of ξ satisfies $\int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i(\theta_j|\theta_i, \theta_j \in (a, b)) = 0$,

(ii) any horizontal segment $((c, d), \theta_j)$ of ξ satisfies $\int_{\Theta_i} v_j(\theta_i, \theta_j) dF_j(\theta_i|\theta_j, \theta_i \in (c, d)) = 0$.

Theorem (Green and Laffont, 1987)

Any posterior implementable decision rule ϕ *is such that, for some* $\xi \in H$, $\phi(\theta) = \phi^{-}\mathbf{1}\{\theta \text{ is below } \xi\} + \phi^{+}\mathbf{1}\{\theta \text{ is above } \xi\}$ *with* $0 \leq \phi^{-} \leq \phi^{+} \leq 1$.

Figure 2 shows an example of posterior implementable decision rules.

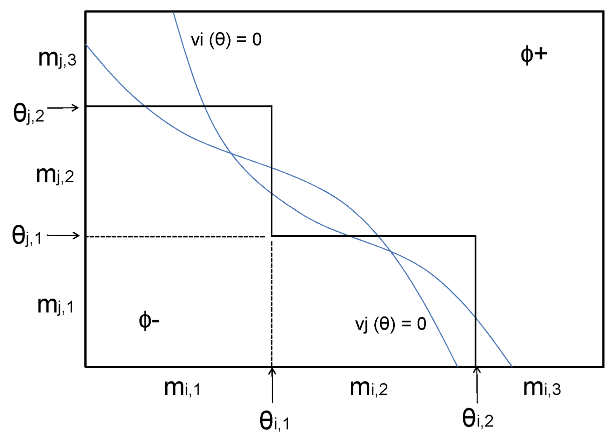

Figure 2. A posterior implementable decision rule with three actions for each agent.

In this example, each agent has three actions, each of which corresponds to a subinterval of types who uses that action in equilibrium. Each line segment in the step function has an associated indifferent type, characterized by the conditions (i) and (ii) that defined the set H. For example, $\theta_{i,1}$ is a solution to $\int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i(\theta_j|\theta_i, m_{j,2}) = \int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i(\theta_j|\theta_i, \theta_j \in (\theta_{j,1}, \theta_{j,2})) = 0$ so that agent i of type $\theta_{i,1}$ is indifferent between $m_{i,1}$ and $m_{i,2}$. Similarly, agent i of type $\theta_{i,2}$ is indifferent between $m_{i,2}$ and $m_{i,3}$.

This characterization allows us to focus on a smaller class of mechanisms. If a mechanism (M, g) has a posterior equilibrium, there is an equivalent mechanism $\left(\widehat{M}, \widehat{g}\right)$ in terms of the resulting decision rule and the equilibrium information structure. An action space $\widehat{M}$ is *a set of subsets* of Θ. An action $\widehat{\Theta}_i \subset \Theta_i$ by agent i is interpreted as "my type is in $\widehat{\Theta}_i$", and it is truthful if his true type is in $\widehat{\Theta}_i$.[6] Moreover, each action $\widehat{m}_i = \widehat{\Theta}_i$ is a closed interval and any two actions share at most one point.

[6]For example, if $\Theta_i \in \widehat{M}$, "my type is in Θ_i" is truthful although it reveals no information.

For the rest of the paper, we use the following convention. Whenever a mechanism has multiple actions for any agent, we label them in an increasing order such that $\max \widehat{\Theta}_{i,k} = \min \widehat{\Theta}_{i,k+1}$ for kth and $k+1$th actions, and denote their boundary point by $\theta_{i,k} \equiv \max \widehat{\Theta}_{i,k} = \min \widehat{\Theta}_{i,k+1}$. If a mechanism in this class has K actions available for agent i and L actions for agent j, we call it a (K, L)-*mechanism*, and the associated posterior implementable decision rule (if it exists) shall be called a (K, L)-*rule*. For any (K, L)-rules, K and L differ at most by one due to Theorem above. While the set of posterior implementable decision rules may look quite restrictive, we note that K and L can be very large and $\phi^- \leq \phi^+$ can take any values in $[0, 1]$. It turns out that studying renegotiation-proofness in this set is not at all trivial.

3 Maximally Informative Decision Rules

Before formally defining MI decision rules, we first investigate properties of posterior implementable decision rules. The results will be used to characterize MI decision rules in the following subsection.

3.1 Structure of posterior implementable rules

First, the following definitions will be useful for our purpose.

Definition 4: *A constant mechanism is* $M_i = \{\Theta_i\}$, $M_j = \{\Theta_j\}$, *and* $g(m) = \phi^0 \in [0, 1]$.

Definition 5: *A dictatorial mechanism for* i *is* $M_i = \{\widehat{\Theta}_{i,1}, \widehat{\Theta}_{i,2}\}$, $M_j = \{\Theta_j\}$ *and* $g(m) = \phi^- 1\{m_i = \widehat{\Theta}_{i,1}\} + \phi^+ 1\{m_i = \widehat{\Theta}_{i,2}\}$ *with* $0 \leq \phi^- < \phi^+ \leq 1$.

Definition 6: *A* $(2, 2)$-*mechanism is* $M_i = \{\widehat{\Theta}_{i,1}, \widehat{\Theta}_{i,2}\}$, $M_j = \{\widehat{\Theta}_{j,1}, \widehat{\Theta}_{j,2}\}$ *with* $0 \leq \phi^- < \phi^+ \leq 1$, *and it is either*

Low type: $g(m) = \phi^- \mathbf{1}\left\{m = \left(\widehat{\Theta}_{i,1}, \widehat{\Theta}_{j,1}\right)\right\} + \phi^+ \mathbf{1}\{otherwise\}$ *or*

High type: $g(m) = \phi^+ \mathbf{1}\left\{m = \left(\widehat{\Theta}_{i,2}, \widehat{\Theta}_{j,2}\right)\right\} + \phi^- \mathbf{1}\{otherwise\}$.

A constant mechanism always has a trivial posterior equilibrium and implements a $(1,1)$-rule, but reveals no information.[7] The other two types of mechanisms may or may not have a posterior equilibrium. If a dictatorial mechanism for i has a posterior equilibrium, it implements a $(2,1)$-rule and partially reveals agent i's type but reveals nothing about agent j's type. If a $(2,2)$-mechanism has a posterior equilibrium, it implements a $(2,2)$-rule and it partially reveals information to both agents. If it is a low type, the low outcome ϕ^- needs low actions from both agents, while at least one high action implements ϕ^+. If it is a high type, the high outcome ϕ^+ needs high actions from both agents, while at least one low action implements ϕ^-. We say that a (K,K)-mechanism with $K \geq 2$ is a *low (high) type* if no agent can choose the low (high) outcome independent of the other agent's action.[8] The following lemma studies the existence of posterior equilibria.

Lemma 1 (existence of (K,L)-rules)

(i) *If no dictatorial mechanism for i (for j respectively) has a posterior equilibrium, then for all $K \geq 2$, no $(K+1,K)$-mechanism ($(K,K+1)$) has a posterior equilibrium.*

(ii) *If no $(2,2)$-mechanism has a posterior equilibrium, then for all $K \geq 3$, then no (K,K)-mechanism has a posterior equilibrium.*

(iii) *A dictatorial mechanism for i can implement a $(2,1)$-rule if and only if $\exists \theta_i' \in \left(\underline{\theta}_i, \overline{\theta}_i\right)$ s.t. $\int_{\Theta_j} v_i(\theta_i', \theta_j) dF_i(\theta_j|\theta_i') = 0$.*

(iv) *A $(2,2)$-mechanism of a low type can implement a $(2,2)$-rule if and only if $\exists \theta' = \left(\theta_i', \theta_j'\right) \in \left(\underline{\theta}_i, \overline{\theta}_i\right) \times \left(\underline{\theta}_j, \overline{\theta}_j\right)$ s.t.*

$$\int_{\Theta_j} v_i(\theta_i', \theta_j) dF_i\left(\theta_j|\theta_i', \theta_j \in \left(\underline{\theta}_j, \theta_j'\right)\right) = \int_{\Theta_i} v_j(\theta_i, \theta_j') dF_j\left(\theta_i|\theta_j', \theta_i \in \left(\underline{\theta}_i, \theta_i'\right)\right) = 0.$$

[7] A constant mechanism is the only case in which, if a direct mechanism $M_i = \Theta_i$ is used instead of $M_i = \{\Theta_i\}$, every agent is indifferent among any actions after actions are made public. Hence, there is a posterior equilibrium in a pure strategy where type is perfectly revealed. We rule out this perfect information revelation because it trivializes any incentive issues.

[8] Figure 2 shows a low type. By Theorem, any (K,K)-mechanism is either a low type or a high type.

(v) *A* $(2,2)$-*mechanism of a high type can implement a* $(2,2)$-rule *if and only if* $\exists \left(\theta_i', \theta_j'\right) \in \left(\underline{\theta}_i, \overline{\theta}_i\right) \times \left(\underline{\theta}_j, \overline{\theta}_j\right)$ *s.t.*

$$\int_{\Theta_j} v_i(\theta_i', \theta_j) dF_i\left(\theta_j | \theta_i', \theta_j \in \left(\theta_j', \overline{\theta}_j\right)\right) = \int_{\Theta_i} v_j(\theta_i, \theta_j') dF_j\left(\theta_i | \theta_j', \theta_i \in \left(\theta_i', \overline{\theta}_i\right)\right) = 0.$$

Lemma 1 (i) and **(ii)** show that dictatorial and $(2,2)$-mechanisms are the key mechanisms in terms of the existence of posterior equilibria. For any information to be revealed in a posterior equilibrium, either dictatorial or $(2,2)$-mechanisms must have a posterior equilibrium. **Lemma 1 (iii)** through **(v)** present conditions for the existence of posterior equilibria in these mechanisms. The conditions state that there must be indifferent types that are consistent with an equilibrium information structure. Note that a $(2,1)$-rule represents a situation where a dictator i can choose any outcome (ϕ^- or ϕ^+) based only on his prior belief about the other agent's type.[9] Agent j learns about the dictator's type through his choice, although she is forced to accept the dictator's choice. All $(K+1, K)$-rules share the same property that only agent i can choose either one of two outcomes ϕ^- and ϕ^+ independent of the agent j's action, while a symmetric argument applies to $(K, K+1)$-rules. On the other hand, a $(2,2)$-rule is characterized by "equal rights" given to both agents because both agents can choose one particular outcome independent of what the other agent's action, while they need to cooperate in order to implement the other outcome.[10] Also, both agents learn new information in equilibrium. All (K, K)-rules share the same property that both agents can choose the same outcome independent of the other agent's action, while they need to coordinate their actions to choose the other outcome.

A natural question is whether a $(2,2)$-rule exists when both $(2,1)$- and $(1,2)$-rules exist. This is a situation where both agents may insist on his/her dictatorial mechanism, but someone (a third party) can suggest a $(2,2)$-rule as a compromising alternative. Such a suggestion

[9] Agent i can choose an outcome ϕ^- by choosing $\widehat{\Theta}_{i,1}$ and ϕ^+ by choosing $\widehat{\Theta}_{i,2}$.

[10] For a low type $(2,2)$-rule, agent i (j) can choose an outcome ϕ^+ by choosing $\widehat{\Theta}_{i,2}$ ($\widehat{\Theta}_{j,2}$) regardless of agent j's (i's) action. To implement ϕ^-, coordinated actions $(\widehat{\Theta}_{i,1}, \widehat{\Theta}_{j,1})$ are required.

not only put both parties on more equal footing but also facilitates more communication between them. When is such a suggestion possible? What is a nature of compromise? The next lemma uncovers some connections between dictatorial and $(2,2)$-rules.

Lemma 2 (dictatorial and $(2,2)$-rules)

Suppose both $(2,1)$- and $(1,2)$-rules exist. Let θ_i^d be the indifference type in the $(2,1)$-rule and θ_j^d be that in the $(1,2)$-rule.

(i) *If there are multiple ($k \geq 1$) $(2,2)$-rules of a low-type (respectively high), then k indifference points $\{(\theta_{i,1}^1, \theta_{j,1}^1), .., (\theta_{i,1}^k, \theta_{j,1}^k)\}$ lie in $(\theta_i^d, \overline{\theta}_i) \times (\theta_j^d, \overline{\theta}_j)$ ($(\underline{\theta}_i, \theta_i^d) \times (\underline{\theta}_j, \theta_j^d)$). If they are ordered by $\theta_{i,1}^1 < ... < \theta_{i,1}^k$, then $\theta_{j,1}^1 > ... > \theta_{j,1}^k$.*

Next, consider agent i with $\overline{\theta}_i$ (respectively $\underline{\theta}_i$) facing agent j with a type set $[\underline{\theta}_j, \theta_j^d]$ ($[\theta_j^d, \overline{\theta}_j]$) and agent j with $\overline{\theta}_j$ ($\underline{\theta}_j$) facing agent i with a type set $[\underline{\theta}_i, \theta_i^d]$ ($[\theta_i^d, \overline{\theta}_i]$).

(ii) *If both agents strictly prefer a higher outcome or both strictly prefer a lower outcome, then k is an odd number.*

(iii) *If one agent weakly prefers a higher outcome while the other agent weakly prefers a lower outcome, and preference is strict for at least one agent, then k is zero (non-existence) or an even number.*

(iv) *If both agents are indifferent for any outcomes, then k can be any number.*

Lemma 2 (i) shows a trade-off between a $(2,2)$-rule of a low type and a $(2,1)$-rule. From agent i's point of view, in the $(2,2)$-rule he cannot choose the low outcome ϕ^- unilaterally, which he could do in the $(2,1)$-rule. Also, while he can still choose ϕ^+ unilaterally in the $(2,2)$-rule, he does so with a smaller probability than he would in the $(2,1)$-rule. The same applies to agent j if we compare the $(2,2)$-rule with a $(1,2)$-rule. Therefore, both agents make a compromise relative to his/her dictatorial rule. In return, both agents learn new information and are given a chance to coordinate their actions. **Lemma 2** also provides necessary conditions for the non-existence and uniqueness of $(2,2)$-rules given that either agent can be a dictator. If we rule out a non-generic case **(iv)**, conditions in **(ii)** and **(iii)**

become necessary and sufficient. Hence, if no $(2,2)$-rule of a low-type exists, then it must be the case that the highest type agents facing the low-type dictator *disagree* on whether they prefer d_1 to d_0 or not. If there is a unique $(2,2)$-rule of a low-type, then they must agree on the matter. **Figure 3** illustrates a case where multiple $(2,2)$-rules of a low type exist, while **Figure 4** illustrates a case where they do not exist.

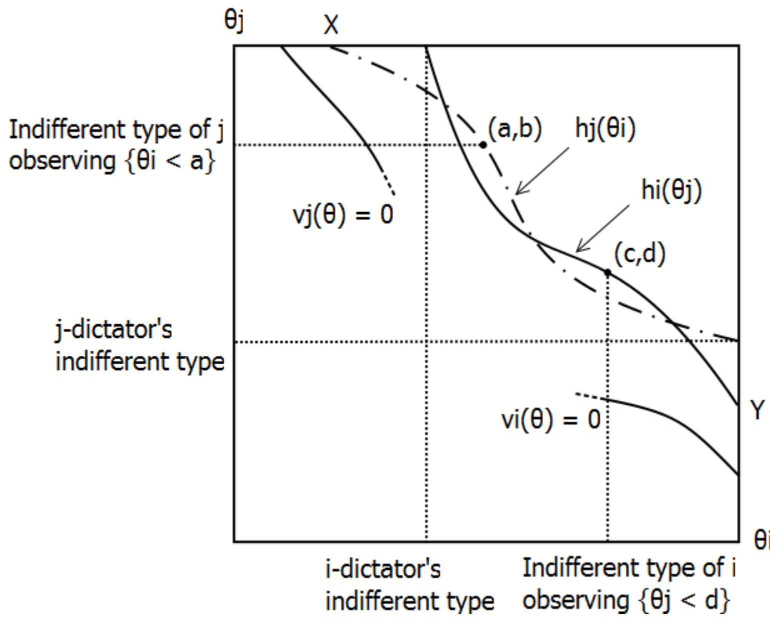

Figure 3. Co-existence of dictatorial and (2,2)-rules of a low type.

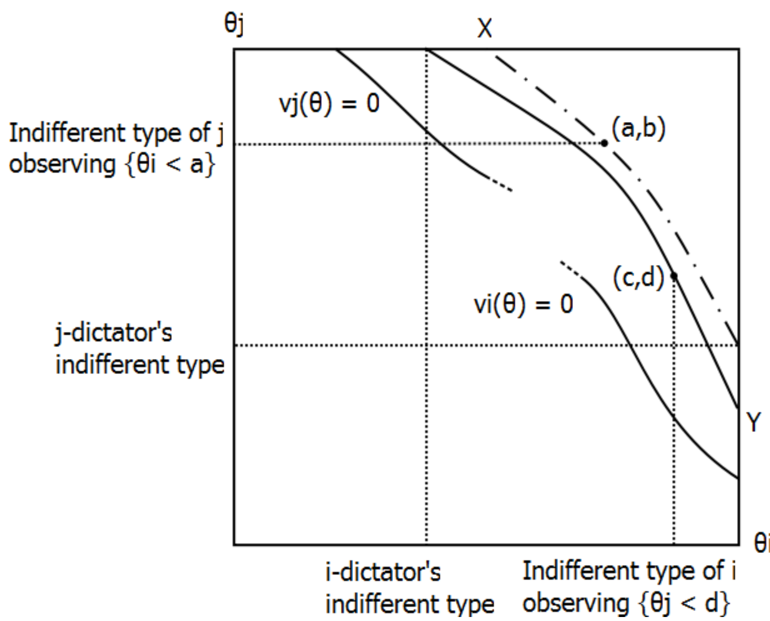

Figure 4. Non-existence of (2,2)-rules of a low type.

In **Figures 3** and **4**, a dashed line with the end point X is a function $h_j : \Theta_i \to \Theta_j$ defined by

$$h_j(x_i) \equiv \left\{ \theta_j \in \Theta_j | \int_{\Theta_i} v_j(\theta_i, \theta_j) \, dF_j(\theta_i | \theta_j, \theta_i \in [\underline{\theta}_i, x_i]) = 0 \right\}$$

and a solid line with the end point Y is a function $h_i : \Theta_j \to \Theta_i$ defined by

$$h_i(x_j) \equiv \left\{ \theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta_i, \theta_j) \, dF_i\left(\theta_j | \theta_i, \theta_j \in \left[\underline{\theta}_j, x_j\right]\right) = 0 \right\}.$$

The intersections of h_i and h_j represent $(2,2)$-rules of a low type. Points X and Y in **Figure 3** show that $\overline{\theta}_i$ facing $\widehat{\Theta}_{j,1}$ and $\overline{\theta}_j$ facing $\widehat{\Theta}_{i,1}$ both prefer a higher outcome ("agreement"), while X and Y in **Figure 4** show that only $\overline{\theta}_i$ prefers a higher outcome ("disagreement"). For the latter case to happen, the ex ante characteristics of two agents must be sufficiently different. Note that the multiple $(2,2)$-rules exist when the two functions h_i and h_j have multiple intersections. The slope of h_i and h_j represents the *expected* marginal rate of substitution of types evaluated at the points of indifference, where expectation is taken over the other agent's type set with a different cutoff level. Therefore, if two agents are sufficiently alike and the expected marginal rate of substitution does not fluctuate much as the cutoff level changes (say h_i and h_j are close to linear), then there exists a unique $(2,2)$-rule. This is formally stated below. Say agents are *symmetric* if $\Theta_i = \Theta_j = \left[\underline{\theta}, \overline{\theta}\right]$, $F_i(\cdot|a) = F_j(\cdot|a)$ $\forall a \in \left[\underline{\theta}, \overline{\theta}\right]$, and $v_i(a,b) = v_j(b,a)$ $\forall (a,b) \in \left[\underline{\theta}, \overline{\theta}\right]^2$. With symmetry, a $(2,1)$-rule exists if and only if $(1,2)$-rule also exists. Let the indifference type denoted by $\theta_i^d = \theta_j^d = \theta^d \in \left(\underline{\theta}, \overline{\theta}\right)$. Also, since h_i and h_j defined above are identical with symmetry, denote both by $h : \left[\underline{\theta}, \overline{\theta}\right] \to \left[\underline{\theta}, \overline{\theta}\right]$.

Lemma 3 ($(2,2)$-rules with symmetry)

Suppose that agents are symmetric and a $(2,1)$-rule with the indifferent type θ^d exists.

(i) *A $(2,2)$-rule of a low type with $\theta_{i,1} = \theta_{j,1} = \theta^* > \theta^d$ exists.*

Next, consider agent with $\overline{\theta}$ facing the other agent with a type set $\left[\underline{\theta}, \theta^d\right]$.

(ii) *If this agent prefers a lower outcome, then:*

(a) *θ^* is the only $(2,2)$-rule of a low type if and only if $\forall \theta_i > \theta^*$ $h(\theta_i) < h^{-1}(\theta_i)$.*

(b) $\theta^* \in \left(\frac{\theta^d+\bar{\theta}}{2}, \bar{\theta}\right)$ *if* h *is concave.*

(iii) *If this agent prefers a higher outcome, then:*

(a) θ^* *is the only* $(2,2)$*-rule of a low type if and only if* $\forall \theta_i < \theta^*\ h(\theta_i) < h^{-1}(\theta_i)$.

(b) $\theta^* \in \left(\theta^d, \frac{\theta^d+\bar{\theta}}{2}\right)$ *if* h *is convex.*

(iv) *If this agent is indifferent for any outcomes, then:*

(a) θ^* *is the only* $(2,2)$*-rule of a low type if and only if* $\forall \theta_i \in \left(\theta^*, \bar{\theta}\right)\ h(\theta_i) \neq h^{-1}(\theta_i)$.

(b) $\theta^* \in \left(\theta^d, \frac{\theta^d+\bar{\theta}}{2}\right]$ *(*$\in \left[\frac{\theta^d+\bar{\theta}}{2}, \bar{\theta}\right)$*) if* h *is concave (convex).*

An analogous result for a $(2,2)$-rule of a high type also holds but is not presented. From the converse of **Lemma 3**, given symmetric agents, if there is no symmetric $(2,2)$-rule, then there is neither $(2,1)$- nor $(2,1)$-rule. By **Lemma 1**, this implies that posterior implementable decision rules must be constant. Hence, with symmetry, information revelation without commitment to actions is possible if and only if $(2,2)$-rules exist.

3.2 Posterior individual rationality

When agents cannot commit to a mechanism, after playing a given mechanism, they may want to play another mechanism as long as they can hold onto the outcome from the initial mechanism as a status quo. An alternative mechanism could be proposed by the mechanism designer (who fails to commit to a mechanism for various reasons), or by competing mechanism designers who try to steal "business". To develop the idea, suppose that there is an outcome $\phi^0 \in [0,1]$ as a status quo and a posterior implementable decision rule ϕ is proposed to challenge ϕ^0. If at least one agent strictly prefers ϕ^0 to a new outcome after actions are made public in the implementation of ϕ, then ϕ should not be considered as improvement over ϕ^0 and hence is rejected. For ϕ to be improvement over ϕ^0, such a rejection of ϕ in favor of ϕ^0 should never happen. This posterior individual rationality is formally stated below.

Definition 7: *A posterior implementable decision rule* ϕ *is posterior individually rational (PIR) relative to a status quo outcome* ϕ^0 *if every agent of any type weakly prefers an* *outcome of ϕ to ϕ^0 and at least one agent of some type strictly prefers ϕ to ϕ^0 in a posterior equilibrium that implements ϕ.*

The definition above implicitly imposes restrictions on the underlying renegotiation game. First, a challenging rule must be posterior implementable. This seems natural because our objective is to study information revelation without commitment to actions. Second, proposal of the new rule is exogenous. This shuts down learning from the endogenous selection of a mechanism, and allows us to focus on learning in a mechanism. Third, improvement is determined *after agents played two mechanisms*. Thus, beliefs are allowed to be updated in the course of renegotiation. Again, this seems natural given our objective of relaxing commitment to a mechanism. Because agents do not commit to a mechanism, they can compare two outcomes after playing two mechanisms.

PIR puts a further restriction on the set of posterior implementable decision rules H. The following lemma characterizes the nature of PIR.

Lemma 4 (posterior individual rationality)

(i) *For any $K \geq 1$, a $(K+1, K)$-rule is PIR relative to $\phi^0 \in [0,1]$ if and only if either one of the following conditions* (a)-(c) *holds*:

(a) $\phi^- = \phi^0 < \phi^+$. *Agent j strictly prefers a higher outcome, for all θ_j after observing $\widehat{\Theta}_{i,K+1}$, and for some θ_j after observing $\widehat{\Theta}_{i,1}$.*

(b) $\phi^- < \phi^0 = \phi^+$. *Agent j strictly prefers a lower outcome, for some θ_j after observing $\widehat{\Theta}_{i,K+1}$, and for all θ_j after observing $\widehat{\Theta}_{i,1}$.*

(c) $\phi^- \leq \phi^0 \leq \phi^+$ *and* $\phi^- < \phi^+$. *For all θ_j, agent j strictly prefers, a higher outcome after observing $\widehat{\Theta}_{i,K+1}$, and a lower outcome after observing $\widehat{\Theta}_{i,1}$.*

(ii) *For any $K \geq 2$, a (K, K)-rule of a low (high respectively) type is PIR relative to ϕ^0 if $\phi^- < \phi^0 = \phi^+$ ($\phi^- = \phi^0 < \phi^+$).*

(iii) *Suppose that a $(2,1)$-rule and a $(K+1, K)$-rule with $K \geq 2$ exist. If a $(2,1)$-rule is PIR relative to ϕ^0, then there is a $(K+1, K)$-rule PIR relative to ϕ^0.*

Lemma 4 (i) provides conditions under which $(K+1, K)$-rules are PIR. Consider $K = 1$ and agent j facing a dictator i. After observing a high action $\widehat{\Theta}_{i,2}$ (hence a high outcome ϕ^+), agent j's indifference type becomes lower. However, if agent j's type is below the new indifferent type, she would still prefer a lower outcome. This imposes $\phi^+ = \phi^0$, because if $\phi^0 < \phi^+$, types below the new indifferent type veto ϕ^+ and revert back to ϕ^0. Hence, unless agent j's type set below the new indifferent type is empty, the PIR rule ϕ must have $\phi^- < \phi^0 = \phi^+$. This is covered in **(b)**. Note that this would be impossible if $\phi^0 = 0$. Similarly, after observing a low action $\widehat{\Theta}_{i,1}$, agent j's indifferent type becomes higher. Unless agent j's type set above the new indifferent type is empty, the PIR rule ϕ must have $\phi^- = \phi^0 < \phi^+$. This is covered in **(a),** and would be impossible if $\phi^0 = 1$. Finally, if agent j's two indifferent types after observing the dictator's actions both lie in the interior of Θ_j, then both $\phi^+ = \phi^0$ and $\phi^- = \phi^0$ must be satisfied. This means that there is no dictatorial rule that is PIR relative to any status quo outcome $\phi^0 \in [0, 1]$. Taken together, a $(K + 1, K)$-rule is *not* PIR relative to ϕ^0 if and only if (a)' $\phi^0 = 1$ and agent j's indifference type after observing $\widehat{\Theta}_{i,1}$ lies in the interior of Θ_j or (b)' $\phi^0 = 0$ and agent j's indifference type after observing $\widehat{\Theta}_{i,K+1}$ lies in the interior of Θ_j or (c)' both indifferent types lie in the interior of Θ_j.

On the other hand, **Lemma 4 (ii)** shows that a (K, K)-rule of a low (high) type, whenever it exists, can be made PIR relative to ϕ^0 by setting ϕ^- and ϕ^+ appropriately as long as $0 < \phi^0$ ($\phi^0 < 1$). **Lemma 4 (iii)** shows that given the existence of $(K + 1, K)$-rules, if information revelation by a $(2, 1)$-rule is unanimously appreciated (i.e., does not result in the rejection of the associated outcome), information revelation by a $(K + 1, K)$-rule is also unanimously appreciated. This result is not particularly helpful in terms of checking the absence of $(K + 1, K)$-rules that are PIR. However, the proof of this result shows that it is sufficient to check that, among all $(K + 1, K)$-rules, the one with the largest value of the highest indifference type and the one with the smallest value of the lowest indifference type are not PIR. If such is a case, then all other $(K + 1, K)$-rules would have agent j of some type rejecting the outcome after observing the lowest or the highest action of agent i.

3.3 MI decision rules

As we discussed above, a new mechanism is proposed state by state *revealed in a posterior equilibrium.* In practice, once some information is revealed, an alternative mechanism *defined only in the revealed information state* can be proposed. As more information is revealed in the original mechanism, it becomes easier to construct an improvement, because the new mechanism is subject to less incentive constraints. For the renegotiation-proofness, this is a bad news. Hence, for the original mechanism to be robust to posterior renegotiation, it should not reveal too much information. On the other hand, an outcome in the new mechanism must be ratified relative to the status quo outcome also state by state revealed in a *new* posterior equilibrium. This restricts the set of mechanisms considered which can improve on the status quo. These two opposing forces make our solution concept non-trivial.

We denote by $\Theta_\phi = \left\{\widehat{\Theta}_{i,k} \times \widehat{\Theta}_{j,l}\right\}_{k=1,..,K,l=1,..,L}$ a partition of Θ created by a (K, L)-rule. Because ϕ can realize only one of these $K \times L$ rectangle type sets, for each possible $\widehat{\Theta} = \widehat{\Theta}_{i,k} \times \widehat{\Theta}_{j,l} \in \Theta_\phi$, potential improvement is a posterior implementable decision rule defined on $\widehat{\Theta}$. Importantly. Assumptions 1 and 2 still hold if we replace Θ with any $\widehat{\Theta} \in \Theta_\phi$. Because each $\widehat{\Theta}$ is a rectangle, all the properties of posterior implementable rules defined on Θ apply to the posterior implementable rules defined on $\widehat{\Theta}$ by relabelling Θ with $\widehat{\Theta}$.

Definition 8: *Let ϕ be a posterior implementable decision rule with a partition Θ_ϕ. Say ϕ is NOT maximally informative (MI) if $\exists \widehat{\Theta} \in \Theta_\phi$ s.t. there is a decision rule $\widehat{\phi}$ defined on $\widehat{\Theta}$ such that:*

- **(a)** *$\widehat{\phi}$ is posterior implementable and PIR relative to $\phi\left(\widehat{\Theta}\right)$ and*
- **(b)** *$\widehat{\phi}$ is not constant.*

Maximal informativeness is defined by the absence of "posterior improvement" with information revelation. The definition requires that there is no equilibrium in which the improvement occurs.[11] It also requires that challenging $\widehat{\phi}$ must be *non-constant* rules. This

[11] This is similar to strong renegotiation-proofness in Maskin and Tirole (1992). This form of domination

reflects our motivation of studying maximal amount of information revelation, and can be relaxed.[12] There are two more subtle points: (i) we do not impose a "credibility" restriction on challenging $\widehat{\phi}$ and (ii) comparison is made between outcomes, not mechanisms. If we impose a restriction that a challenging $\widehat{\phi}$ must also be robust to a new challenge, the concept of MI becomes a recursive one. We leave this investigation for future work. Also, if we consider competing firms proposing a mechanism as mentioned in the introduction, requiring robustness to a one-shot renegotiation seems reasonable in many situations. For example, suppose that a firm finds a profitable opportunity (a mechanism that will be accepted by agents) but is aware that with some probability another firm might steal its business later. We argue that this firm might take a chance and offer the mechanism anyway. So it is not entirely vacuous to ask the existence/absence of such mechanism as a threat. For the second point, some might argue that posterior comparison should be made between mechanisms, not outcomes. In our renegotiation game, agents learn by playing two mechanisms. In stead of comparing two outcomes, they may be allowed to *replay a mechanism* given new information. This does not affect the incentive compatibility in the new mechanism (because of posterior implementability), while it might affect the incentive compatibility in the original mechanism. In this paper, we side-step this potentially interesting issue by assuming that agents are allowed to hold on to the outcome of the original mechanism, but not allowed to replay the mechanism. Given our view that a posterior implementable decision rule represents an outcome of an open-ended negotiation process, it seems natural to rule out "starting it all over" after two rounds of such negotiation.

The following lemma provides a reason why requiring no posterior improvement in the above definition may be reasonable. A problem is that if the posterior improvement is rationally anticipated, the original decision rule will not be incentive compatible.

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was not studied in Forges (1994). In her environment, $\mathcal{E}_0$ redefined with a general improving mechanism would be similar to our notion.

[12] If we drop (b) in Definition 8, MI rules must have $\phi^- = 0$ and $\phi^+ = 1$ in states where preferences of two agents are aligned.

Lemma 5 (posterior renegotiation and posterior implementability)

Suppose that a (K, L)-rule ϕ with $K + L \geq 3$ is not MI. If an improvement $\widehat{\phi}$ on some $\widehat{\Theta} \in \Theta_\phi$ as described in Definition 8 is rationally anticipated, then at least one agent will not choose a posterior equilibrium action of ϕ with positive probability.

Proof: First, if neither $v_i(\theta) = 0$ nor $v_j(\theta) = 0$ lies in the interior of $\widehat{\Theta}$, there cannot be an improvement as in Definition 8. Hence, there is at least one agent who has an indifferent type as a boundary of $\widehat{\Theta}$. Suppose it is agent i and denote by θ_i^* his indifferent type after observing $\widehat{\Theta}_j$. Because no $(2,1)$-rule exists on $\widehat{\Theta}$, the improvement $\widehat{\phi}$ on $\widehat{\Theta}$ must refine $\widehat{\Theta}_j$. If a higher (lower) type set than $\widehat{\Theta}_j$ is revealed, agent i of type θ_i^* strictly prefers a higher (lower) outcome. Since at least one of the two outcomes must be changed but no agent of any type is made worse off, agent i of type θ_i^* strictly prefers the new outcome with some probability and weakly prefers it with the remaining probability. Therefore, if agent i of type θ_i^* anticipates $\widehat{\phi}$, he is no longer indifferent between two actions $\widehat{\Theta}_i$ and $\widehat{\Theta}_i'$ that shares θ_i^* with $\widehat{\Theta}_i$, but would strictly prefer $\widehat{\Theta}_i$ to $\widehat{\Theta}_i'$. Hence, there is a positive measure of types in $\widehat{\Theta}_i'$ in the neighborhood of θ_i^*, who would also prefer $\widehat{\Theta}_i$ to $\widehat{\Theta}_i'$. ■

Lemma 5 shows that the possibility of posterior improvement, when rationally anticipated by agents, creates incentive to lie in the (otherwise) posterior implementable decision rule. As Forges (1994) points out, it is a general observation that posterior improvement disturbs incentive compatibility.[13] The next proposition is the main result of the paper.

Proposition:

(i) *If a (K, L)-rule is MI, then $K + L \leq 5$.*

(ii) (a) *A $(1,1)$-rule is MI if $(2,1)$-, $(1,2)$-, $(2,2)$-rules do not exist.*

(b) *A $(1,1)$-rule with $\phi^0 \in (0,1)$ is MI if and only if $(2,2)$-rules do not exist and for all $K \geq 1$, $(K+1, K)$- and $(K, K+1)$-rules are not PIR relative to ϕ^0.*

(iii) (a) *A $(2,1)$-rule is MI if $(1,2)$-rules on $\widehat{\Theta}_{i,1}$ and $\widehat{\Theta}_{i,2}$, $(2,2)$-rules of a high type*

[13] See the discussion in section 5.2 in Forges (1994).

on $\widehat{\Theta}_{i,1}$, and $(2,2)$-rules of a low type on $\widehat{\Theta}_{i,2}$ do not exist.

(b) *A $(2,1)$-rule is MI if and only if the following conditions are satisfied:*

(I) *For all $K \geq 1$, $(K, K+1)$-rules are not PIR, relative to ϕ^- on $\widehat{\Theta}_{i,1}$ and relative to ϕ^+ on $\widehat{\Theta}_{i,2}$.*

(II) *$(2,2)$-rules of a high type on $\widehat{\Theta}_{i,1}$ and $(2,2)$-rules of a low type on $\widehat{\Theta}_{i,2}$ do not exist.*

(iv) (a) *A $(2,2)$-rule of a low type is MI if $(1,2)$-rules on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,1}$, $(2,2)$-rules of a high type on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,1}$, and $(2,2)$-rules of a low type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,1}$ do not exist.*

(b) *A $(2,2)$-rule of a low type is MI if only if the following conditions are satisfied:*

(I) For all $K \geq 1$, *$(K, K+1)$-rules on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,1}$ are not PIR relative to ϕ^+.*

(II) *$(2,2)$-rules of a high type on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,1}$ and $(2,2)$-rules of a low type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,1}$ do not exist.*

(v) (a) *A $(3,2)$-rule is MI if and only if $(2,2)$-rules of a low type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,2}$ and $(2,2)$-rules of a high type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,1}$ do not exist.*

(b) *A $(3,2)$-rule is MI only if $\{\theta \in \Theta | v_j(\theta) = 0\} \subset \widehat{\Theta}_{i,2} \times \Theta_j$.*

Proposition characterizes MI decision rules. **(iii)** and **(v)** have symmetric counterparts for $(1,2)$- and $(2,3)$-rules but they are omitted. A key feature of MI decision rules is that they cannot have more than five actions. This is because whenever two agents have revealed that they are "middle" types, it is possible to make both agents happier by revealing more information. Considering that there is a continuum of types, and also that there can be a posterior implementable decision rules with infinite number of partitions[14], the result shows that information revelation is significantly reduced without commitment to a mechanism. The set of MI decision rules is small, because they must be robust to decision rules *defined on the revealed information state.* An alternative decision rule is applied to a smaller set of types, hence it is subject to less incentive constraints. This dynamic consideration restricts a set of renegotiation-proof decision rules.

[14] See page 84 (a discussion about an accumulation point in Figure 7) in Green and Laffont (1987).

Sufficient conditions stated in part **(a)** of **Proposition (ii)** to **(iv)** are not necessary, because it is possible that a $(2,1)$- or $(1,2)$-rule exists but not PIR. Note that these are the cases where the rejection of the second outcome can reveal information, if agents actually play in the second mechanism. However, if sufficient conditions hold, such learning from rejection never occurs. Necessary and sufficient conditions stated in part **(b)** of **Proposition (ii)** to **(iv)** may look hard to check. However, as we discussed after **Lemma 4**, it is enough to check that the one with *the largest indifferent type* and the one with *the smallest indifferent type* for agent i (j) are not PIR. Finally, **Lemma 1, 4 and Proposition** taken together identify environments where (non-trivial) MI rules exist. For example, with the appropriate choice of v_i, v_j and F, it is not difficult to construct an example of a MI $(3,2)$-rule.

4 Conclusion

How much information we can collect and reveal subject to incentive constraints is an important issue, but surprisingly little is known once we leave the Revelation Principle and the implicit commitment assumptions that lie behind it. This paper relaxes these assumptions and presents the concept of maximally informative decision rules. The results indicate that renegotiation-proofness puts significant restrictions on the amount of information revelation.

To further investigate the property of information revelation without commitment, it is important to obtain a characterization of posterior implementation in general environments. Also, the assumption that all actions are simultaneously chosen and made public constrains the equilibrium information structure. If any arbitrary observation pattern (including private observation) can be specified as a part of a mechanism, more information structures become possible in equilibrium. Allowing general patterns of information revelation will expand the set of implementable rules and will affect the set of maximally informative decision rules. Also, a continuum of types and costless renegotiation makes the set of MI decision rules small in our environment. Studying how finite types and costly renegotiation can expand this set seems important for many applications. Finally, by allowing agents to adjust the value of (ϕ^-, ϕ^+), the model could be extended to study dynamic learning and coordination.[15] Relative to an exogenous proposal by a third party, this would make improvement more "rigid", hence likely to enlarge the set of MI rules.

5 Proofs

Proof of Lemma 1.

(i) If a $(K+1, K)$-mechanism has a posterior equilibrium,

$\theta_{i,1} = \left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta_i,\theta_j) dF_i(\theta_j|\theta_i, \theta_j \in \widehat{\Theta}_{j,K}) = 0\right\}$ and

$\theta_{i,K} = \left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta_i,\theta_j) dF_i(\theta_j|\theta_i, \theta_j \in \widehat{\Theta}_{j,1}) = 0\right\}$ lie in the interior of Θ_i. Because $\widehat{\Theta}_{j,1}$ is the lowest action, $\widehat{\Theta}_{j,K}$ is the highest action, and $v_i(\theta)$ is strictly increasing in θ, $\theta_i^* \equiv \left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta_i,\theta_j) dF_i(\theta_j|\theta_i) = 0\right\} \in (\theta_{i,1}, \theta_{i,K})$. This is an indifferent type of i given his prior belief. Hence, $\widehat{\Theta}_{i,1} = [\underline{\theta}_i, \theta_i^*]$ and $\widehat{\Theta}_{i,2} = \left[\theta_i^*, \overline{\theta}_i\right]$ has a posterior equilibrium. The proof is symmetric for a $(K, K+1)$-mechanism.

(ii) Consider a (K, K)-mechanism of a low type (as in Figure 2). If it has a posterior equilibrium, both $\theta_{i,1} = \left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta_i,\theta_j) dF_i(\theta_j|\theta_i, \theta_j \in \widehat{\Theta}_{j,K-1}) = 0\right\}$ and

$\theta_{i,K-1} = \left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta_i,\theta_j) dF_i(\theta_j|\theta_i, \theta_j \in \widehat{\Theta}_{j,1}) = 0\right\}$ lie in the interior of Θ_i. Similarly, both $\theta_{j,1} = \left\{\theta_j \in \Theta_j | \int_{\Theta_i} v_j(\theta_i,\theta_j) dF_j(\theta_i|\theta_j, \theta_i \in \widehat{\Theta}_{i,K-1}) = 0\right\}$ and

$\theta_{j,K-1} = \left\{\theta_j \in \Theta_j | \int_{\Theta_i} v_j(\theta_i,\theta_j) dF_j(\theta_i|\theta_j, \theta_i \in \widehat{\Theta}_{i,1}) = 0\right\}$ lie in the interior of Θ_j. Let $X_i \equiv \overset{K-1}{\underset{k=1}{\cup}} \widehat{\Theta}_{i,k}$ and $X_j \equiv \overset{K-1}{\underset{k=1}{\cup}} \widehat{\Theta}_{j,k}$. Because $\widehat{\Theta}_{j,1}$ is the lowest action in X_j, $\widehat{\Theta}_{j,K-1}$ is the highest action in X_j, and $v_i(\theta)$ is strictly increasing in θ, $x_i^* \equiv \left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta_i,\theta_j) dF_i(\theta_j|\theta_i, \theta_j \in X_j) = 0\right\}$ lies in the interior of X_i. Similarly, $x_j^* \equiv \left\{\theta_j \in \Theta_j | \int_{\Theta_i} v_j(\theta_i,\theta_j) dF_j(\theta_i|\theta_j, \theta_i \in X_i) = 0\right\}$ lies in the interior of X_j. Define $h_i : X_j \to X_i$ by $h_i(x) = \left\{\theta_i \in X_i | \int_{\Theta_j} v_i(\theta_i,\theta_j) dF_i(\theta_j|\theta_i, \theta_j \in [\underline{\theta}_j, x]) = 0\right\}$ $\forall x \in X_j$. Similarly define $h_j : X_i \to X_j$ by $h_j(x) = \left\{\theta_j \in X_j | \int_{\Theta_i} v_j(\theta_i,\theta_j) dF_j(\theta_i|\theta_j, \theta_i \in [\underline{\theta}_i, x]) = 0\right\}$ $\forall x \in X_i$. Note that $h_i(\cdot)$ and $h_j(\cdot)$ are continuous, strictly decreasing, $h_i(\theta_{j,1}) = \theta_{i,K-1}$, $h_j(\theta_{i,1}) = \theta_{j,K-1}$, $h_i(\theta_{j,K-1}) = x_i^*$, and $h_j(\theta_{i,K-1}) = x_j^*$. Hence, a mapping $t : X_i \times X_j \to$

[15] Watson (1999) is a related work in a dynamic environment.

$X_i \times X_j$ defined by $t(x_i, x_j) = (h_i(x_j), h_j(x_i))$ has at least one fixed point. This fixed point constitutes a $(2,2)$-rule of low type. The proof is symmetric for a high type.

(iii) (if) By Assumptions 1 and 2, $\theta_i \lesseqgtr \theta_i' \Leftrightarrow \int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i(\theta_j|\theta_i) \lesseqgtr 0$. Agent i with $\theta_i < \theta_i'$ prefers a higher outcome while agents with $\theta_i > \theta_i'$ prefers a lower outcome. Therefore, a mechanism with $\widehat{\Theta}_{i,1} = [\underline{\theta}_i, \theta_i']$ leading to ϕ^- and $\widehat{\Theta}_{i,2} = \left[\theta_i', \overline{\theta}_i\right]$ leading to $\phi^+ > \phi^-$ has a posterior equilibrium.

(only if) If the condition is not satisfied, it must be either $\forall \theta_i \in \Theta_i$, $\int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i(\theta_j|\theta_i) \geq 0$ or $\forall \theta_i \in \Theta_i$, $\int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i(\theta_j|\theta_i) \leq 0$. For the former case, agent i always prefers a higher outcome and $\widehat{\Theta}_{i,1}$ would not be chosen. For the latter case,agent i always prefers a lower outcome and $\widehat{\Theta}_{i,2}$ would not be chosen.

(iv) (if) For agent i observing $\theta_j \in \left(\underline{\theta}_j, \theta_j'\right)$, $\theta_i \lesseqgtr \theta_i' \Leftrightarrow \int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i\left(\theta_j|\theta_i, \theta_j \in \left(\underline{\theta}_j, \theta_j'\right)\right) \lesseqgtr 0$. Hence, agent i with $\theta_i < \theta_i'$ prefers a lower outcome and optimally chooses $\widehat{\Theta}_{i,1}$ while agent i with $\theta_i > \theta_i'$ prefers a higher outcome and optimally chooses $\widehat{\Theta}_{i,2}$. For agent i observing $\theta_j \in \left(\theta_j', \overline{\theta}_j\right)$, the outcome ϕ^+ is independent of his action and choosing $\widehat{\Theta}_{i,1}$ or $\widehat{\Theta}_{i,2}$ truthfully is optimal.

(only if) If the condition is not satisfied, it must be one of the six cases:

(a) $\forall \theta_i \in \Theta_i$, $\int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i\left(\theta_j|\theta_i, \theta_j \in \left(\underline{\theta}_j, \theta_j'\right)\right) \geq 0$ for all $\theta_j' \in \left(\underline{\theta}_j, \overline{\theta}_j\right)$,

(b) $\forall \theta_i \in \Theta_i$, $\int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i\left(\theta_j|\theta_i, \theta_j \in \left(\underline{\theta}_j, \theta_j'\right)\right) \leq 0$ for all $\theta_j' \in \left(\underline{\theta}_j, \overline{\theta}_j\right)$,

(c) $\forall \theta_j' \in \left(\underline{\theta}_j, \overline{\theta}_j\right)$ s.t. $\left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i\left(\theta_j|\theta_i, \theta_j \in \left(\underline{\theta}_j, \theta_j'\right)\right) = 0\right\} \equiv k\left(\theta_j'\right) \in \left(\underline{\theta}_i, \overline{\theta}_i\right)$, it holds $\int_{\Theta_i} v_j(\theta_i, \theta_j') dF_j\left(\theta_i|\theta_j', \theta_i \in \left(\underline{\theta}_i, k\left(\theta_j'\right)\right)\right) = c \neq 0$.

Cases (d), (e), (f) are symmetric cases for agent j. For (a) and (b), agent i always prefers one outcome to the other (ϕ^+ for (a) and ϕ^- for (b)), so there cannot be a $(2,2)$-posterior equilibrium. For (c), first note that $k\left(\theta_j'\right)$ is a candidate for $\theta_{i,1}$, and that if θ_j' which satisfies $k\left(\theta_j'\right) \in \left(\underline{\theta}_i, \overline{\theta}_i\right)$ exists, it is unique and $k(\cdot)$ is decreasing. Consider agent j with θ_j slightly smaller than θ_j' if $c > 0$ and slightly larger than θ_j' if $c < 0$. In the former case, there exists a positive measure of types $\theta_j < \theta_j'$ such that $\int_{\Theta_i} v_j(\theta_i, \theta_j) dF_j\left(\theta_i|\theta_j, \theta_i \in \left(\underline{\theta}_i, k\left(\theta_j'\right)\right)\right) > 0$. They would prefer a higher outcome and hence choose $\widehat{\Theta}_{j,2} = \left[\theta_j', \overline{\theta}_j\right]$ rather than $\widehat{\Theta}_{j,1} =$

$\left[\underline{\theta}_j, \theta'_j\right]$ as they are supposed to. In the latter case, there exists a positive measure of types $\theta_j > \theta'_j$ such that $\int_{\Theta_i} v_j(\theta_i, \theta_j) dF_j\left(\theta_i|\theta_j, \theta_i \in \left(\underline{\theta}_i, k\left(\theta'_j\right)\right)\right) < 0$. They would prefer a lower outcome and hence choose $\widehat{\Theta}_{j,1}$ rather than $\widehat{\Theta}_{j,2}$. The remaining cases are symmetric.

(v) Symmetric with (iv). ■

Proof of Lemma 2.

(i) Define $h_i : \Theta_j \to \Theta_i$ by $h_i(x_j) \equiv \left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i\left(\theta_j|\theta_i, \theta_j \in \left[\underline{\theta}_j, x_j\right]\right) = 0\right\}$ and $h_j : \Theta_i \to \Theta_j$ by $h_j(x_i) \equiv \left\{\theta_j \in \Theta_j | \int_{\Theta_i} v_j(\theta_i, \theta_j) dF_j(\theta_i|\theta_j, \theta_i \in [\underline{\theta}_i, x_i]) = 0\right\}$. Then $h_i\left(\overline{\theta}_j\right) = \theta_i^d$ and h_i is strictly decreasing. Similarly, $h_j\left(\overline{\theta}_i\right) = \theta_j^d$ and h_j is strictly decreasing. By **Lemma 1**, $(2,2)$-rules of a low type exist is and only if h_i and h_j have intersections. By construction of h_i and h_j, all the intersections must lie in $\left(\theta_i^d, \overline{\theta}_i\right) \times \left(\theta_j^d, \overline{\theta}_j\right)$ and satisfy the last property.

(ii) The stated condition implies either (a) $h_i\left(\theta_j^d\right) < \overline{\theta}_i$ and $h_j\left(\theta_i^d\right) < \overline{\theta}_j$ (this is **Figure 3**) or (b) $\exists\theta_i > \theta_i^d$ s.t. $h_j(\theta_i) = \overline{\theta}_j$ and $\exists\theta_j > \theta_j^d$ s.t. $h_i(\theta_j) = \overline{\theta}_i$. Either way, because h_i and h_j are strictly decreasing, they must cross each other for odd number of times in $\left[\theta_i^d, \overline{\theta}_i\right] \times \left[\theta_j^d, \overline{\theta}_j\right]$.

(iii) The stated condition implies either (a) $h_i\left(\theta_j^d\right) \leq \overline{\theta}_i$ and $\exists\theta_i \geq \theta_i^d$ s.t. $h_j(\theta_i) = \overline{\theta}_j$ and at least one inequality is strict, or (b) $h_j\left(\theta_i^d\right) \leq \overline{\theta}_j$ and $\exists\theta_j \geq \theta_j^d$ s.t. $h_i(\theta_j) = \overline{\theta}_i$ and at least one inequality is strict. Either way, h_i and h_j must cross each other for even number of times in $\left[\theta_i^d, \overline{\theta}_i\right] \times \left[\theta_j^d, \overline{\theta}_j\right]$.

(iv) The stated condition implies that h_i and h_j intersect at two points $\left(\theta_i^d, \overline{\theta}_j\right)$ and $\left(\overline{\theta}_i, \theta_j^d\right)$, which do not constitute a $(2,2)$-rules. Two strictly decreasing curves sharing the same end points can intersect in the middle any number of times. ■

Proof of Lemma 3.

(i) When agents are symmetric, case **(iii)** in **Lemma 2** cannot happen because h_i and h_j are located symmetrically with respect to a straight line connecting two points $(\underline{\theta}, \underline{\theta})$ and $\left(\overline{\theta}, \overline{\theta}\right)$ on $\left[\underline{\theta}, \overline{\theta}\right]^2$. Call this line the 45 degree line. Because h_i and h_j are strictly decreasing and cross the 45 degree line, they must cross each other on the 45 degree line only once. This is θ^*, and by **Lemma 2 (i)**, $\theta^* > \theta^d$.

(ii) $h_i(\theta_j)$ starts from the interior of a segment $\{\overline{\theta}, [\theta^d, \overline{\theta}]\}$ and monotonically decreases in θ_j to a point $(\theta^d, \overline{\theta})$, while $h_j(\theta_i)$ starts from the interior of a segment $\{[\theta^d, \overline{\theta}], \overline{\theta}\}$ and monotonically decreases in θ_i to a point $(\overline{\theta}, \theta^d)$. If the condition $\forall\theta_i > \theta^*\ h(\theta_i) < h^{-1}(\theta_i)$ holds, h_i and h_j do not cross below the 45 degree line. By symmetry, θ^* is the only intersection. If the condition does not hold, h_i and h_j have an intersection below and above the 45 degree line and θ^* is not a unique $(2,2)$-rule. If h is concave, both h_i and h_j must lie above the straight line connecting two points $(\theta^d, \overline{\theta})$ and $(\overline{\theta}, \theta^d)$. Call this line the negative 45 degree line. Because the point $\left(\frac{\theta^d+\overline{\theta}}{2}, \frac{\theta^d+\overline{\theta}}{2}\right)$ is on the negative 45 line, $\theta^* \in \left(\frac{\theta^d+\overline{\theta}}{2}, \overline{\theta}\right)$.

(iii) $h_i(\theta_j)$ starts from the interior of a segment $\{\overline{\theta}, [\underline{\theta}, \theta^d]\}$ and decreases in θ_j to a point $(\theta^d, \overline{\theta})$, while $h_j(\theta_i)$ starts from the interior of a segment $\{[\underline{\theta}, \theta^d], \overline{\theta}\}$ and decreases in θ_i to a point $(\overline{\theta}, \theta^d)$. The rest of the proof is analogous to (ii) and hence omitted.

(iv) $h_i(\theta_j)$ starts from a point $(\overline{\theta}, \theta^d)$ and decreases in θ_j to a point $(\theta^d, \overline{\theta})$, while $h_j(\theta_i)$ starts from a point $(\theta^d, \overline{\theta})$ and decreases in θ_i to a point $(\overline{\theta}, \theta^d)$. The rest of the proof is analogous to (ii) and hence omitted. ■

Proof of Lemma 4.

(i) (if) For each case, agent i is strictly better off for at least one action and weakly better off for the other action relative to ϕ^0. For (a), agent j is made strictly better off when $\phi^+ > \phi^0$ occurs after observing $\widehat{\Theta}_{i,2}, .., \widehat{\Theta}_{i,K+1}$ and is made indifferent between ϕ^- and ϕ^0 for all other cases because $\phi^- = \phi^0$. For (b), agent j is made strictly better off when $\phi^- < \phi^0$ occurs after observing $\widehat{\Theta}_{i,1}, .., \widehat{\Theta}_{i,K}$ and is made indifferent between ϕ^+ and ϕ^0 for all other cases because $\phi^+ = \phi^0$. For (c), agent j is made strictly better off for at least one of the outcomes and not worse off for the other outcome.

(only if) If $\phi^0 < \phi^-$ or $\phi^+ < \phi^0$, agent i of some type (types in $\widehat{\Theta}_{i,1}$ for $\phi^0 < \phi^-$ and types in $\widehat{\Theta}_{i,K+1}$ for $\phi^+ < \phi^0$) would veto the new outcome and revert back to ϕ^0. If $\phi^- = \phi^0 < \phi^+$ but (a) does not hold, either case (c) holds or agent j of some type strictly prefers a lower outcome after observing $\widehat{\Theta}_{i,K+1}$ and would veto ϕ^+ to revert back to ϕ^0. If $\phi^- < \phi^0 = \phi^+$ but (b) does not hold, either case (c) holds or agent j of some type strictly prefers a higher outcome after observing $\widehat{\Theta}_{i,1}$ and would veto ϕ^- to revert back to ϕ^0. If $\phi^- \leq \phi^0 \leq \phi^+$ and $\phi^- < \phi^+$ but (c) does not hold, then either that agent j of some type strictly prefers a lower outcome after observing $\widehat{\Theta}_{i,K+1}$ and would veto ϕ^+ to revert back to ϕ^0 or that agent j of some type strictly prefers a higher outcome after observing $\widehat{\Theta}_{i,1}$ and would veto ϕ^- to revert back to ϕ^0. Hence, if none of (a)(b)(c) holds, ϕ is not PIR relative to ϕ^0.

(ii) In a (K, K)-rule of a low type, agents choosing actions that lead to ϕ^- strictly prefer a lower outcome after observing actions. So they strictly prefer ϕ^- to $\phi^0 > \phi^-$. Agents choosing the highest actions together strictly prefer a higher outcome but they are indifferent between ϕ^+ and $\phi^0 = \phi^+$. If only one agent chooses the highest action, the outcome is ϕ^+. He/she strictly prefers a higher outcome and is indifferent between ϕ^+ and $\phi^0 = \phi^+$. The other agent who did not choose the highest action may prefer a higher or a lower outcome, but he/she is indifferent between ϕ^+ and $\phi^0 = \phi^+$. The proof for a high type is symmetric.

(iii) A high action $\left[\theta_i^d, \overline{\theta}_i\right]$ in a $(2,1)$-rule and the highest action $\widehat{\Theta}_{i,K+1} = \left[\theta_{i,K}, \overline{\theta}_i\right]$ in a $(K+1, K)$-rule satisfy $\theta_i^d < \theta_{i,K}$, because otherwise $\theta_{i,K}$ cannot be an indifferent type after observing a type set of agent j, $\widehat{\Theta}_{j,1}$, which is worse than Θ_j. Therefore, if agent i after observing $\left[\theta_i^d, \overline{\theta}_i\right]$ prefers a higher outcome, he also prefers it after observing $\widehat{\Theta}_{i,K+1}$. Similarly, a low action $\left[\underline{\theta}_i, \theta_i^d\right]$ in a $(2,1)$-rule and the lowest action $\widehat{\Theta}_{i,1} = [\underline{\theta}_i, \theta_{i,1}]$ in a $(K+1, K)$-rule satisfy $\theta_{i,1} < \theta_i^d$. Hence, if agent i after observing $\left[\underline{\theta}, \theta_i^d\right]$ prefers a lower outcome, he also prefers it after observing $\widehat{\Theta}_{i,1}$. Therefore, if one of (a)(b)(c) in **(i)** holds for a $(2,1)$-rule, it also holds for a $(K+1, K)$-rule. ■

Proof of Proposition.

(i) Consider a (K, L)-rule with $K + L \geq 6$. At least one rectangle $\widehat{\Theta}_{i,k} \times \widehat{\Theta}_{j,l}$ on which the high outcome ϕ^+ is chosen is characterized by

$$\underline{\theta}_i' \equiv \min \widehat{\Theta}_{i,k} = \left\{\theta_i \in \Theta_i \middle| \int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i(\theta_j | \theta_i, \theta_j \in \widehat{\Theta}_{j,l}) = 0\right\},$$
$$\overline{\theta}_i' \equiv \max \widehat{\Theta}_{i,k} = \left\{\theta_i \in \Theta_i \middle| \int_{\Theta_j} v_i(\theta_i, \theta_j) dF_i(\theta_j | \theta_i, \theta_j \in \widehat{\Theta}_{j,l-1}) = 0\right\},$$

$\underline{\theta}'_j \equiv \min \widehat{\Theta}_{j,l} = \left\{\theta_j \in \Theta_j | \int_{\Theta_i} v_j(\theta_i,\theta_j) dF_j(\theta_i|\theta_j, \theta_i \in \widehat{\Theta}_{i,k}) = 0\right\}$,
$\overline{\theta}'_j \equiv \max \widehat{\Theta}_{j,l} = \left\{\theta_j \in \Theta_j | \int_{\Theta_i} v_j(\theta_i,\theta_j) dF_j(\theta_i|\theta_j, \theta_i \in \widehat{\Theta}_{i,k-1}) = 0\right\}$.

Define $h_i : \widehat{\Theta}_{j,l} \to \widehat{\Theta}_{i,k}$ by $h_i(x) \equiv \left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta_i,\theta_j) dF_i(\theta_j|\theta_i, \theta_j \in [\underline{\theta}'_j, x]) = 0\right\}$ for $x \in \widehat{\Theta}_{j,l}$. Similarly define $h_j : \widehat{\Theta}_{i,k} \to \widehat{\Theta}_{j,l}$ by $h_j(x) \equiv \left\{\theta_j \in \Theta_j | \int_{\Theta_i} v_j(\theta_i,\theta_j) dF_j(\theta_i|\theta_j, \theta_i \in [\underline{\theta}'_i, x]) = 0\right\}$ for $x \in \widehat{\Theta}_{i,k}$. Note that $h_i(\cdot)$ and $h_j(\cdot)$ are continuous, strictly decreasing, $h_i\left(\overline{\theta}'_j\right) = \underline{\theta}'_i$ and $h_j\left(\overline{\theta}'_i\right) = \underline{\theta}'_j$. Also, $h_i\left(\underline{\theta}'_j\right) = \left\{\theta_i \in \Theta_i | v_i\left(\theta_i, \underline{\theta}'_j\right) = 0\right\} \in \left(\underline{\theta}'_i, \overline{\theta}'_i\right)$, i.e., $v_i(\theta) = 0$ goes through a horizontal segment $\left\{\theta | \theta_i \in \widehat{\Theta}_{i,k}, \theta_j = \underline{\theta}'_j\right\}$, because $v_i(\theta) = 0$ goes through two vertical segments $\left\{\theta | \theta_i = \underline{\theta}'_i, \theta_j \in \widehat{\Theta}_{j,l}\right\}$ and $\left\{\theta | \theta_i = \overline{\theta}'_i, \theta_j \in \widehat{\Theta}_{j,l-1}\right\}$ (otherwise $\underline{\theta}'_i$ and $\overline{\theta}'_i$ would not be indifferent types). Similarly, $h_j\left(\underline{\theta}'_i\right) = \{\theta_j \in \Theta_j | v_j\left(\underline{\theta}'_i, \theta_j\right) = 0\} \in \left(\underline{\theta}'_j, \overline{\theta}'_j\right)$, i.e., $v_j(\theta) = 0$ goes through a vertical segment $\left\{\theta | \theta_i = \underline{\theta}'_i, \theta_j \in \widehat{\Theta}_{j,l}\right\}$ for the similar reason. Hence, a mapping $t : \widehat{\Theta}_{i,k} \times \widehat{\Theta}_{j,l} \to \widehat{\Theta}_{i,k} \times \widehat{\Theta}_{j,l}$ defined by $t(x_i, x_j) = (h_i(x_j), h_j(x_i))$ has at least one fixed point. Let $x^* = (x_i^*, x_j^*) \in \widehat{\Theta}_{i,k} \times \widehat{\Theta}_{j,l}$ be such a fixed point. Consider a decision rule defined on $\widehat{\Theta}_{i,k} \times \widehat{\Theta}_{j,l}$ that takes $\phi^-(< \phi^+)$ on the area below and the left to x^* and takes ϕ^+ on the remaining area. This is a $(2,2)$-rule defined on $\widehat{\Theta}_{i,k} \times \widehat{\Theta}_{j,k}$ and it is PIR relative to ϕ^+ on $\widehat{\Theta}_{i,k} \times \widehat{\Theta}_{j,k}$. Therefore, a (K,L)-rule with $K + L \geq 6$ is not MI.

(ii) (a) By **Lemma 1**, constant rules are the only posterior implementable decision rules they are MI by **Definition 8**.[16]

(b) Sufficiency is immediate by **Lemma 1**. If a $(2,2)$-rule exists, by **Lemma 4 (ii)**, it can always be made PIR relative to any $\phi^0 \in (0,1)$ (set $\phi^- < \phi^0 = \phi^+$ for a low type and $\phi^- = \phi^0 < \phi^+$ for a high type). Hence, the condition that no $(2,2)$-rule exists is also necessary. If $\exists K \geq 1$ s.t. either a $(K+1,K)$- or a $(K,K+1)$-rule is PIR relative to ϕ^0, then by **Definition 8** a constant rule with ϕ^0 is not MI. Hence the second condition is also necessary.

(iii) (a) There is no $(2,1)$-rules on $\widehat{\Theta}_{i,1}$ and $\widehat{\Theta}_{i,2}$, because $\theta_{i,1} = \max \widehat{\Theta}_{i,1} = \min \widehat{\Theta}_{i,2}$ is the unique indifferent type of i after observing Θ_j. There is no $(2,2)$-rules of low type on $\widehat{\Theta}_{i,1}$

[16] Suppose that **Definition 8** is modified such that maximal informativeness does not allow an improvement by a constant rule too. In this case, constant rules with $\phi^0 \in (0,1)$ are MI only if agents' ex ante preferences are not aligned, while constant rules with $\phi^0 \in \{0,1\}$ are MI if ex ante preferences are aligned ($\phi^0 = 0$ $(= 1)$ is MI if both prefer a lower (higher) outcome).

because an indifferent type of i after observing any type sets lower than Θ_j must be greater than $\theta_{i,1}$. Similarly, there is no $(2,2)$-rule of a high type on $\widehat{\Theta}_{i,2}$. Hence, by **Lemma 1**, the condition is sufficient.

(b) This proof is similar to (ii) (b).

(iv) (a) First, because $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,2}$ lies above both $v_i(\theta) = 0$ and $v_j(\theta) = 0$, there cannot be any improvement on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,2}$. Second, there is no $(2,1)$-rule on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,1}$ (because $\theta_{i,1}$ is an indifferent type after observing $\widehat{\Theta}_{j,1}$) and no $(1,2)$-rule on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,2}$ (because $\theta_{j,1}$ is an indifferent type after observing $\widehat{\Theta}_{i,1}$). Third, there is no $(2,1)$-, $(1,2)$-, $(2,2)$-rules of a low type on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,1}$ (for $(2,2)$-rules, notice that an indifferent type of i after observing a type set lower than $\widehat{\Theta}_{j,1}$ is greater than $\theta_{i,1}$). Hence, it suffices to show that if there is no $(2,2)$-rule of a high type on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,1}$, then there is neither $(2,1)$-rule nor $(2,2)$-rule on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,2}$. Because $v_j(\theta) = 0$ crosses a horizontal segment $\{(\underline{\theta}_i, \theta_{i,1}), \theta_{j,1}\}$ and $v_i(\theta) = 0$ crosses a vertical segment $\left\{\theta_{i,1}, \widehat{\Theta}_{j,1}\right\}$ and both are decreasing, if $v_i(\theta) = 0$ crosses a horizontal segment $\{(\underline{\theta}_i, \theta_{i,1}), \theta_{j,1}\}$, then there is a $(2,2)$-rule of a high type on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,1}$. Too see this, construct $h_i(x) \equiv \left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta) dF_i(\theta_j|\theta_i, \theta_j \in [x, \theta_{j,1}]) = 0\right\}$ connecting $h_i\left(\underline{\theta}_j\right) = \theta_{i,1}$ and $h_i(\theta_{j,1}) \in (\underline{\theta}_i, \theta_{i,1})$ and $h_j(x) \equiv \left\{\theta_j \in \Theta_j | \int_{\Theta_i} v_j(\theta) dF_i(\theta_i|\theta_j, \theta_i \in [x, \theta_{i,1}]) = 0\right\}$ connecting $h_j(\underline{\theta}_i) = \theta_{j,1}$ and $h_j(\theta_{i,1}) \in \left(\underline{\theta}_j, \theta_{j,1}\right)$. They must cross at least once. Therefore, if there is no $(2,2)$-rule of a high type on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,1}$, then $v_i(\theta) = 0$ does not cross a horizontal segment $\{(\underline{\theta}_i, \theta_{i,1}), \theta_{j,1}\}$. This implies that $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,2}$ is above $v_i(\theta) = 0$ and that there cannot be an indifferent type of i after observing any subset of $\widehat{\Theta}_{j,2}$. Hence, there is neither $(2,1)$-rule nor $(2,2)$-rule on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,2}$.

(b) This proof is similar to (ii) (b).

(v) (a) First, note that in a $(3,2)$-rule $v_i(\theta) = 0$ crosses two vertical segments while $v_j(\theta) = 0$ crosses a horizontal segment. Because neither line goes through $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,1}$ and $\widehat{\Theta}_{i,3} \times \widehat{\Theta}_{j,2}$, there cannot be any improvement on these sets. Second, on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,2}$, there cannot be $(2,1)$-, $(1,2)$-, and $(2,2)$-rules of a high type (consider $\theta_{i,1}$ and $\theta_{j,1}$). Similarly, there cannot be $(2,1)$-, $(1,2)$-, and $(2,2)$-rules of a low type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,1}$ (consider $\theta_{i,2}$ and $\theta_{j,1}$). Third, on $\widehat{\Theta}_{i,3} \times \widehat{\Theta}_{j,1}$, there cannot be $(2,1)$- and $(2,2)$-rules of a high type (consider $\theta_{i,2}$). Similarly, there cannot be $(2,1)$- and $(2,2)$-rules of a low type on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,2}$ (consider $\theta_{i,1}$). Hence, it suffices to show: if neither $(2,2)$-rule of a low type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,2}$ nor $(2,2)$-rule of a high type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,1}$ exists, then there is neither $(1,2)$- nor $(2,2)$-rules of a low type on $\widehat{\Theta}_{i,3} \times \widehat{\Theta}_{j,1}$ and there is neither $(1,2)$- nor $(2,2)$-rules of a high type on $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,2}$. Notice that if $v_j(\theta) = 0$ crosses the left vertical segment $\left\{\theta_{i,1}, \left(\theta_{j,1}, \overline{\theta}_j\right)\right\}$, then there exists a $(2,2)$-rule of a low type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,2}$. Too see this, construct $h_i(x) \equiv \left\{\theta_i \in \Theta_i | \int_{\Theta_j} v_i(\theta) dF_i(\theta_j | \theta_i, \theta_j \in [\theta_{j,1}, x]) = 0\right\}$ connecting $h_i\left(\overline{\theta}_j\right) = \theta_{i,1}$ and $h_i\left(\theta_{j,1}\right) \in (\theta_{i,1}, \theta_{i,2})$ and $h_j(x) \equiv \left\{\theta_j \in \Theta_j | \int_{\Theta_i} v_j(\theta) dF_i(\theta_i | \theta_j, \theta_i \in [\theta_{i,1}, x]) = 0\right\}$ connecting $h_j\left(\theta_{i,2}\right) = \theta_{j,1}$ and $h_j\left(\theta_{i,1}\right) \in \left(\theta_{j,1}, \overline{\theta}_j\right)$. They must cross at least once. Similarly, if $v_j(\theta) = 0$ crosses the right vertical segment $\left\{\theta_{i,2}, \left(\underline{\theta}_j, \theta_{j,1}\right)\right\}$, then there exists a $(2,2)$-rule of a high type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,1}$. Therefore, if neither $(2,2)$-rule of a low type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,2}$ nor $(2,2)$-rule of a high type on $\widehat{\Theta}_{i,2} \times \widehat{\Theta}_{j,1}$ exists, then $v_j(\theta) = 0$ crosses none of the two vertical segments. This implies that $v_j(\theta) = 0$ crosses neither $\widehat{\Theta}_{i,3} \times \widehat{\Theta}_{j,1}$ nor $\widehat{\Theta}_{i,1} \times \widehat{\Theta}_{j,2}$. Hence, there can be neither $(1,2)$- nor $(2,2)$-rules on these type sets.

(b) The condition says $v_j(\theta) = 0$ crosses none of the two vertical segments, which was proved in (a). ■

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