TECHNOLOGICAL CHANGE AND WAGE DIFFERENTIALS RESULTS AND POLICY IMPLICATIONS FROM A DYNAMIC INTERTEMPORAL GENERAL EQUILIBRIUM MODEL

by

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Results and Policy Implications from a Dynamic Intertemporal
General Equilibrium Model

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Abstract
The effect of technological change on wage differentials between skilled and unskilled labour has been extensively investigated. However, the existing literature provides controversial results. This paper provides insights into the relationship between technological change and wage differentials by constructing a DIGE model of a closed economy. This model suggests a range of policy implications.

The main results are: first, a technology upgrade increases wage differentials by a larger percentage than the size of the shock in the short run and reduces wage differentials in the long run. Second, government would expect an increasing short-run and a decreasing long-run wage differential if it increases the tax rate. Third, the effect of increased education investment on wage differentials is unambiguous in the short run. However, in the long run wage differentials decrease.

JEL classification: C61; C68; D10; D91; J31
1. Introduction

The effect of technological change on wage differentials between skilled and unskilled labour has been extensively investigated. A range of empirical studies take a stand on skill-biased technological change being the reason for raising demand for skilled labour relative to the demand for unskilled labour, therefore leading to an increase in wage differentials¹. Some studies, such as Heckman et al. (1998) and Lloyd-Ellis (1999) employ a general equilibrium model to capture the effect from both skilled labour supply and demand. The existing literature provides controversial results, as Lloyd-Ellis (1999) pointed out "Considerable microeconomic evidence finds a positive relationship between the introduction of new technologies into production and the returns to skill... In contrast, recent macroeconomic experience suggests that technological change and wage inequality have been negatively correlated over time".

This paper attempts to investigate this issue by using a new method, a dynamic intertemporal general equilibrium (DIGE) model. A relatively comprehensive theoretical framework of a closed economy is constructed. It is a one-good, two-labour (skilled and unskilled), and three-agent (firms, households, and government) model. All the key variables, e.g. wages and domestic quantities of different types of labour, are endogenous and are determined by the forces of supply and demand in their corresponding markets. By using this model, both the short and long run effects of the technological change on

wage differentials can be illuminated. Since government is embedded in this model, policy implications can be directly extracted from the simulation. The results from this theoretical model can also be compared with those from the empirical studies.

There are four sections in this paper. It begins with a brief overview of the literature and methodology in Section 1. Section 2 outlines the theoretical framework of a DIGE model. Section 3 presents the simulation and Section 4 concludes.

2. The Model

The framework of this model is as follows: firms produce the good by hiring physical capital together with skilled and unskilled labour, they then sell this good to the households for consumption, to the government for education capital investment and to themselves for physical capital investment. The objective of each firm is to maximize its intertemporal profit. The ownership of firms belongs to households. The household supplies unskilled labour to firms and skilled labour to both firms and the government in order to earn wages which, together with the dividends from renting physical capital to firms, finance the purchase of goods and education. Leisure is consumed by the household with an opportunity cost of not working. The objective of households is to maximize utility by an optimal distribution of consumption between the good and leisure. The government buys the good from the firms and transforms it into education capital. This capital is combined with skilled labour hired by the government to produce education. The role of government as an education supplier is essential. This model
attempts to capture the reality of government supplying education in consideration of the associated externalities. The government balances its budget by collecting labour income tax\(^2\) and selling education to the household. The accumulation of physical capital, skill formation, education capital and financial assets drives the dynamic evolution of the economy over time.

2.1 Firms

The production function of the representative firm is assumed to be Cobb-Douglas as follows\(^3\)

\[
Q_t = A \cdot K^\alpha \cdot L_s^\beta \cdot L_u^{1-\alpha-\beta},
\]

where \(Q_t\) is the output, \(A\) is the technology parameter or so-called total factor productivity, and \(K, L_s, L_u\) are respectively the physical capital stock, the skilled and the unskilled labour hired by firms. \(\alpha, \beta,\) and \(1-\alpha-\beta\) state the shares of the three employed inputs.

Capital accumulation depends on the rate of fixed capital formation \(J\) and the rate of depreciation \(\delta\).

\[
dK/dt = J_t - \delta \cdot K_t,
\]

Under the assumption of rising marginal costs of installation in the investment process, the total investment expenditure \(I\) is

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\(^2\)To avoid unnecessary complexities, a subsidy rate on investment and a tax rate on financial dividends is assumed to be balanced out.

\(^3\)A CES production function for both the good and education is also examined and it yields the similar conclusion of wage differentials with a technological shock and a taxation shock.
where $\phi$ is a positive parameter, and $(\phi/2)(J/K_i)$ is the unit cost of adjustment, which is assumed to be a linear function of the rate of capital formation.

The current value Hamiltonian function is employed to solve the above autonomous one state variable system with $\lambda$ as the shadow price of capital. By solving the first-order differential equation and applying the transversality condition, the shadow price of capital becomes

$$
\lambda(t) = \int_t^\infty [Q_K + (\phi/2)(J/K)^2] e^{(r+\delta)s} \cdot ds.
$$

where $Q_K$ is the marginal product of capital and $(\phi/2)(J/K)^2$ is the marginal product of capital in reducing adjustment costs in investment at each point in time. Therefore, $\lambda$ is the increment to the real value of the firm from a unit increase in its investment at time $t$.

2.2 Households

The aim of the household is to maximize its intertemporal utility subject to several constraints.

Max. $\int_0^\infty U(C_i, L_i) \cdot e^{\delta t} \cdot dt$

Subject to

1. $\frac{dF}{dt} = r F_i + (1 - \tau) \cdot (W_{i,t} L_{a,t} + W_{h,t} L_{a,t}) - C_i - P_{E,t} \cdot S_{E,t}$

2. $\frac{dL_i}{dt} = J_{i,t} - \delta L_{a,t}$

3. $S_{E,t} = J_{E,t} \cdot [1 + \frac{\Phi}{2} \cdot \frac{J_{E,t}}{L_{E,t}}]$
\( F_t = \lambda_t \cdot K_t \)

where \( C_t \) is the consumption of the good, \( l_t \) is the leisure taking, \( \theta \) is the rate of time preference, \( r \) is the interest rate, \( F_t \) is financial assets, \( \tau \) is the tax rate, \( W_s \) and \( W_u \) are, respectively the wage rates of skilled and unskilled labour, \( P_{E,t} \) is the price of one unit of education, \( S_{E,t} \) is the amount of education bought, \( J_{t,t} \) is fixed skill formation, \( \delta_s \) is the depreciation rate of skill, \( L_{s,t} \) is the amount of skilled labour, and \( \Phi \) is the adjustment cost parameter.

Equation (5) is the household’s budget constraint. Equation (6) shows that net skill accumulation is skill depreciation subtracted from fixed skill formation. Equation (7) states that education investment depends on fixed skill formation and an adjustment cost function. The adjustment cost relies on the ratio of fixed skill formation to skilled labour. If skilled labour is increasing, the adjustment cost is decreasing. This is plausible due to the spillover effect among labour.

The current value Hamiltonian function is employed to solve the above autonomous two-state variables system with \( \mu_1 \) and \( \mu_2 \) as the respective shadow prices for the financial asset and skill. The shadow price of skill is greater than the shadow price of the financial asset, because the total cost of forming a unit of skill is greater than that of accumulating one unit of financial asset, due to the adjustment cost of skill formation. If the shadow price of skill is not greater than that of the financial asset, the household would like to defer spending on skill formation and instead accumulate financial assets for future consumption.
Applying the transversality condition to the shadow price of skill, $\mu_2$, results in

$$\mu_2(t) = \int_t^\infty \{ \mu_1 \cdot \left[ (1 - \tau) \cdot W + P \cdot \frac{\Phi}{2} \cdot \left( \frac{J}{L} \right)^2 + U_{L, t} \} \cdot e^{- (0 + \delta s) \cdot t} \} \cdot dt$$

Equation (9) states that the shadow price of skill is equal to the present discounted value of future marginal utility. The first component of the shadow price of skill contains the marginal utility of consuming goods, the after-tax skilled wage, and the reduction of the adjustment cost in education investment. It provides the gross increment of utility the household can get from supplying one additional unit of skilled labour. The second part is the marginal disutility of offering one unit of skilled labour. Combining these two gives the net utility the household can achieve by supplying one unit of skilled labor. It substantiates the essence of $\mu_2$.

2.3 Government

Government is a supplier of education. The education production function is

$$E = f(K_{E,t}, L_{G,t})$$

where $E$ is the education supply, $K_{E,t}$ is the education capital, and $L_{G,t}$ is skilled labour working for government. The interpretation of $K_{E,t}$ could be the hardware associated with schooling, e.g., classroom, equipment, etc. $L_{G,t}$ could be the software associated with schooling, e.g., teachers, administrators, etc.

Education capital accumulates via governmental investment in education as follows

$$\frac{dK_{E}}{dt} = I_{E,t} \cdot \delta_{E} \cdot K_{E,t}$$
where $I_{E,t}^G$ is the government investment in education, and $\delta_E$ is the depreciation rate of education capital.

2.4 Steady State

A full model in a steady state is presented in Appendix 1. This model provides the relationship between skilled and unskilled wage in a steady state as follows

\[ W_s = W_u + \frac{P_E}{(1 - \tau)} \cdot (\theta + \theta \cdot \Phi \cdot \delta_s + \delta_s + \frac{1}{2} \cdot \Phi \cdot \delta_s^2). \]

The expression of equation (12) is independent of the functional form of both the utility and production functions. It provides a rigorous theoretical result for wage differentials. The relationship between the skilled and unskilled wage depends on the rate of time preference, the depreciation rate of skill, the skill adjustment cost parameter, the tax rate and the price of education. A higher skill adjustment cost, skill depreciation rate, or time preference, all tend to boost the wage differential. The reason why higher skill adjustment costs and higher skill depreciation rates raise wage differentials is straightforward. The reason for a larger time preference having this effect is that the rate of time preference counts because an investment in skill formation takes time to repay. A larger time preference involves a larger adjustment cost for skill formation, therefore a patient household will expect a higher skilled wage.

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4 Due to the complicated framework, it is not possible to solve for a reduced form of wage differentials.

5 A detailed proof is available from the author.
By equation (12), a technological improvement in the production pushes down the price of goods. This motivates the government to cut the tax rate and this decreases wage differentials. The government plays an important role in wage differentials in the context of the education price and taxation. The government controls education investment, and therefore, impinges on education price. If the government increases education investment, thereby decreasing the education price, it can lead to a decreased wage differential. A cut in government education investment leads to an increased wage differential. What matters in a general equilibrium is the interactive effect of the education price and the tax rate. Intuitively, a higher tax rate makes it possible for government to lower the education price. In a steady state, since skill formation catches up with the skill demand, a decreasing demand for education drives down the education price. More detailed information on policy implications will be obtained by the simulation results reported in the next section.

3 Simulation

A shock to technology (A) in the form of an improvement by 1 per cent is examined. An increase in government spending on education and an increase in tax rates are also analysed to investigate their effect on wage differentials. Following convention, the ratio of the skilled to the unskilled wage is used as the measure of wage differentials. Except for the interest rate and the tax rate in absolute percentage change, all endogenous variables are expressed as the percentage change relative to the baseline. Production can be skilled labour intensive, unskilled labour intensive, or capital intensive. Since this
model is robust, and all three cases end with the same conclusion for the wage differential with the technology shock, only one case is presented. The following simulation examines the case where production is skilled labour intensive. Appendix 2 presents the results of the case where production is unskilled labour intensive. The parameters and exogenous variables settings are as follows:

\[
\begin{align*}
\alpha &= 0.3; & \theta &= 0.1; & \xi &= 0.6; & \tau &= 0.3; \\
\beta &= 0.4; & \delta_h &= 0.1; & \delta_B &= 0.1; & I^G_f &= 10 \\
\delta &= 0.2; & \Phi &= 1; & T &= 8760; & \phi &= 1; \\
\gamma &= 0.7; & A &= 1.
\end{align*}
\]

Figure 1 shows the transition of wage differentials following the technology shock. A technology improvement enlarges the wage differential by 3.5 per cent in the short run, but reduces the wage differential by 0.015 per cent in the long run. It shows that technological change has a relatively large impact, with a larger size than that of the shock, on increasing wage differentials in the short run. This enlarging impact on wage differentials dies out in the long run.

Figure 1 Wage Differentials by Technology Shock

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6 This is one setting which avoids corner solutions and provides a reasonable converging time length.
The reason for this is that, in the short run, a technology improvement raises the demand for skilled labour. Since skill formation takes time, this shortage cannot be immediately filled. In the long run, with an affordable cost of skill formation, the skilled labour supply eventually catches up with the amount demanded.

Figure 2 illustrates the effect on wage differentials of an increased tax rate and increased government spending on education. In the short run, an increase in the tax rate by 1 percentage point (i.e. from 30% to 31%) raises the wage differential by around 1.25 percent due to the jump in the skilled wage and the fall in the unskilled wage. The reason for the short-run boost to the skilled wage is the higher government demand for skilled labour to produce education. The short-run fall in the unskilled wage is due to a drop in goods production caused by a crowding out effect of higher taxation, resulting in lower demand for unskilled labour. In the long run, due to the glut of skilled labour, the skilled wage falls below the baseline, and the unskilled wage increases above the baseline due to the relative scarcity of unskilled labour, so that the wage differential drops about 0.01 percent below the baseline. An increase in government education investment by 1 percent pushes down the skilled wage and boosts the unskilled wage, resulting in reduced wage differentials. The effect in reducing wage differentials grows larger, to around 1.6 percent, in the long run due to the time needed to complete the skill formation. It shows that an increase in government education investment will have a larger impact on wage differentials in the long run than in the short run.
An interactive effect on wage differentials from shocks to technology and taxation as well as shocks to technology and government education investment can be analyzed by using the above results to provide interesting policy implications.

A combined shock of an improved technology and an increased tax rate results in an increased short-run and a decreased long-run wage differential. An upgrading technology combined with a reduction of tax rate shock results in a decreased short-run and an increased long-run wage differential. This result follows Figure 1 and 2, which show a change of the tax rate generating a larger effect on wage differentials than a shock of technology. Thus the effect from a cut in tax rate may offset the effect from a technology upgrade and dominate the net effect on wage differentials.

Since increased education investment takes time to show its effect on wage differentials, a combined shock of a technology upgrade and increased government education investment presents a short run effect dominated by the technology improvement, and a long run effect dominated by the education investment. That is, wage differentials increase in the short run. In the long run, with an increase in education spending by
government, wage differentials decrease, and with a reduction in education investment, wage differentials increase. The simulation results of these combined shocks are shown in Appendix 3.

4 Conclusion

This paper provides insights into the relationship between technological change and wage differentials by constructing a DICE model of a closed economy. Several policy implications are also captured by this model.

The main results are: first, a technology upgrade raises wage differentials by a larger percentage than the size of the shock in the short run and reduces wage differentials in the long run. The intuition for this is that skill formation takes time and is unable to fill the shortage of skilled labour due to a technology improvement in the short run. However, as long as the adjustment cost of skill is affordable by the household, the skill supply eventually catches up with the demand in the long run. In contrast to the existing literature which emphasizes the role of skill biased technological change in raising wage differentials, this paper provides another argument such as the adjustment cost of skill playing a deterministic role in the long run effect.

Second, government would expect an increasing short-run and a decreasing long-run wage differential if it increases the tax rate. A higher tax income enables the government
to spend more on education, therefore, in the long run, the skilled wage drops due to a glut of skill labour and the unskilled wage rises due to a scarcity of unskilled labour.

Third, an increase in government education investment provides education at a cheaper price, therefore a larger amount of skilled labour can be formed. The effect of increased education investment on wage differentials takes time to settle since education itself is a time consuming process. Hence, in the short run, the effect is not obvious. In the long run, wage differentials decrease due to a glut of skilled labour and a scarcity of unskilled labour.

Fourth, the interactive effect on wage differentials of a combined improvement in technology and an increase in the tax rate follows the above result, i.e. an increased short-run but a decreased long-run wage differential. However, a reduction of the tax rate may generate an effect to offset the effect of a technology upgrade and ends with a decreasing short-run and an increasing long-run wage differential. This depends on the size of the tax rate shock.

Fifth, the effect of education investment on wage differentials takes time to carry out. A combined shock of a technology upgrade and increased government education investment presents a short run effect dominated by the technology improvement and a long run effect dominated by the education investment. That is, wage differentials increase in the short run. In the long run, with increased education spending by government, wage
differentials decrease, and with a reduction of education investment, wage differentials increase.
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Appendix 1

Model in Steady State

Equations

\[ Q = A \cdot K^\alpha \cdot L_s^\beta \cdot L_u^{1-\alpha-\beta} \]

\[ J_t = \delta \cdot K_t \]

\[ I = J \cdot (1 + \phi \delta/2) \]

\[ Q_{Ls} = W_s/P \]

\[ Q_{Lu} = W_u/P \]

\[ \lambda = 1 + \phi \delta \]

\[ Q_K = (r + \delta)\lambda - \phi \delta^2/2 \]

\[ 0 = \tau_1 \cdot F_1 + (1-\tau_u) \cdot (W_s \cdot L_{s,1} + W_u \cdot L_{u,1}) - C_t \cdot P_{E,t} \cdot S_{E,t} \]

\[ J_{s,1} = \delta_{s,1} \cdot L_{s,1} \]

\[ F_1 = \lambda \cdot K_t \]

\[ S_{E,t} = J_{s,1} \cdot (1+\delta_u) \]

\[ I_t = T - L_{s,1} - L_{u,1} \]

\[ U_C = \mu_1 \]

\[ U_{L_{1,1}} = -\mu_1 \cdot (1 - \tau) \cdot W_u \]

\[ \mu_2 = \mu_1 \cdot P_E \cdot (1+\phi \delta) \]

\[ \tau_u = \theta \]

\[ U_{L_{1,1}} = (\theta + \delta_u) \cdot \mu_2 \cdot \mu_1 \cdot [((1 - \tau) \cdot W_s + P_E \cdot (\phi \cdot \delta^2)/2)] \]

\[ L_s = L_s^G + L_s^F \]

\[ S_{E,t} = K_{E,t}^G \cdot L_s^{G_{s,t}} \]

\[ I_{E,t}^G = \delta_{E,t} \cdot K_{E,t} \]

\[ I_{E,t}^G + W_s \cdot L_{s,t} = \tau \cdot (W_s \cdot L_{s,t} + W_u \cdot L_{u,t}) + P_E \cdot S_{E,t} \]

\[ Q_t = C + I_{E,t}^G + 1 \]
Appendix 3

**Wage Differential - Technology & Tax**

- Technology Up 1% Tax Down 0.5%
- Technology Up 1% Tax Down 1.5%
- Technology Up 1% Tax Up 1%

**Wage Differential - Technology & Education**

- Technology Up 1% Education Down 0.5%
- Technology Up 1% Education Down 1.5%
- Technology Up 1% Education Up 1%
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<td>12</td>
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<td>Some Market Effects of E-Commerce</td>
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<td>18</td>
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<td>Hsiao-chuan Chang</td>
<td>Technological Change and Wage Differentials Results and Policy Implications from a Dynamic Intertemporal General Equilibrium Model</td>
<td>June 2001</td>
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