

ISSN 0819-2642  
ISBN 0 7340 2628 5



**THE UNIVERSITY OF MELBOURNE**  
**DEPARTMENT OF ECONOMICS**

**RESEARCH PAPER NUMBER 970**

**SEPTEMBER 2006**

**A MODEL OF ADJUSTMENT COSTS  
AND ASYMMETRIC PRICE TRANSMISSION**

by

Tim Helm

Department of Economics  
The University of Melbourne  
Melbourne Victoria 3010  
Australia.

# A model of adjustment costs and asymmetric price transmission

Tim Helm  
The University of Melbourne

18 September 2006

## **Abstract**

This paper constructs a dynamic oligopoly model in order to examine vertical price transmission in the presence of asymmetric input search costs. By assuming these adjustment costs are convex in the rate of change of output, time-dependent price adjustment paths are derived for positive and negative upstream price shocks. A particular form of adjustment cost function is proposed, and is used to examine relationships between market characteristics and the magnitude, speed, and asymmetry of price transmission. Among other results, it is found that higher levels of market power are associated with smaller magnitudes, faster speeds, and less asymmetry. The absolute size of the adjustment costs, as well as their asymmetry, also influences the degree of pricing asymmetry.

# 1 Introduction

This paper examines ‘asymmetric price transmission’, the phenomenon of faster pass-through of input price increases compared with decreases. The tendency for downstream prices to rise faster than they fall is gaining widespread public and academic interest, with farmer and consumer advocates and competition authorities increasingly scrutinising certain high profile markets. Asymmetric price transmission is common: econometric studies have observed it in a range of markets, from agricultural goods to gasoline, and sometimes at several stages in the supply chain.<sup>1</sup> It is also poorly understood: standard theories of competitive and oligopolistic markets which focus on shifts in equilibria provide little help in explaining why the processes of adjustment to these equilibria should be asymmetric. The theoretical literature on asymmetric transmission is limited to a small collection of models and ideas drawing, usually, on aspects of imperfect competition and strategic interactions between firms, or on some form of asymmetric adjustment costs.

The goal of the paper is to formalise one strand of the latter theory – that price transmission asymmetry results from asymmetric adjustment costs – within the setting of a dynamic oligopoly model. Adding a time dimension to the model enables emphasis to be placed on asymmetry in the speed rather than the final magnitude of price transmission. In this setup an adjustment cost faced by each firm, convex in the rate of change of output, slows movement to equilibrium following an input price shock. The structure of the industry is left quite general, with functional forms for demand and costs largely unspecified, and instead approximated by elasticities. However, it proves necessary to place several restrictive assumptions on the information and beliefs held by the firm. This is done purely for tractability, allowing difficulties inherent in solving differential games to be bypassed without assuming one of the specific forms for which solutions are known.

---

<sup>1</sup>Peltzman (2000) finds asymmetry in two-thirds of 77 consumer and 165 producer goods surveyed, and concludes that asymmetry can be ‘fairly labelled as a stylized fact’ of markets. Borenstein, Cameron and Gilbert’s (1997) study of the gasoline supply chain uses four pricing points and finds asymmetry at several stages. See also Meyer and von Cramon-Taubadel’s (2004) survey for discussion of empirical techniques and results.

A second goal of the paper is to generate predictions about the influence of industry characteristics on the speed and magnitude of downstream transmission, and about the degree of price transmission asymmetry resulting from asymmetries in adjustment costs. By observing the sensitivity of the price adjustment path to the adjustment cost, some tentative conclusions can be drawn about the influence of asymmetric adjustment costs on price transmission, and the role of other parameters.

The final contribution of the paper is to suggest a form for an adjustment cost function to be used in such dynamic modelling. This function is characterised by a single parameter (interpreted as a type of elasticity), and is employed in the numerical simulations, while a more general two-parameter version is suggested but not used.

The organisation of the paper is as follows. The next section contains a brief discussion of asymmetric price transmission, focussing on adjustment cost-based theories. Section 3 presents the price transmission model, with an emphasis on the differences in method from those usually employed for dynamic games. Section 4 introduces a particular form for the adjustment cost function with an explanation of its properties. In Section 5 the model and adjustment cost function are used to examine the determinants of the speed, magnitude, and asymmetry of price transmission.

## **2 Adjustment costs and asymmetric price transmission**

While the logic of the idea that larger costs of adjustment cause slower or smaller price responses is obvious, identifying plausible sources of asymmetric adjustment costs is more difficult. Several explanations based around inventory management have been proposed,<sup>2</sup> while other models have produced asymmetric pricing by interacting a symmetric repricing cost with other market characteristics. For example, Ball and Mankiw's (1994) menu cost model predicts asymmetry where there is trend inflation, while spatial competition and symmetric menu costs creates asymmetric pricing in Azzam (1999).

---

<sup>2</sup>See Reagan and Weitzman (1982), Borenstein *et al.* (1997), and Ward (1982).

The model presented here closely matches an ‘asymmetric input search cost’ hypothesis. Under this hypothesis, firms face higher costs of acquiring inputs, through search costs and price premia, than of shedding inputs. The effect of this asymmetric cost will be different rates of output expansion and contraction in response to equal sized negative and positive input price shocks, with associated asymmetry in price changes (Peltzman (2000, p494)). When the input search costs satisfy certain conditions, only the speed of price adjustment is affected – the long-run price response is identical in magnitude for both types of upstream shock. In the classification of Meyer and von Cramon-Taubadel (2004), this hypothesis concerns vertical (rather than spatial), positive asymmetry (input price increases transmit faster), in the speed, rather than the magnitude, of adjustment to upstream shocks.

While the distinction is not often clearly made, the implications of asymmetry in the speed and in the magnitude of price transmission are quite different. Where equally-sized increases and decreases in a price lead to unequal long-run changes in a related price (magnitude asymmetry), divergence between the two prices will occur. This possibility seems implausible, at least for fluctuating price series, since it implies that the long-run relationship between the prices must also be changing.<sup>3</sup> More often (in the markets commonly studied) prices exhibit asymmetric rates of adjustment with symmetric long-run magnitudes of transmission.<sup>4</sup> This suggests that using a time-dependent model to examine adjustment paths may prove more useful than focussing only on equilibria.

Asymmetry in the timing or size of price changes along the supply chain clearly has implications for the welfare of the groups involved, but whether this causes net welfare losses depends crucially on the cause of the asym-

---

<sup>3</sup>Magnitude asymmetry may be more likely when fluctuations are biased in one direction, so that the price series exhibit a long-run trend. Morrisset (1998) alludes to this possibility when discussing transmission between world commodity prices and final-good retail prices.

<sup>4</sup>Although Peltzman finds that asymmetry persists for as long as the estimated lag coefficients are significant, tests of the long-run effects of asymmetry show that input and output prices move together. That is, the two price series are in long-run equilibrium (see 2000, p486). Similarly, Borenstein *et al.* (1997) find that crude oil price increases lead to significantly higher gasoline prices at 2 and 4 weeks than crude price decreases, with the difference being insignificant by 8 weeks.

metry. Of the two broad categories of explanation for asymmetric pricing – adjustment costs and market power – only asymmetry due to abuse of market power leads to a deadweight welfare loss.<sup>5</sup> This suggests an important role for economics in developing tests to distinguish between causes of asymmetry, and to measure distributional effects, in addition to documenting its existence.

Most attempts to detect causes of asymmetry have been fairly ad hoc. One approach involves searching for correlations between the degree of asymmetry and easily measured market characteristics (such as the concentration ratio, or the input price volatility). More sophisticated approaches have attempted to rule out explanations on the basis of where or when asymmetry is observed. In their study of the gasoline supply chain, Borenstein *et al.* (1997) argue that not all potential explanations for asymmetry are relevant at each stage. For example, theories based around costly consumer search and local market power cannot be relevant in determining price adjustment in the spot market, where the group of buyers is small and well-informed. Miller and Hayenga (2001) test symmetry of transmission separately for high and low frequency price cycles. Explanations which are relevant only for high frequency price changes (consumer search, for example) or low frequency cycles (inventory dynamics) can then be ruled out when symmetric transmission is observed in these frequency bands.

The model and simulations presented here similarly attempt to generate testable predictions about how asymmetry in adjustment costs might be manifested in price transmission. Predictions are made about the pattern of asymmetry over time, the maximum degree of pricing asymmetry given asymmetry in costs, and the mitigating or exacerbating effects of various parameters. The important qualifications in the next section about the types of cost, the time horizon, and the assumed behaviour and knowledge of firms should be borne in mind.

---

<sup>5</sup>See Meyer and von Cramon-Taubadel (2004). They note that asymmetric transmission, when due to market power, also provides a signal of the usual welfare losses from inefficiently high prices.

### 3 Modelling price adjustment

The key optimisation problem of the model is a profit-maximising differential game introduced in subsection 3.2, the solution of which is a time-dependent path for each firm’s quantity choice. However, differential games are difficult to solve – solutions are only known for games with certain functional forms, as discussed in subsection 3.3. To retain the generality of a model without specific functional forms (like a dynamic analog of a static equilibrium displacement model), the differential game is simplified to a set of single player optimisation problems. The informational assumptions needed to achieve this are discussed in subsection 3.4, and the optimal solution is found in subsection 3.5.

To study transmission of input prices it is necessary to connect profit maximising behaviour for the firm, as described by the dynamic model, with a cost minimising choice of inputs. A static framework describing input choice and production is presented in subsection 3.1, and manipulated in the usual comparative static analysis fashion in subsection 3.6, closely following McCorriston, Morgan, and Rayner (1998). The result of overlaying these dynamic and static models is a closed form solution for the price which depends only on various elasticity parameters, the adjustment cost, and time. This is presented at the end of subsection 3.6, with a brief discussion in subsection 3.7 regarding the proper interpretation and relevance of the model.

#### 3.1 Industry structure

Consider a downstream industry with quantity-setting firms, producing a single good from two inputs. This ‘food industry’ is assumed to encompass all intermediate stages of the supply chain between primary producers and consumers (processing, distribution, and retailing). The agricultural input and a marketing input (representing all other inputs and processing costs) are used to produce the final good according to the firm’s variable-proportions production technology  $f$ , so that:

$$q_i = f(q_i^A, q_i^M) \tag{1}$$

It is assumed that  $f$  exhibits constant returns to scale, and that all firms share the same technology. Input prices  $\bar{p}^A$  and  $\bar{p}^M$  are exogenously determined.<sup>6</sup> The inverse retail demand function is defined in terms of industry quantity  $Q$  as:

$$p = p(Q) \quad (2)$$

Each firm chooses an input mix to minimise total costs ( $\bar{p}^A q_i^A + \bar{p}^M q_i^M$ ) subject to the constraint  $f(q_i^A, q_i^M) \geq q_i$ , yielding cost minimisation conditions:

$$\bar{p}^A = c_i \cdot f_A \quad (3)$$

$$\bar{p}^M = c_i \cdot f_M \quad (4)$$

where  $c_i$  is marginal cost, the first derivative of the cost function  $C(q_i; \bar{p}^A, \bar{p}^M)$ . Since  $f$  exhibits constant returns to scale,  $c_i$  is a constant (not dependent on  $q_i$ ).

### 3.2 The firm's problem as a differential game

Suppose that the industry is initially in equilibrium at  $t = 0$  and that there is a shock to the agricultural input price  $\bar{p}^A$ . Firm  $i$ , who is assumed to maximise the present discounted value of profits over an infinite horizon, faces the problem:

$$\max_{q_i} J(q_i) = \int_{t=0}^{\infty} e^{-rt} (p(Q) \cdot q_i - C(q_i) - A(\dot{q}_i, q_i)) dt \quad (5)$$

---

<sup>6</sup>In all respects but this, equations (1) to (4) and the equilibrium displacement modelling method in subsection 3.6 follow the price transmission model of McCorriston *et al.* (1998). Their model assumed input supply functions were not perfectly elastic (input prices were endogenously determined), so that the elasticity of price transmission depended on the input supply elasticities, among other parameters. The price transmission elasticity was defined as the ratio of proportional changes in equilibrium values of retail and agricultural input prices ( $\tau = d \ln p / d \ln p^A$ ). However if their static model is viewed as shorthand for a dynamic process, so that the equilibria correspond to the steady states of some dynamic system, then the ability to interpret  $\tau$  as ‘the proportional change in  $p$  following a 1 percent shock to  $p^A$ ’ is lost. For with endogenous input prices, such an initial shock causes substitution in production away from the agricultural input and a subsequent lowering of its price, so that the proportional change in  $p^A$  once equilibrium is reached is less than 1 percent. Hence  $\tau$  as defined above does not match the intuitive understanding of price transmission elasticity, as described in the quotation marks, with a larger discrepancy the less elastic the input supply. To get around this conceptual problem, and reduce the algebra somewhat, the model presented assumes exogenous input prices.



where profits in each period simply equal revenue minus costs, including the adjustment cost  $A$  which depends on both the level and the rate of change of output.

To match  $A$  most closely with the notion of an input search cost, there should be some restrictions on its form.<sup>7</sup> Firstly, in the absence of inflation, anticipation of future shocks, or any kind of uncertainty about the duration of the price change, neither a constant or concave cost is sufficient to ensure a smooth adjustment path to equilibrium. In this model, to avoid instantaneous jumps in  $q_i$  the adjustment cost must be convex in  $\dot{q}_i$  ( $A_{\dot{q}_i \dot{q}_i}(\dot{q}_i, q_i) > 0$ ). Secondly, in order for adjustment to take place at all, even for small shocks,  $A(0, q_i) = 0$  is necessary. Finally the conditions  $A_{q_i}(0, q_i) = 0$  and  $A_{\dot{q}_i}(0, q_i) = 0$  are imposed, the latter ensuring that the magnitude of price transmission is symmetric, even when the parameters of  $A$  differ for positive and negative shocks.<sup>8</sup>

The set of optimisation problems (equation (5), and its counterparts for other firms) constitute a differential game, with the adjustment cost generating the ‘structural dynamics’ of the game. That is, the adjustment cost (rather than sticky prices or some other friction) makes movement to steady state time-dependent rather than instantaneous. This follows Driskill and McCafferty’s (1989) linear-demand quadratic-cost duopoly model, and Dockner’s (1992) generalised version.

A differential game consists of a set of optimal control problems where the payoffs for each player, and the law of motion of the state variable, depend on the actions taken by all players. The objective in (5) is an optimal control problem where  $x_i = \dot{q}_i$  is player  $i$ ’s control variable,  $q = (q_1, \dots, q_i, \dots, q_N)$  is the  $N$ -dimensional state variable, and  $\dot{q} = (x_1, \dots, x_i, \dots, x_N)$  is the law of motion. Because each player’s payoff  $J(q_i)$  depends on elements of the state beyond their control (the  $q_j$ ’s, for  $j \neq i$ ), this is a differential game rather than a single-player dynamic optimisation problem.

---

<sup>7</sup>A more accurate, but probably less tractable, representation of the input search cost theory would impose the adjustment cost on the changes in the inputs,  $\dot{q}_i^A$  and  $\dot{q}_i^M$ , which are determined implicitly in this maximisation problem via the cost minimisation conditions.

<sup>8</sup>This condition is commonly used in oligopoly theory, for example by Dockner (1992).

### 3.3 Concepts in dynamic games

A Nash equilibrium in such games is a set of strategies, one for each player, such that no player wishes to deviate given the strategies played by the others. There are several options available for the choice of strategy space, with the relevance of each dependent on the informational assumptions concerning the players.<sup>9</sup> The two most commonly employed in dynamic games are the ‘open-loop’ and ‘closed-loop’ strategy spaces. When all players are aware of the structure of the game and the payoff functions for other firms, but are either mutually forced to commit to an output path from the outset, or unable to observe or infer the value of the state at any  $t > 0$ , then they select strategies from the open-loop strategy space. An open-loop, or time-dependent strategy is a function  $x_i(t)$  which determines the value of the control according only to the time, not to the value of the state or any other information existing at  $t$  (the price, for instance). Assuming open-loop strategies in a dynamic game reduces the true time-dimension of the game – it essentially becomes a single-shot game, although the outcome is still a dynamic process.

If commitment is impossible, and the state is observable throughout time, then each firm plays a state-dependent strategy,  $x_i(q, t)$ , known as a closed-loop (or Markovian) strategy. Markovian strategies are also ‘memory-less’ since they condition action on the current state but not on the history of the game, so that trigger strategies are excluded.<sup>10</sup>

Finding equilibrium strategies for differential games entails simultaneously solving  $N$  interrelated optimal control problems, which is notoriously difficult even when simple time-dependent strategies are used. The Pontryagin necessary conditions derived from the firms optimal control problems specify a differential equation system, from which the steady states of the dynamic system can usually be found relatively simply.<sup>11</sup> However analyti-

---

<sup>9</sup>For more on this, see Dockner, Jørgenson, van Long and Sorger (2000, s2.3, 3.5 & 4.1).

<sup>10</sup>Open and closed loop strategy spaces are also not the only possibilities available. For example, another potential strategy space is that which allows players a choice between time-dependent and state-dependent strategies. This would be appropriate for a game context where players had the option of precommitment available to them.

<sup>11</sup>Driskill and McCafferty (1989) find the steady states of a game similar to that pre-

cal solutions to this differential equation system are known only for certain classes of game (and usually a value function approach to solving these games is preferred). For example, the linear-quadratic class of games can be shown to have closed-loop equilibrium strategies which are linear functions of the state variables.<sup>12</sup>

### 3.4 Information, beliefs and strategy space assumptions

A shortcut way of solving the firm's problem above, without restricting it to fit into a particular class of games, is to assume an informational context and associated strategy space which reduces the game to a set of single-player dynamic optimisation problems. First, assume that firms are unable to observe or infer their rival's outputs, so that their strategies (rates of change of output) are forced to be simple time-dependent rather than state-dependent functions.

Second, assume that firms are not fully rational, in the sense of being able to calculate their rivals' reaction functions. Instead, let each firm conjecture a response of the rest of the industry, expressed as a fixed elasticity, to their own change in output. Thus firm  $i$  believes that a proportional change in own output will instantaneously lead to a certain proportional change in industry output, given by  $\theta_i = \frac{dQ}{dq_i} \frac{q_i}{Q}$ . As in the static model of McCorriston *et al.* (1998) the conjectural elasticity is assumed to be symmetric across firms, and is used as a proxy for market power. The parameter ranges between  $\theta = 0$  for competitive firms, who believe their own output is negligible in comparison to the industry, to  $\theta = 1$  for a monopolist.

These two assumptions, taken in tandem, imply an unusual (somewhat paranoid) belief on the part of the firm: each cannot observe the others'

---

sented above, and Dockner (1992) demonstrates the equivalence of these steady states, for open and closed loop strategies, to certain static conjectural variations equilibria.

<sup>12</sup>See Figuières, Jean-Marie, Quérou and Tidball (2004, p61). Because  $p(Q)$  in equation (5) is not necessarily linear, and  $A$  depends on  $q_i$  as well as  $\dot{q}_i$ , the game specified above does not belong to the class they examine. See also Dockner *et al.* (2000, ch4) for discussion of the theory behind the Pontryagin maximum principle and value function approaches to solving differential games.

moves, but believes the others can observe theirs and will react.

Finally, it is assumed that firms are fully informed about the demand function and aware of the price  $p$  at  $t = 0$ , but cannot observe the evolution of  $p$  through time. They are thus unable to infer the actions of their rivals and update their prior belief (the conjecture  $\theta$ ), and hence have no gain in re-optimising at a later date (that is, no desire to change the production path selected at  $t = 0$ ).

### 3.5 Dynamic optimisation problem

With these three assumptions the differential game becomes a set of  $N$  independent optimal control problems of the form:

$$\max_{x_i} J(x_i) = \int_{t=0}^{\infty} e^{-rt} (p(Q) \cdot q_i - C(q_i) - A(x_i, q_i)) dt \quad (6)$$

where  $x_i = \dot{q}_i$ ,  $Q$  is conjectured to depend on  $q_i$  according to  $\theta = \frac{dQ}{dq_i} \frac{q_i}{Q}$ , and  $q_i(0) = q_{i0}$ , for  $i = 1, \dots, N$ . With a unidimensional state variable ( $q_i$ ) and a single control ( $x_i$ ), this problem can be manipulated in the usual way to yield a system of two differential equations in  $q_i$  and  $x_i$  (see Appendix):

$$\begin{aligned} \dot{x}_i &= \frac{1}{A_{x_i x_i}} \left( r A_{x_i} + A_{q_i} - A_{x_i q_i} \cdot x_i - p \left( 1 - \frac{\theta}{\eta} \right) + c_i \right) \\ \dot{q}_i &= x_i \end{aligned} \quad (7)$$

where  $\eta$  is the absolute value of the elasticity of demand ( $\eta = \left| \frac{dQ}{dp} \frac{p}{Q} \right|$ ).

The steady state value of firm  $i$ 's output is found by setting  $\dot{q}_i = \dot{x}_i = x_i = 0$ , bearing in mind the conditions imposed on  $A$  above:

$$p \left( 1 - \frac{\theta}{\eta} \right) = c_i \quad (8)$$

The steady state output,  $q_i^*$ , is given implicitly by (8), via  $p$  which is a function of industry output  $Q$ .

The equations in (7) are non-linear and of an analytically unsolvable type, unless highly specific assumptions are made about functional forms. Hence a linear approximation around the steady state of the system is used. The

solution – an optimal strategy for firm  $i$  given its information, beliefs, and payoff – is:

$$q_i(t) = q_i^* + (q_{i0} - q_i^*)e^{\lambda_2 t} \quad (9)$$

or, in terms of proportional changes from the initial quantity:

$$d \ln q_i(t) = d \ln q_i^* (1 - e^{\lambda_2 t}) \quad (10)$$

The term  $\lambda_2$  ( $< 0$ ) is an eigenvalue of the linearised system, given by:

$$\lambda_2 = \frac{1}{2} \left( r - \sqrt{r^2 + 4K} \right) \quad (11)$$

where  $K = \frac{1}{A_{x_i x_i}^*} \left( r A_{x_i q_i}^* + A_{q_i q_i}^* + \frac{c_i}{q_i^*} \frac{\theta}{\eta} (1 + \mu) \right)$  (see Appendix for details).<sup>13</sup> The expression  $d \ln q_i^*$  denotes the proportional shift in firm  $i$ 's steady state output value caused by the shock. Equation (10) is the key result of the dynamic modelling – a closed form solution for the adjustment path of the firm's output.

### 3.6 Static framework

To transform this output timepath into an expression describing price adjustment, the techniques of equilibrium displacement modelling are used. In this method, structural equations of a static model are given without specifying functional forms, but with elasticities, assumed constant, describing the relationships between variables. The system of structural equations is totally differentiated, giving a set of expressions relating proportional changes in variables. These are jointly solved for the changes in endogenous variables as a function of elasticities and changes in exogenous variables.

With equations (1) through (4), and (8) describing relationships between variables in steady state, differentiation yields a system of 5 equations relating proportional changes in 6 endogenous variables ( $q_i$ ,  $q_i^A$ ,  $q_i^M$ ,  $p$ ,  $Q$ , and  $c_i$ ) and 1 exogenous variable ( $\bar{p}^A$ ) (see McCorrison *et al.* (1998)):

$$d \ln q_i = \alpha d \ln q_i^A + \beta d \ln q_i^M \quad (12)$$

---

<sup>13</sup>Parameter  $\mu$  is defined as  $\mu = \frac{\omega \theta}{\eta - \theta}$ , where  $\omega = \frac{d \eta}{d p} \frac{p}{\eta}$ . The value of  $\omega$  depends on the form of the demand curve. For example, a constant-elasticity demand curve has  $\omega = 0$ , and linear demand implies  $\omega = 1 + \eta$ .

$$d \ln p = -\frac{1}{\eta} d \ln Q \quad (13)$$

$$d \ln \bar{p}^A = d \ln c_i - \frac{\beta}{\sigma} (d \ln q_i^A - d \ln q_i^M) \quad (14)$$

$$0 = d \ln c_i + \frac{\alpha}{\sigma} (d \ln q_i^A - d \ln q_i^M) \quad (15)$$

$$d \ln p = \frac{d \ln c_i}{1 + \mu} \quad (16)$$

The parameters are defined as follows:  $\alpha$  ( $\beta$ ) is the elasticity of output with respect to the agricultural (marketing) input,  $\alpha = f_A \frac{q^A}{q}$  ( $\beta = f_M \frac{q^M}{q}$ ). These two parameters can also be interpreted as cost shares of each input, with  $\alpha + \beta = 1$ .<sup>14</sup> The elasticity of substitution between inputs in production is  $\sigma$  ( $\sigma = \frac{f_A \cdot f_M}{f_{AM} \cdot q}$ ). These parameters, and the three defined earlier ( $\theta$ ,  $\eta$ , and  $\omega$ ), are all positive.

Firms in this game have the same cost function, adjustment costs, and discount rate, leading to identical output paths and steady state values. This symmetry implies that the proportional change in industry output equals that of the firm. Hence equation (13) is rewritten as  $d \ln p = -\frac{1}{\eta} d \ln q_i$ , and the system becomes:

$$\begin{bmatrix} 1 & -\alpha & -\beta & 0 & 0 \\ 1 & 0 & 0 & 0 & -\eta \\ 0 & -\frac{\beta}{\sigma} & \frac{\beta}{\sigma} & 1 & 0 \\ 0 & \frac{\alpha}{\sigma} & -\frac{\alpha}{\sigma} & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 + \mu \end{bmatrix} \begin{pmatrix} d \ln q_i \\ d \ln q_i^A \\ d \ln q_i^M \\ d \ln c_i \\ d \ln p \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ d \ln \bar{p}^A \\ 0 \\ 0 \end{pmatrix} \quad (17)$$

Solving for the proportional change in steady state output of the firm, in terms of the shock to the agricultural input price, gives:

$$d \ln q_i^* = \frac{-\alpha \eta}{1 + \mu} d \ln \bar{p}^A \quad (18)$$

<sup>14</sup>Since  $\bar{p}^A = c_i \cdot f_A$  and  $\bar{p}^M = c_i \cdot f_M$ , according to the cost minimisation conditions (3) and (4),  $\alpha$  ( $\beta$ ) can be written  $\alpha = \frac{\bar{p}^A q_i^A}{c_i q_i}$  ( $\beta = \frac{\bar{p}^M q_i^M}{c_i q_i}$ ). Constant returns to scale in production implies constant marginal cost, so that  $c_i q_i = C(q_i)$ , and thus  $\alpha$  and  $\beta$  represent cost shares.

By making use of equation (18) and  $d \ln q_i(t) = -\eta \cdot d \ln p(t)$ , the output timepath (10) is transformed into the desired result, an adjustment path for the retail price:

$$d \ln p(t) = \frac{\alpha}{1 + \mu} d \ln \bar{p}^A (1 - e^{\lambda_2 t}) \quad (19)$$

### 3.7 Scope and relevance of model

Subsections 3.5 and 3.6 derived a path of adjustment over time of a downstream price to an upstream (input) price shock. However, the use of various assumptions and simplifications in this derivation raises the question of which situations the model is applicable to, and how equation (19) should be interpreted.

First, capital is fixed in this model. The firm's only choice is over input levels, implying that the model is best interpreted as describing short run changes. A long-run response to a permanent input price change could potentially involve either capital investment (thus altering the production technology), or entry and exit of firms, neither of which are accounted for here. Moreover, the assumption of non-observability of rival's outputs or the industry price (subsection 3.4) seems to require that quantity adjustment occurs quickly.

Second, the applicability of the modelling methods used require the shock to the system to be small: equilibrium displacement modelling assumes fixed elasticities, which in reality only holds for incrementally small changes.<sup>15</sup> The linearised version of the non-linear differential equation system (7) is also only likely to be a good approximation in a small neighbourhood of the steady state. Neither of these caveats necessarily preclude the relevance of the model for the input search cost hypothesis being examined.

## 4 Adjustment cost function

This section introduces a single-parameter adjustment cost function in order to simplify comparisons of adjustment cost sizes in the numerical simulations

---

<sup>15</sup>See Piggott (1992) for discussion of this condition and of the overall merits of the equilibrium displacement modelling approach.

below. This function satisfies the restrictions introduced earlier and reduces the complexity of the eigenvalue expression, as shown in subsection 4.2. The parameter is easily interpreted as a type of elasticity, and the function may prove useful in future modelling using general functional forms. A more general two-parameter version is given in subsection 4.3.

## 4.1 One parameter form

Suppose the level of adjustment costs is given by the function:

$$A^\delta(x_i, q_i) = \delta c_i q_i \left| \frac{x_i}{q_i} \right|^2 \quad (20)$$

where  $x_i$  is the rate of change of  $q_i$ . The level of adjustment cost incurred is the square of (the absolute value of) the proportional rate of change, scaled up by the firm's total cost  $c_i q_i$  and a positive parameter  $\delta$ .

The function  $A^\delta$  satisfies the restrictions introduced in subsection 3.2 and employed in the subsequent working –  $A^\delta(0, q_i) = A_{q_i}^\delta(0, q_i) = A_{x_i}^\delta(0, q_i) = 0$ , and  $A_{x_i x_i}^\delta(x_i, q_i) > 0$ .

The adjustment cost parameter  $\delta$  can be interpreted in two equivalent ways. First, suppose that the proportional rate of change of output is  $\frac{x_i}{q_i}$ , so that  $x_i$  units are brought into production over a single period of time. The marginal production cost of each new unit is  $c_i$ , and the adjustment cost per new unit is  $A^\delta(x_i, q_i)/x_i$ . Then the adjustment cost per new unit, as a proportion of the production cost, is  $A^\delta(x_i, q_i)/c_i x_i = \delta \left| \frac{x_i}{q_i} \right|^2$ .<sup>16</sup> Thus the first interpretation of  $\delta$  is as a multiplicative scalar which maps the percentage output change into the percentage premium paid to produce each additional unit. For instance, if increasing output by 10 percent over a period added 20 percent to the cost of production of those new units, then this could be modelled with the function above and the parameter choice  $\delta = 2$ .

A second interpretation follows naturally from the first: since the product of  $\delta$  and the proportional change in output equals the proportional change

---

<sup>16</sup>For shorthand, the adjustment cost per new unit, as a proportion of production cost  $c_i$ , could be referred to as the ‘relative adjustment cost’.



in unit cost for the marginal units, then  $\delta$  actually represents the elasticity of the effective marginal cost with respect to output.<sup>17</sup>

## 4.2 Eigenvalue with one parameter form

The function  $A^\delta$  has the properties that  $A_{x_i q_i}^{\delta*} = A_{x_i q_i}^\delta(0, q_i^*) = 0$  and  $A_{q_i q_i}^{\delta*} = 0$ . The second partial derivative of  $A^\delta$  with respect to  $x_i$  is  $A_{x_i x_i}^\delta(x_i, q_i) = 2\delta c_i/q_i$ , which at the steady state is  $A_{x_i x_i}^{\delta*}(0, q_i^*) = 2\delta c_i/q_i^*$ . Substituting these three expressions into the eigenvalue expression (11) yields:

$$\lambda_2 = \frac{1}{2} \left( r - \sqrt{r^2 + \frac{2\theta}{\delta\eta}(1+\mu)} \right) < 0 \quad (21)$$

## 4.3 Two parameter form

One limitation of the function in equation (20) is that the convexity of the cost is fixed – a doubling of the proportional rate of change of output will double the relative adjustment cost. A more general adjustment cost function with similar properties could be specified, with the parameter  $\rho$  ( $\rho > 0$ ) representing the sensitivity of relative adjustment costs to the proportional speed of output change:

$$A^{\delta,\rho}(x_i, q_i) = \delta c_i q_i \left| \frac{x_i}{q_i} \right|^{1+\rho} \quad (22)$$

The elasticity of effective marginal cost with respect to output is  $\delta \left| \frac{x_i}{q_i} \right|^{\rho-1}$ .

## 5 Determinants of speed, magnitude, and asymmetry of price transmission

The next subsection derives a set of expressions relating various food industry parameters to the speed and magnitude of price transmission. These confirm several of the relationships found by McCorriston *et al.* (1998) regarding

---

<sup>17</sup>The qualifier ‘effective’ is used to clearly distinguish the cost of the marginal unit, which includes an adjustment cost, from the marginal production cost  $c_i$  which does not change.

transmission magnitudes. Subsection 5.2 graphically demonstrates how the parameters and the adjustment cost determine the evolution of the price over time and the asymmetry between upwards and downwards price adjustments.

## 5.1 Market characteristics and speed/magnitude

The eigenvalue  $\lambda_2$  in the equation (19) determines the speed of adjustment, with a larger (more negative) eigenvalue causing faster adjustment. In turn, the term  $\frac{2}{\delta} \frac{\theta}{\eta} (1 + \mu)$  largely determines the size of the eigenvalue, when adjustment cost function  $A^\delta$  is used. The magnitude of price transmission – the long-run elasticity of  $p$  with respect to  $\bar{p}^A$  – can also be derived from (19) by setting  $t = \infty$ . This price transmission elasticity is  $\frac{d \ln p^*}{d \ln \bar{p}^A} = \frac{\alpha}{1 + \mu}$ . Several simple relationships between parameter values and speeds and magnitudes of adjustment can now be observed.

First, it is straightforward to see that increasing the adjustment cost parameter  $\delta$  lowers  $\frac{2}{\delta} \frac{\theta}{\eta} (1 + \mu)$ , making the eigenvalue smaller in magnitude and thus slowing adjustment to steady state. This parameter has no influence on the magnitude of price transmission. In contrast, the agricultural input cost share  $\alpha$  has no effect on the speed of price transmission but largely determines the magnitude.

A higher discount rate  $r$  has no effect on the magnitude of price transmission, but increases the speed:

$$\frac{\partial \lambda_2}{\partial r} = \frac{r}{4(2\lambda_2 - r)} < 0 \quad (23)$$

The influence of the market power parameter is shown through the partial derivative:

$$\frac{\partial \lambda_2}{\partial \theta} = \frac{1}{2(2\lambda_2 - r)} \frac{1}{\delta \eta} \left( (1 + \mu) + \frac{\omega \eta \theta}{(\eta - \theta)^2} \right) < 0 \quad (24)$$

Assuming that the parameters fall inside the stable range which ensures  $1 + \mu > 0$  (see Appendix), the derivative is negative, implying that higher values of  $\theta$  cause faster adjustment to steady state. The market power parameter also influences the magnitude of transmission, according to the derivative:

$$\frac{\partial}{\partial \theta} \left( \frac{d \ln p^*}{d \ln \bar{p}^A} \right) = \frac{-\alpha}{(1 + \mu)^2} \frac{\omega \eta}{(\eta - \theta)^2} < 0 \quad (25)$$

which is negative for all parameter values (except for  $\omega = 0$ ). Thus when the firm's conjecture, assumed to be a proxy for market power in the industry, is larger, price transmission is faster but of a smaller magnitude.

The derivative of the eigenvalue with respect to  $\eta$  is:

$$\frac{\partial \lambda_2}{\partial \eta} = \frac{1}{2(2\lambda_2 - r)} \frac{\theta}{\delta \eta} \left( -\frac{1 + \mu}{\eta} + \frac{\omega \theta}{(\eta - \theta)^2} \right) \quad (26)$$

The sign of this derivative is not unambiguous – it depends on the precise form of the demand curve. When  $\omega > \frac{(\eta - \theta)^2}{\theta(1 + \mu)}$ , the derivative is negative, so that more elastic demand implies faster price transmission. More elastic demand also reduces the magnitude of price transmission, as shown by the derivative:

$$\frac{\partial}{\partial \eta} \left( \frac{d \ln p^*}{d \ln \bar{p}^A} \right) = \frac{-\alpha}{(1 + \mu)^2} \frac{\omega \theta}{(\eta - \theta)^2} < 0 \quad (27)$$

Finally, the derivative with respect to  $\omega$  is:

$$\frac{\partial \lambda_2}{\partial \omega} = \frac{1}{2(2\lambda_2 - r)} \frac{\theta^2}{\delta \eta (\eta - \theta)} \quad (28)$$

which is negative for  $\eta > \theta$  and positive for  $\eta < \theta$ . The magnitude of price transmission is affected by the curvature of demand as shown by the derivative:

$$\frac{\partial}{\partial \omega} \left( \frac{d \ln p^*}{d \ln \bar{p}^A} \right) = \frac{-\alpha}{(1 + \mu)^2} \frac{\omega}{\eta - \theta} \quad (29)$$

which is negative (positive) for  $\eta > \theta$  ( $\eta < \theta$ ). Higher  $\omega$  is generally associated with a faster speed and smaller magnitude of price transmission. However in the unusual case of very low demand elasticity ( $\eta < \theta$ ) this conclusion is reversed, at least for the small range of demand curves ( $\omega < \frac{\theta - \eta}{\theta}$ ) which are able to be considered in this framework.<sup>18</sup>

The effect on price transmission of increasing the values of each parameter are summarised in Table 1.

## 5.2 Adjustment cost size, market power, and asymmetry

This subsection simulates the responses of  $p$  over time to 1 percent increases and decreases in  $\bar{p}^A$  at  $t = 0$ . The plots of the proportional change in the

<sup>18</sup>See the Appendix for discussion of parameter values and stability.

Table 1: Impact of parameters on speed and magnitude of price transmission

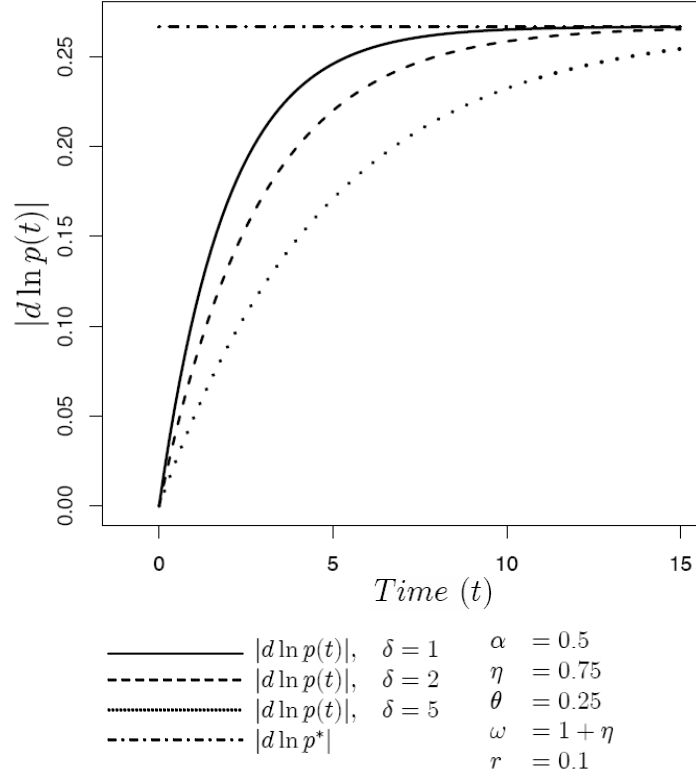
Parameter	Magnitude	Speed
$\delta$	– (no effect)	↓ (slower)
$\alpha$	↑ (larger)	– (no effect)
$r$	– (no effect)	↑ (faster)
$\theta$	↓ (smaller)	↑ (faster)
$\eta$	↓ (smaller)	↑ (faster) if $\omega > \frac{(\eta-\theta)^2}{\theta(1+\eta-\theta)}$ ↓ (slower) if $\omega < \frac{(\eta-\theta)^2}{\theta(1+\eta-\theta)}$
$\omega$	↓ (smaller) if $\eta > \theta$ ↑ (larger) if $\eta < \theta$	↑ (faster) if $\eta > \theta$ ↓ (slower) if $\eta < \theta$

price,  $d \ln p(t)$ , are intended to resemble the usual presentation of results in empirical studies of asymmetric pricing. In such studies, estimated coefficients on current and lagged input price changes, lagged output price changes, and error correction terms are used to plot the separate effects of 1 percent increases and decreases in the input price. Here equation (19) and the eigenvalue expression (equation (21)) are used to predict the impact of such shocks. When different adjustment cost parameters, denoted by  $\delta^+$  and  $\delta^-$ , apply to positive and negative input price changes, separate price adjustment paths can be observed.

Various adjustment paths for the retail price, corresponding to different values of the adjustment cost parameter, are shown in Figure 1. When  $\delta = 5$  (dotted line) the retail price appears to take approximately twice as long to reach the new equilibrium value as when  $\delta = 1$  (the bold line). These lines are intended to represent the unequal responses to positive and negative shocks. If expanding output incurs a greater adjustment cost than contracting output, as hypothesised, then the bold and dashed/dotted lines represent proportional output price responses to cost increases and decreases, respectively.

Figure 2 adds two curves showing the difference between proportional price responses at time  $t$ ,  $|d \ln p^+(t)| - |d \ln p^-(t)|$  (graphically, the vertical distance between bold and dashed/dotted lines). Two ratios of downwards- and upwards- adjustment costs are used:  $\delta^- = 2\delta^+$  and  $\delta^- = 5\delta^+$ . When the

Figure 1: Output price response ( $|d \ln p(t)|$ ) to 1 percent input price shock, for various  $\delta$



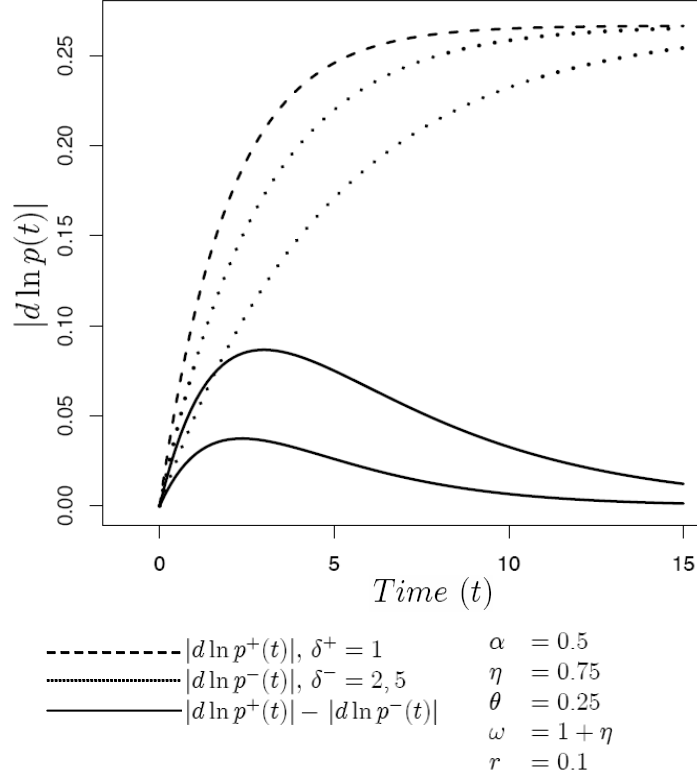
cost of expanding output is double that for output contractions, percentage price responses to the 1 percent shock seem to differ by up to 0.04 percentage points, or around 15 percent of the final price transmission magnitude of 0.27 percent. With expansion costs five times as high, the maximum difference is 0.08 percentage points (approximately 30 percent of the long-run magnitude).

The absolute size, as well as the asymmetry, of adjustment costs appears to influence the asymmetry of price transmission. Figure 3 demonstrates this, showing the asymmetry through time for various sized adjustment costs, with a fixed ratio of  $\delta^- = 5\delta^+$ .<sup>19</sup>

Since higher adjustment costs slow transmission of price changes, it is unsurprising that the ‘peak’ of asymmetry in Figure 3 occurs later for higher

<sup>19</sup>All the lines in Figure 3 correspond to the bold lines of Figure 2. That is, each shows the difference in proportional price responses for positive and negative shocks.

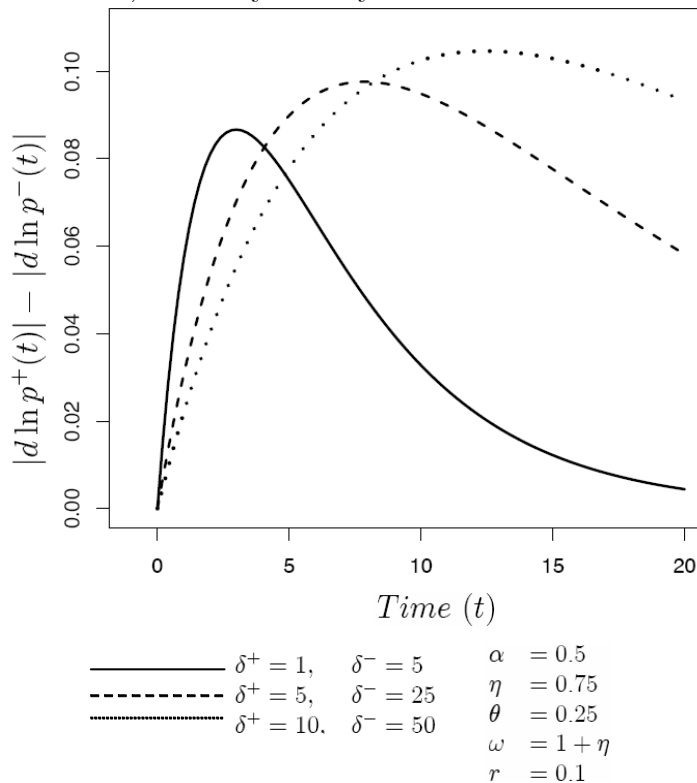
Figure 2: Asymmetry in output price response ( $|d \ln p^+(t)| - |d \ln p^-(t)|$ ), for various  $\delta$



values of  $\delta^+$  (the dashed and dotted lines). A more important feature of the graph is that higher adjustment costs are associated with larger pricing asymmetry, for any given asymmetry in adjustment costs (the dotted line peaks highest, the bold line lowest). This raises the possibility that asymmetric pricing may be more prevalent in industries with large frictional costs than in other industries, even where the asymmetry of costs is the same.

One further relationship is between the market power parameter  $\theta$  and the degree of asymmetry in price transmission. Of interest is whether the presence of market power influences the degree of adjustment cost-induced asymmetry, and whether this effect mitigates or exacerbates the tendencies suggested by market power based theories of asymmetric pricing. Figure 4 plots asymmetry of price transmission for several values of  $\theta$  and  $\omega$ . Adjustment costs are asymmetric with  $\delta^+ = 1$  and  $\delta^- = 5$ , and the two sets of lines

Figure 3: Asymmetry in price response ( $|d \ln p^+(t)| - |d \ln p^-(t)|$ ): various adjustment cost sizes, fixed asymmetry of costs.

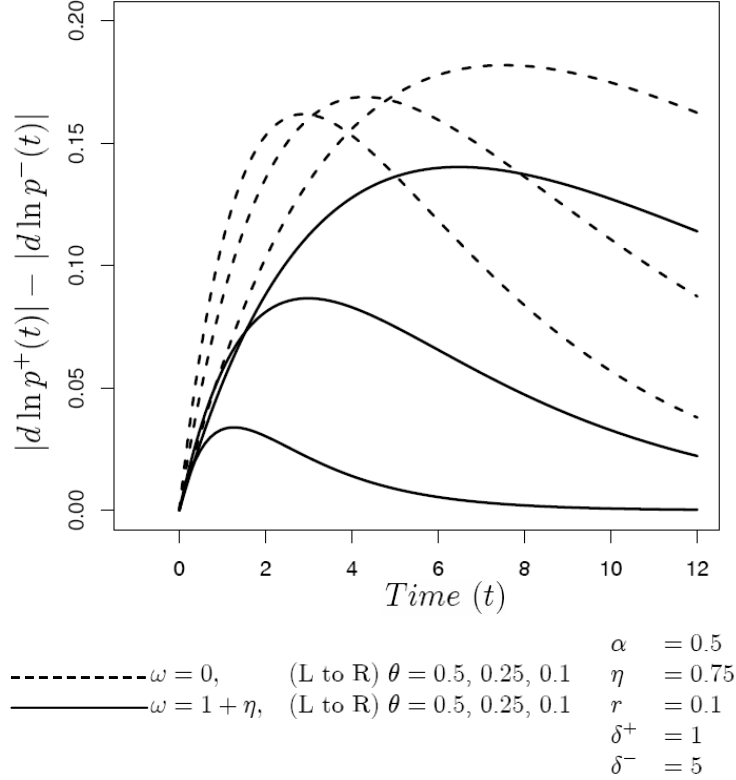


correspond to different types of demand curve, with  $\omega = 1 + \eta$  (linear demand) in bold and  $\omega = 0$  (constant elasticity of demand) shown as a dashed line.

Subsection 5.1 showed that the speed of price adjustment increases with  $\theta$ , explaining why the curves peak sooner for higher  $\theta$ . Figure 4 demonstrates that the maximum degree of asymmetry is actually higher in less concentrated industries, but that the strength of this relationship is heavily dependent on the shape of the demand curve. The higher the value of  $\omega$ , the greater the asymmetry-dampening effect of  $\theta$ .

The implication of this result is that there appears to be no simple additive relationship between market power-induced asymmetry in pricing and adjustment cost-induced asymmetry. Concentration may create the conditions for positive asymmetric price transmission (according to market power

Figure 4: Asymmetry of price response ( $|d \ln p^+(t)| - |d \ln p^-(t)|$ ): linear and constant-elasticity demands, various  $\theta$



based theories), but it also mitigates the asymmetry caused by asymmetric adjustment costs, with the strength of this countervailing effect dependent on the form of the demand curve.

## 6 Conclusions

Econometric studies have suggested that price adjustment processes are typically asymmetric in their speed, but symmetric in terms of long-run equilibrium changes. This behaviour has been modelled here by deriving a fully specified adjustment path for downstream prices following an upstream price shock. Input search costs slowed adjustment, and asymmetry in these costs generated the asymmetric pricing behaviour. Three key features characterised the modelling methods used.



First, in order to link input price shocks with output price changes a dynamic profit maximisation problem was superimposed on a static input-cost minimisation model. The dynamic problem was the source of the time-dependent adjustment path. The static model, in conjunction with the steady state condition from the dynamic problem, allowed calculation of changes in steady states in terms of the size of the input price shock.

Second, in the style of an equilibrium displacement model functional forms were left largely unspecified, so that the price adjustment path could be expressed in terms of various elasticity parameters, the adjustment cost, and the time. By the implicit assumption of fixed elasticity values, and by the use of a linear approximation to solve a differential equation, the model's scope was limited to small shocks. Constant returns to scale in production and certain restrictions on the form of the adjustment cost were also assumed.

Third, the firm's profit maximisation problem was specified as a differential game of oligopoly, but was actually solved by reducing the game to a simpler optimal control problem via an unusual choice of information and beliefs. The quantity choices of rivals and the evolution of the price were assumed unobservable, and each firm adopted a conjecture about strategic responses to their own quantity choice.

Each of these methods has strengths and shortcomings which suggest modifications and extensions for future research. Most importantly, the dynamic model needs to be grounded in realistic, rational firm behaviour. The current behavioural assumptions are probably too narrow to describe well informed oligopolistic firms, and the incorrect conjectures made by firms are not plausible in the context of repeated games and learning. The challenge is to combine the usefulness of the general functional form approach and the application to price transmission with an acceptable game theoretic characterisation of beliefs and strategies. A useful intermediate step might be adapting a dynamic oligopoly game for which solutions are known (such as Dockner's (1992) model) to the price transmission issue.

Some useful insights into the determinants of price transmission magnitudes, speeds, and asymmetry have been gained. The most interesting of these concerned the firm's conjecture, assumed to be a market power proxy.

Higher market power values simultaneously lowered the transmission magnitude, accelerated adjustment to steady state, and reduced asymmetry in pricing. The latter relationship suggests a more complex role for competition than is usually predicted by market power based theories of asymmetric transmission. Several relationships between demand curve parameters and the speed of transmission were also derived, and the numerical simulations demonstrated a positive connection between absolute adjustment cost size and price transmission asymmetry.

Finally, a convex adjustment cost function was proposed which is easily interpreted and may be useful for dynamic oligopoly modelling. The single parameter in this function represents the elasticity of marginal costs with respect to output. Since changes in output incur an adjustment cost as well as changing the total production cost, this elasticity is a useful way of describing the burden of the adjustment cost.

The theoretical and empirical strands of the asymmetric pricing literature are not well integrated, and a potential way to draw these together is by treating firm interactions as dynamic processes. Explicitly modelling price adjustment paths allows theoretical predictions and empirical results to be compared in the same terms, even if formal tests to detect causes of asymmetry are still some way off. Although there are obvious areas for refinement of the model presented, it seems that the approach used offers much potential for insights into price transmission.

## 7 Appendix

The next subsection derives a two-equation differential equation system from the control problem in equation (6), using Pontryagin's maximum principle (see Leonard and van Long (1992)). Because this system is not analytically solvable, subsection 7.2 constructs a linear approximation to the true equations. Subsection 7.3 discusses the stability properties of this linearised system, and a closed form solution is found.

### 7.1 Derivation of equation system (7)

The optimal control  $x_i$ , given the maximisation problem (6), must satisfy Pontryagin's maximum principle. The current value Hamiltonian is:

$$H(x_i, q_i, \phi_i, t) = p(Q) \cdot q_i - C(q_i) - A(x_i, q_i) + \phi_i x_i \quad (30)$$

where  $\phi_i$  is the co-state variable, and the maximum condition and adjoint equation are:

$$\frac{\partial H}{\partial x_i} = 0 \Rightarrow A_{x_i} = \phi_i \quad (31)$$

and:

$$\begin{aligned} \dot{\phi}_i &= r\phi_i - \frac{\partial H}{\partial q_i} \\ &= r\phi_i - p\left(1 - \frac{\theta}{\eta}\right) + c_i + A_{q_i} \end{aligned} \quad (32)$$

respectively. The latter equation uses the fact that the conjectured derivative of  $p(Q)$  with respect to  $q_i$ , given the conjecture  $\theta = \frac{dQ}{dq_i} \frac{q_i}{Q}$ , is:

$$\frac{dp}{dq_i} = \frac{dp}{dQ} \frac{dQ}{dq_i} = \frac{dp}{dQ} \frac{Q}{p} \frac{dQ}{dq_i} \frac{q_i}{Q} \frac{p}{q_i} = -\frac{\theta}{\eta} \frac{p}{q_i} \quad (33)$$

where  $\eta$  is the absolute value of the elasticity of demand ( $\eta = \left| \frac{dQ}{dp} \frac{p}{Q} \right|$ ).

To obtain a differential equation system in  $x_i$  and  $q_i$ , the maximum condition is differentiated with respect to  $t$ , giving:

$$A_{x_i q_i} \cdot \dot{q}_i + A_{x_i x_i} \cdot \dot{x}_i = \dot{\phi}_i \quad (34)$$

When this expression and the original maximum condition are substituted for  $\dot{\phi}_i$  and  $\phi_i$  in the adjoint equation, it becomes:

$$A_{x_i q_i} \cdot \dot{q}_i + A_{x_i x_i} \cdot \dot{x}_i = rA_{x_i} - p\left(1 - \frac{\theta}{\eta}\right) + c_i + A_{q_i} \quad (35)$$

which can be rearranged to:

$$\dot{x}_i = \frac{1}{A_{x_i x_i}} \left( rA_{x_i} + A_{q_i} - A_{x_i q_i} \cdot x_i - p\left(1 - \frac{\theta}{\eta}\right) + c_i \right) \quad (36)$$

This equation, and the law of motion of the state ( $\dot{q}_i = x_i$ ) form the system (7).

## 7.2 Linearisation of equations (7)

A linear approximation of (7) around the steady state given by  $x_i^* = 0$  and equation (8) is given by the first order Taylor series expansion:

$$\begin{aligned} \dot{x}_i &\approx g_{x_i}^1(x_i^*, q_i^*) (x_i - x_i^*) + g_{q_i}^1(x_i^*, q_i^*) (q_i - q_i^*) \\ \dot{q}_i &\approx g_{x_i}^2(x_i^*, q_i^*) (x_i - x_i^*) + g_{q_i}^2(x_i^*, q_i^*) (q_i - q_i^*) \end{aligned} \quad (37)$$

where:

$$\begin{aligned} g^1(x_i, q_i) &= \frac{1}{A_{x_i x_i}} \left( rA_{x_i} + A_{q_i} - A_{x_i q_i} \cdot x_i - p\left(1 - \frac{\theta}{\eta}\right) + c_i \right) \\ g^2(x_i, q_i) &= x_i \end{aligned} \quad (38)$$

The first partial derivative of  $g^1$  is:

$$\begin{aligned} g_{x_i}^1(x_i, q_i) &= -\frac{A_{x_i x_i x_i}}{A_{x_i x_i}^2} \left( rA_{x_i} + A_{q_i} - A_{x_i q_i} \cdot x_i - p\left(1 - \frac{\theta}{\eta}\right) + c_i \right) \\ &\quad + \frac{1}{A_{x_i x_i}} (rA_{x_i x_i} + A_{q_i x_i} - A_{x_i q_i x_i} \cdot x_i - A_{x_i q_i}) \end{aligned} \quad (39)$$

Evaluated at steady state, the first bracketed term disappears, since this term is just the steady state condition. In the second bracketed term,  $A_{q_i x_i}$  and  $A_{x_i q_i}$  cancel each other, so at the steady state ( $x_i^* = 0$ ) this term reduces to  $rA_{x_i x_i}^*$  and the whole expression becomes:

$$g_{x_i}^1(x_i^*, q_i^*) = r \quad (40)$$

The second partial derivative of  $g^1$  is given below. Since the demand curve may not have constant elasticity, firms take this into account in their optimisation. The change in elasticity along the demand curve is reflected in the term  $\omega = \frac{d\eta}{dp} \frac{p}{\eta}$ , and the value of  $\omega$  depends on the form of the demand curve ( $\omega = 0$  for a constant-elasticity demand curve,  $\omega = 1$  for a semi-logarithmic demand, and linear demand implies  $\omega = 1 + \eta$ ).

$$g_{q_i}^1(x_i, q_i) = -\frac{A_{x_i x_i q_i}}{A_{x_i x_i}^2} \left( r A_{x_i} + A_{q_i} - A_{x_i q_i} \cdot x_i - p \left( 1 - \frac{\theta}{\eta} \right) + c_i \right) + \frac{1}{A_{x_i x_i}} \left( r A_{x_i q_i} + A_{q_i q_i} - A_{x_i q_i q_i} \cdot x_i - \frac{dp}{dq_i} \left( 1 - \frac{\theta}{\eta} \right) - p \frac{\theta}{\eta^2} \frac{d\eta}{dp} \frac{dp}{dq_i} \right) \quad (41)$$

Substituting  $\frac{dp}{dq_i} = -\frac{\theta}{\eta} \frac{p}{q_i}$  and  $\omega = \frac{d\eta}{dp} \frac{p}{\eta}$ , the second bracketed term becomes:

$$\begin{aligned} & r A_{x_i q_i} + A_{q_i q_i} - A_{x_i q_i q_i} \cdot x_i + \frac{\theta}{\eta} \frac{p}{q_i} \left( 1 - \frac{\theta}{\eta} \right) + p \frac{\theta}{\eta^2} \frac{d\eta}{dp} \frac{\theta}{\eta} \frac{p}{q_i} \\ &= r A_{x_i q_i} + A_{q_i q_i} - A_{x_i q_i q_i} \cdot x_i + \frac{p}{q_i} \left( \frac{\theta(\eta - \theta) + \theta^2 \omega}{\eta^2} \right) \end{aligned} \quad (42)$$

Evaluated at the steady state the first bracketed term disappears as before, and substituting  $p = \left( \frac{\eta}{\eta - \theta} \right) c_i$  (the steady state expression) the partial derivative becomes:

$$\begin{aligned} g_{q_i}^1(x_i^*, q_i^*) &= \frac{1}{A_{x_i x_i}^*} \left( r A_{x_i q_i}^* + A_{q_i q_i}^* + \frac{c_i}{q_i} \left( \frac{\eta}{\eta - \theta} \frac{\theta(\eta - \theta) + \theta^2 \omega}{\eta^2} \right) \right) \\ &= \frac{1}{A_{x_i x_i}^*} \left( r A_{x_i q_i}^* + A_{q_i q_i}^* + \frac{c_i}{q_i^*} \frac{\theta}{\eta} (1 + \mu) \right) \end{aligned} \quad (43)$$

where  $\mu = \frac{\omega \theta}{\eta - \theta}$ .

The partial derivatives of  $g^2$  are  $g_{x_i}^2(x_i, q_i) = 1$  and  $g_{q_i}^2(x_i, q_i) = 0$ .

### 7.3 Derivation of equation (9)

The linearised equation system is thus:

$$\begin{pmatrix} \dot{x}_i \\ \dot{q}_i \end{pmatrix} = \begin{bmatrix} r & K \\ 1 & 0 \end{bmatrix} \begin{pmatrix} x_i - x_i^* \\ q_i - q_i^* \end{pmatrix} \quad (44)$$

where  $K = \frac{1}{A_{x_i x_i}^*} \left( r A_{x_i q_i}^* + A_{q_i q_i}^* + \frac{c_i}{q_i^*} \frac{\theta}{\eta} (1 + \mu) \right)$ .

The determinant of the matrix is  $-K$ , but without an explicit formula for the adjustment cost the stability properties of this system are difficult to establish. Proceeding under the assumption that the partial derivatives  $A_{x_i q_i}$  and  $A_{q_i q_i}$  take zero value at the steady state output level,<sup>20</sup> it is clear that the stability properties are entirely determined by the term  $\frac{1}{A_{x_i x_i}^*} \frac{c_i}{q_i^*} \frac{\theta}{\eta} (1 + \mu)$ . Therefore as long as  $A$  is convex at the steady state ( $A_{x_i x_i}^* > 0$ ), the sign of  $1 + \mu$  determines whether the system is saddlepath stable or globally unstable. A sufficient condition for  $1 + \mu$  to be positive and the determinant to be negative, implying saddlepath stability, is that  $\eta > \theta$ . Even when this does not hold, it is necessary that the elasticity of demand  $\eta$  fall within the range  $\theta(1 - \omega) < \eta < \theta$  for  $1 + \mu$  to be negative and the system to be unstable. Instability thus requires, firstly, that demand be highly inelastic or the market be very concentrated (the right hand side inequality). The left hand side inequality says that even in such situations a certain curvature of the demand curve is required – it must be flatter to some extent than a constant-elasticity demand curve ( $\omega > \frac{\theta - \eta}{\theta}$  must hold). It should be remembered also that for any adjustment cost function where  $A_{x_i q_i}^* = A_{q_i q_i}^* = 0$  fails, the stable range of parameters will be different to that calculated here.

Eigenvalues of the matrix in (44) are given by the solutions to the characteristic equation:

$$c(\lambda) = -\lambda(r - \lambda) - K \quad (45)$$

which are:

$$\lambda_1 = \frac{1}{2} \left( r + \sqrt{r^2 + 4K} \right) > 0 \quad (46)$$

$$\lambda_2 = \frac{1}{2} \left( r - \sqrt{r^2 + 4K} \right) < 0 \quad (47)$$

and which have the signs indicated, so long as saddlepath stability holds.

Using an eigenvector representation of the closed-form solution:

$$\begin{pmatrix} q_i(t) - q_i^* \\ x_i(t) \end{pmatrix} = \begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix} \begin{pmatrix} A_1 e^{\lambda_1 t} \\ A_2 e^{\lambda_2 t} \end{pmatrix} \quad (48)$$

---

<sup>20</sup>This assumption is satisfied by the adjustment cost function introduced in section (4), as is the requirement for convexity at the steady state ( $A_{x_i x_i}^* > 0$ ).

the output timepath for firm  $i$  is:

$$q_i(t) = q_i^* + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \quad (49)$$

The requirement of a convergent timepath in a saddlepath stable system dictates that the coefficient on the positive eigenvalue ( $A_1$ ) be set to 0, and the initial value of the firm's output  $q_i(0) = q_{i0}$  determines the other coefficient to be  $A_2 = q_{i0} - q_i^*$ . The convergent solution is thus:

$$q_i(t) = q_i^* + (q_{i0} - q_i^*) e^{\lambda_2 t} \quad (50)$$

or, in terms of percentage changes from the initial output value:

$$\frac{q_i(t) - q_{i0}}{q_{i0}} = \left( \frac{q_i^* - q_{i0}}{q_{i0}} \right) (1 - e^{\lambda_2 t}) \quad (51)$$

In log notation the proportional change in firm  $i$ 's output from its initial value at time  $t$ ,  $d \ln q_i(t)$ , is a function of the change in steady state output  $d \ln q_i^*$  and the stable eigenvalue,  $\lambda_2$ :

$$d \ln q_i(t) = d \ln q_i^* (1 - e^{\lambda_2 t}) \quad (52)$$

## References

- [1] Azzam, A.M. (1999), "Asymmetry and rigidity in farm-retail price transmission", *American Journal of Agricultural Economics* 81(3): 525-533
- [2] Ball, L.M., and Mankiw, N.G. (1994), "Asymmetric price adjustment and economic fluctuations", *The Economic Journal* 104(424): 247-261
- [3] Borenstein, S., Cameron, A.C., and Gilbert, R. (1997), "Do gasoline prices respond asymmetrically to crude oil price changes?", *Quarterly Journal of Economics* 112(1): 305-339
- [4] Dockner, E.J. (1992), "A dynamic theory of conjectural variations", *Journal of Industrial Economics* 40(4): 377-395
- [5] Dockner, E.J., Jørgenson, S., van Long, N., and Sorger, G. (2000), *Differential games in Economics and Management Science*, Cambridge: Cambridge University Press
- [6] Driskill, R.A., and McCafferty, S. (1989), "Dynamic duopoly with adjustment costs: A differential game approach", *Journal of Economic Theory* 49(2): 324-338
- [7] Figuières, C., Jean-Marie, A., Quérou, N., and Tidball, M. (2004), *Theory of conjectural variations*, Singapore: World Scientific
- [8] Léonard, D., van Long, N. (1992), *Optimal control theory and static optimization in economics*, Cambridge: Cambridge University Press
- [9] McCorriston, S., Morgan, C.W., and Rayner, A.J. (1998), "Processing technology, market power and price transmission", *Journal of Agricultural Economics* 49(2): 185-201
- [10] Meyer, J., and von Cramon-Taubadel, S. (2004), "Asymmetric price transmission: a survey", *Journal of Agricultural Economics* 55(3): 581-611



- [11] Miller, D.J., and Hayenga, M.L. (2001), "Price cycles and asymmetric price transmission in the U.S. pork market", *American Journal of Agricultural Economics* 83(3): 551-562
- [12] Morriset, J. (1998), "Unfair trade? The increasing gap between world and domestic prices in commodity markets during the past 25 years", *World Bank Economic Review* 12(3): 503-526
- [13] Peltzman, S. (2000), "Prices rise faster than they fall", *Journal of Political Economy* 108(3): 466-502
- [14] Piggott, R.R. (1992), "Some old truths revisited", *Australian Journal of Agricultural Economics* 36(2): 117-140
- [15] Reagan, P.B., and Weitzman, M.L. (1982), "Asymmetries in price and quantity adjustments by the competitive firm", *Journal of Economic Theory* 27(2): 410-420
- [16] Ward, R.W. (1982), "Asymmetry in retail, wholesale and shipping point pricing for fresh vegetables", *American Journal of Agricultural Economics* 62(2): 205-212