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When is Seller Price Setting with Linear Fees Optimal for Intermediaries?

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When is Seller Price Setting with Linear Fees Optimal for Intermediaries?*

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Abstract

Mechanisms where sellers set the price and are charged a linear commission fee are widely used by real world intermediaries, e.g. by real estate brokers. Empirically these commission fees exhibit very little variance, both across heterogeneous regional markets and over time. So far, there is no theoretical explanation why such seller price setting mechanisms are used and why the linear fees vary so little. In this paper, we first show that in a Bayesian setup seller price setting with linear fees is revenue equivalent to the intermediary optimal direct mechanism derived by Myerson and Satterthwaite (1983) if and only if the seller's cost is drawn from a generalized power distribution. Whenever such a mechanism is optimal, the fee structure is independent of the distribution from which the buyer's valuation is drawn. Second, we derive the intermediary optimal direct mechanism when there are many buyers and possibly many sellers and we show that with one seller any standard auction with linear fees and reserve price setting by the seller (which are used e.g. by eBay) implements this mechanism if the seller's cost is drawn from a power distribution and if buyers' valuations are identically distributed. Third, we show that when the number of buyers approaches infinity while there is still one seller, seller price setting and price setting by the intermediary are equivalent, intermediary optimal mechanisms.

Keywords: Brokers, linear commission fees, optimal indirect mechanisms.

JEL-Classification: C72, C78, L13

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1 Introduction

In many industries, intermediaries do not buy or sell the goods they trade but rather let the seller set the price and commission a flat percentage fee levied on this price when the object is sold. Examples of industries where such "fee setting mechanisms" are used include real estate brokers, retailers, and art galleries. Empirical research documents the widespread use and the remarkably small variance of percentage fees in real estate brokerage both over time and across different regional markets.[1] Both facts – their linear structure and their small variance – are considered puzzling (see e.g. Hsieh and Moretti, 2003; Levitt and Syverson, 2005). Despite the widespread use and the economic significance of fee setting mechanisms[2] and despite the recent upsurge of interest in intermediation, there exists essentially no theoretical literature on whether and when these mechanisms (with or without linear fees) are desirable from the perspective of the intermediary.[3]

In this paper, we make a first step towards a better understanding of these mechanisms, thereby providing at least a partial solution to what is perceived as puzzling. We assume a Bayesian setup with a monopolistic profit maximizing intermediary designing an exchange mechanism for one buyer and one seller, both having private information about their valuation for the good. This allows us to build on Myerson and Satterthwaite's (1983) results on direct mechanisms that are optimal for the intermediary and to obtain the following. First, we characterize the general mechanism with seller price setting that is optimal for the intermediary. Second, we derive a necessary and sufficient condition on the distribution of the seller's valuation for seller price setting with linear fees to be optimal for the intermediary. Third, whenever seller price setting with linear fees is optimal, the optimal fee is independent of the distribution of the buyer's valuation. Fourth, under only slightly more restrictive assumptions about the inverse hazard rates, we show that

[1]See e.g. Hsieh and Moretti (2003), Levitt and Syverson (2005), Rutherford, Springer, and Yavas (2005) and Hendel, Nevo, and Ortala-Magné (2007).

[2]In 2005 real estate brokerage commissions in the U.S. exceeded \$60 billion (DOJ, 2007).

[3]For the theoretical literature, see e.g. Spulber (1999), Rust and Hall (2003), Caillaud and Jullien (2003) and Rochet and Tirole (2006). The fact that (to the best of our knowledge) no name for this type of mechanism exists only goes to show how little theoretical interest these mechanisms have received. Two papers that provide explanations of when intermediaries may use percentage fees and when they set prices are Hagiu (2006) and Yavas (1992). Hagiu's argument relies on the presence and nature of network externalities, while Yavas' explanation depends on the presence and working of search markets.

setting an ask and a bid price is never optimal for the intermediary.[4] Fifth, we extend the setup to many buyers and possibly many sellers and we characterize the intermediary optimal direct mechanism. For the case with one seller we show that an auction with linear fees and reserve price setting by the seller (which are used e.g. by eBay) implements this mechanism if the seller's valuation is drawn from a power distribution and if buyers' valuations are identically distributed. Last, when the number of buyers approaches infinity while there is still one seller, seller price setting with fees and bid-ask-price setting by the intermediary are equivalent, intermediary optimal mechanisms.

The remainder of this paper is structured as follows. Section 2 lays out the setup. In Section 3, we derive the main results. Section 4 discusses price setting by the intermediary and intermediary optimal mechanisms with many buyers and sellers, and Section 5 concludes.

2 Setup

There is one seller who owns one indivisible good of known quality and one buyer who may want to buy it. The buyer has private information about his valuation of the good v which is drawn from the distribution F with strictly positive density f on the support $[\underline{v}, \bar{v}]$. For brevity, we refer to the seller's valuation of the good, or his opportunity cost of selling the good, as his cost.[5] The seller has private information about his cost c, which is drawn from G with strictly positive g on $[\underline{c}, \bar{c}]$. Let us further define the buyer's virtual valuation function $\Phi(v) := v - (1 - F(v))/f(v)$ and by analogy the seller's *virtual cost function* $\Gamma(c) := c + G(c)/g(c)$. Throughout we make the standard assumption that Φ and Γ are increasing.

The seller and the buyer can only trade through a monopolistic intermediary who has all the bargaining power and can hence choose the trade mechanism. F and G are

[4] This stands in contrast to the standard double marginalization result which says that letting both the upstream monopolist (seller) and the downstream monopolist (intermediary) set a mark-up is always less efficient than letting one entity set all prices (ask and bid price setting in our case). Our results hinge on the private information of the seller about his own valuation for the good (or equivalently, his production costs).

[5] This makes clear that the model also applies to settings where the good has to be produced by the seller at a cost.

common knowledge. All agents are risk neutral, the buyer's utility is $v-p$ and the seller's $p-c$ in case of trade at price p.

We focus on mechanisms that maximize the intermediary's expected profits, and for brevity we call such mechanisms intermediary optimal mechanisms. This allows us to use the results of Myerson and Satterthwaite (1983, Section 5, Theorems 3 and 4) on intermediation.[6] For the sake of expositional clarity, we recapitulate the results of Myerson and Satterthwaite (1983) that are relevant to our analysis:

Lemma 1. *An incentive compatible, interim individually rational mechanism is intermediary optimal if and only if it has the following properties:*

(i) the good is transferred iff $\Phi(v) \geq \Gamma(c)$*,*

(ii) the seller with the highest cost $\bar{c}$ *and the buyer with the lowest valuation* $\underline{v}$ *both have zero expected utility.*

We will present a generalization of this result to multiple buyers and sellers in Lemma 2 below.

3 Optimality of Fee Setting Mechanisms

We confine our attention to indirect mechanisms with the properties that the seller sets the price, the intermediary charges a fee that only depends on the price set by the seller, and the buyer can only decide to reject or accept the price offered. We call mechanisms with these properties "fee setting mechanisms".

Denote the price at which the good is offered to the buyer as $P(c)$, the payment received by the seller as $\kappa(P(c))$ where κ is the "net price function" and $P(c) - \kappa(P(c))$ the fee charged by the intermediary. For simplicity, we maintain the following assumption throughout the rest of the paper:

Assumption 1. $\Phi(\bar{v}) \leq \Gamma(\bar{c})$ *and* $\Phi(\underline{v}) \leq \Gamma(\underline{c})$*.*

[6]Myerson and Satterthwaite (1983) are almost exclusively cited for their impossibility results. A notable exception is Spulber (1999, Ch.7). However, he merely compares the optimal direct mechanism of Myerson and Satterthwaite with price setting by the intermediary, which is never intermediary optimal for a finite number of buyers, as shown in Proposition 4 below.

This assumption ensures that a mechanism satisfying (i) of Lemma 1 also satisfies (ii). Dropping this assumption would make the equations unnecessarily complicated without adding any substantial insights.[7]

We first describe the general optimal fee setting mechanism in Proposition 1.

Proposition 1. *The following fee setting mechanism is intermediary optimal in the class of all incentive compatible, interim individually rational mechanisms. The seller with cost c sets the price $P(c)$, where*

$$P(c) = \Phi^{-1}(\Gamma(c)) \tag{1}$$

for $c \in [\underline{c}, \Gamma^{-1}(\bar{v})]$ and an arbitrary price no less than $\bar{v}$ else. Upon successful sale, the seller gets $\kappa(P(c))$, where

$$\kappa(P(c)) = c + \int_{c}^{P^{-1}(\bar{v})} \frac{1 - F(P(t))}{1 - F(P(c))} dt \tag{2}$$

for $P(c) \leq \bar{v} \Leftrightarrow c \leq P^{-1}(\bar{v})$ and κ arbitrary for $P(c) > \bar{v}$.

Proof. Note that an indirect mechanism that lets the seller set the price corresponds to a direct mechanism with the properties that there are no payments if the good is not exchanged and payments in case of exchange can only be conditioned on the seller's report but not on the buyer's. Therefore, by the revelation principle we can focus our attention to direct mechanisms with these properties. Denote the probability that the good is exchanged depending on reported cost c and reported valuation v as $Q(c, v)$. For the class of mechanisms we consider it is clearly a weakly dominant strategy for the buyer to accept whenever the price is less than or equal to his valuation. Therefore, the seller's expected probability of exchange is $q(c) := E_v[Q(c, v)] = 1 - F(P(c))$ and consequently, trade occurs iff the buyer accepts the offer, i.e. iff $v \geq P(c)$. Combining this with the optimality condition (i) of Lemma 1 and the monotone increasingness of Φ, we get that for an optimal mechanism trade occurs iff $\Phi(v) \geq \Phi(P(c)) = \Gamma(c)$. This gives us (1). Because of Assumption 1 property (i) of Lemma 1 implies (ii).

[7] $\Phi(\underline{v}) > \Gamma(\underline{c})$ is not difficult to accommodate for. A seller would never set a price less than $\underline{v}$ and therefore (ii) holds. For $\Phi(\bar{v}) > \Gamma(\bar{c})$ the intermediary should be able to solve the problem by imposing a price cap: the seller is not allowed to set a price satisfying $\Phi(p) > \Gamma(\bar{c})$. Therefore, the highest cost seller would get zero profits.

Denote a truthfully reporting seller's utility as $U(c) := (\kappa(P(c)) - c)q(c)$. By standard arguments (see e.g. the argument leading up to equation (4) in the proof of Theorem 1 in Myerson and Satterthwaite (1983)) incentive compatibility implies

$$U(c) = U(\bar{c}) + \int_c^{\bar{c}} q(t)dt. \tag{3}$$

We already know that the highest cost seller is not going to sell and hence $U(\bar{c}) = 0$. This is also true for other sellers with sufficiently high cost, namely $P(c) \geq \bar{v}$ or $c \geq P^{-1}(\bar{v}) := \Gamma^{-1}(\Phi(\bar{v}))$. Therefore, the upper limit of the integral can be written as $P^{-1}(\bar{v})$. Equating (3) with $U(c) = (\kappa(P(c)) - c)q(c)$ from its definition and rearranging yields

$$\kappa(P(c)) = c + \int_c^{P^{-1}(\bar{v})} \frac{q(t)}{q(c)} dt, \tag{4}$$

which is equivalent to (2). □

Proposition 1 means that optimality can be achieved even if one does not use information about the buyer's valuation when determining payments in case of trade. Next, we show that setting linear fees is optimal for the intermediary if and only if the seller's cost is drawn from a generalized power distribution. In this case the parameters of the linear fee are fully determined by the distribution of c.

Proposition 2. *The following two statements are equivalent:*

(i) linear fee setting is intermediary optimal, i.e. the net price function can be written as

$$\kappa(p) = \mu p + \lambda, \tag{5}$$

(ii) the seller's cost is drawn from a generalized power distribution G of the form

$$G(c) = \left(\frac{c - \underline{c}}{\bar{c} - \underline{c}}\right)^{\beta} \quad \text{with} \quad \beta > 0, \tag{6}$$

where $\mu = \beta/(\beta + 1)$ and $\lambda = \underline{c}/(1 + \beta)$.

Proof. By the same standard arguments leading to (3) we also get $U'(c) = -q(c)$ almost everywhere because of incentive compatibility. Equating this with the derivative obtained from the definition $U'(c) = [(\kappa(P(c)) - c)q(c)]'$ and rearranging yields

$$\Phi(P(c)) = P(c) - \frac{\kappa(P(c)) - c}{\kappa'(P(c))}. \tag{7}$$

(i) implies (ii) Take $\kappa(p) = \mu p + \lambda$. Then the right hand side of (7) becomes $(c - \lambda)/\mu$. Equating this with $\Gamma(c)$ in order to achieve optimality according to Lemma 1 (i) gives the differential equation $g(c) = G(c)\mu/((1-\mu)c - \lambda)$. With the condition $G(\underline{c}) = 0$ one obtains (6) with $\beta = \mu/(1-\mu)$ and $\underline{c} = \lambda/(1-\mu)$. The upper bound of the support $\bar{c}$ remains arbitrary.

(ii) implies (i) Observe that with the distribution G specified in (6) one has $\Gamma^{-1}(p) = \tilde{\mu}p + \tilde{\lambda}$ with $\tilde{\mu} := \beta/(\beta+1)$ and $\tilde{\lambda} := \underline{c}/(\beta+1)$ and, therefore, $P^{-1}(p) = \Gamma^{-1}(\Phi(p)) = \tilde{\mu}\Phi(p) + \tilde{\lambda}$. Take (7) and replace $P(c)$ with p, c with $P^{-1}(p)$, and Φ by its definition. Rearranging leads to

$$(1 - F(p))(\kappa'(p) - \tilde{\mu}) - f(p)(\kappa(p) - (\tilde{\mu}p + \tilde{\lambda})) = 0. \tag{8}$$

Defining $w(p) := \kappa(p) - (\tilde{\mu}p + \tilde{\lambda})$ equation (8) leads to $[w(p)(1 - F(p))]' = 0$. From (2) it follows that $\kappa(p)$ is not singular at $p = \bar{v}$ (actually $\kappa(\bar{v}) = P^{-1}(\bar{v})$). Since $1 - F(\bar{v}) = 0$ it follows that $w(p) \equiv 0$, i.e. (5) is satisfied with $\mu = \tilde{\mu}$ and $\lambda = \tilde{\lambda}$. $\square$

As the parameters of an optimal linear fee are fully determined by the distribution of the seller's cost G, Corollary 1 follows directly from Proposition 2.

Corollary 1 (Invariance of Linear Fees)**.** *If a linear fee is intermediary optimal for some distributions (G, F), then it will also be optimal for $(G, \tilde{F})$, where $\tilde{F}$ is an arbitrary distribution with an increasing virtual valuation $\tilde{\Phi}$.*

It can also be shown that the reverse implication – in some sense – of Corollary 1 also holds.

Proposition 3. *If a fee $\kappa(p)$ is optimal for a given G and for arbitrary F, then the fee has to be linear and G has to be a generalized power distribution.*

Proof. The optimality condition (i) of Lemma 1 implies $\Phi(P(c)) = \Gamma(c)$. If we want optimality to hold for arbitrary distributions F, and hence for arbitrary functions $P(c)$, equating the right hand side of (7) and $\Gamma(c)$ yields $\Gamma(c) = p - (\kappa(p) - c)/\kappa'(p)$ for arbitrary p. This differential equation in κ has the solution

$$\kappa(p) = \mu(p - \Gamma(c)) + c \tag{9}$$

defined up to a constant μ. If we want this to hold for any c we need $c - \mu\Gamma(c) = \lambda$ for some constant λ, and hence $\Gamma(c) = (c - \lambda)/\mu$. Substituting this back to (9) results in $\kappa(p) = \mu p + \lambda$, i.e. a linear fee. This also implies a generalized power distribution G by Proposition 2. $\square$

As optimality of linear fees implies invariance of the fees with respect to the buyer's distribution, the empirical prediction of Proposition 2 is that whenever profit maximizing intermediaries choose linear fee setting as a mechanism, these fees will be invariant. Clearly, this prediction is consistent with available empirical evidence.

Note also that the upper part of the seller's cost distribution $[P^{-1}(\bar{v}), \bar{c}]$, i.e. those sellers who for sure cannot sell, is irrelevant for the intermediation problem at hand. Therefore, Proposition 2 means that a linear fee only implies a generalized power distribution in the relevant range $[\underline{c}, P^{-1}(\bar{v})]$. Above this range, G can have any shape, provided its virtual cost function is increasing. Corollary 1 and the empirical prediction thus hold not only when the cost distribution is the same over time and across regions, but even if it has only the same shape in the relevant range.

The following interpretation can be given for the different percentage fees observed in different industries. A percentage fee punishes the seller for raising the price (i.e. reporting a higher cost in the corresponding direct mechanism described in Proposition 1) and serves therefore as an incentive for truthful reporting. A low percentage fee implies a high parameter β of the cost distribution, less uncertainty about the seller's cost, and hence less need to incentivize him. Thus a real estate agent – charging a fee of 6% – can be interpreted as having less uncertainty about the seller's own valuation for his house than an auctioneer such as Sotheby's – charging around 20% – about the seller's own valuation for say a painting.[8]

[8]The analogy between the seller setting the transaction price in case of one buyer and the seller setting the reserve price of an auction in case of many buyers is shown in subsection 4.2.

To illustrate our results, let $G(c) = c^\beta$ for $c \in [0,1]$ with $\beta > 0$ and let F be an arbitrary distribution with support $[0,1]$ and increasing Φ, and assume $\kappa(p) = \mu p$. Then the expected profit of the seller with cost c when setting price p is $(\mu p - c)(1 - F(p))$ and the maximizer is given by the first order condition $\Phi(p(c,\mu)) = c/\mu$. Hence, the intermediary's expected profit when using a linear fee setting mechanism with $\kappa(p) = \mu p$ is $\int_0^\mu (1-\mu)P(c,\mu)(1-F(P(c,\mu)))g(c)dc$, where the upper limit of the integral stems from the fact that a seller with $c > \mu$ can never sell profitably. Observing that $P(c,\mu) = P(t)$ with $t = c/\mu$ and substituting variables, this expected profit is equal to $\int_0^1 (1-\mu)P(t)(1-F(P(t)))g(\mu t)\mu dt = \mu^\beta(1-\mu)\int_0^1 P(t)(1-F(P(t)))\beta t^{\beta-1}dt$, where the integral is positive and independent of μ and the equality follows because $g(\mu t) = \beta(\mu t)^{\beta-1}$. Thus, the optimal μ is $\mu = 1/(1+\beta) \Leftrightarrow \beta = \mu/(1+\mu)$. For β equal to one, G is a uniform distribution and the optimal fee is $p - \kappa(p) = p/2$. If F is uniform on $[0,1]$ as well, the seller with cost c sets the price $P(c) = 1/2 + c$ and the intermediary's expected profit is $1/2\int_0^{1/2}(1/2+c)(1/2-c)dc = 1/24$. This is, of course, the same as in the example provided after Theorem 4 in Myerson and Satterthwaite (1983).[9]

It is worth mentioning that analogous results can be obtained for mechanisms where the buyer sets the price and the fee is conditioned on this price. It is for instance optimal for the intermediary to let the buyer set the price and charge a linear fee $\kappa_B(p) - p = (\mu_B - 1)p + \lambda_B$ if and only if the buyer's valuation is distributed with $F(v) = 1 - [(v - \underline{v})/(\bar{v} - \underline{v})]^{\beta_B}$ with $\mu_B = -\beta_B/(1+\beta_B)$ and $\lambda_B = \bar{v}/(1+\beta_B)$.

4 Extensions

4.1 Intermediary Price Setting

Consider now an alternative mechanism, called *price setting*, which is widely used e.g. by stock market and used car dealers: the intermediary sets an ask (or buyer) price p^B and

[9]Observe the following analogy. The double auction described by Chatterjee and Samuelson (1983) satisfies the social optimality condition stated in Myerson and Satterthwaite (1983, Theorem 2) for uniform F and G. The fee setting mechanism described here satisfies the intermediary optimality conditions for a power distribution G and arbitrary F. Observe further that it can never be socially optimal to let the seller (or the buyer) set the price, as can be shown easily with Theorem 2 in Myerson and Satterthwaite (1983).

a bid (or seller) price p^S; if both seller and buyer are willing to trade at these prices, then the intermediary earns the bid-ask spread. Otherwise, there is no trade. The following proposition characterizes the optimal bid and ask price and shows that under fairly weak assumptions price setting is never optimal for the intermediary.

Proposition 4. *Assume the following about the inverse hazard rates of the distributions: $(1 - F(v))/f(v)$ is decreasing and $G(c)/g(c)$ is increasing. Then the optimal ask price p^B and bid price p^S are given by the equations $p^B = \Gamma(p^S)$ and $p^S = \Phi(p^B)$. Further, intermediary price setting is never optimal for the intermediary.*

Proof. The intermediary's expected profit with price setting is $(p^B - p^S)(1 - F(p^B))G(p^S)$. The assumptions about the inverse hazard rates ensure concavity of the profit function. Therefore, the unique maximum is given by the first order conditions. Taking derivatives with respect to p^B and p^S yields $p^S = \Phi(p^B)$ and $p^B = \Gamma(p^S)$. We complete the proof by showing that trade with price setting neither implies nor is implied by trade in the Myerson-Satterthwaite mechanism for arbitrary distributions F and G.

Trade with intermediary price setting, no trade with the Myerson-Satterthwaite optimal mechanism. Take a buyer and a seller for whom trade just occurs with price setting, i.e. valuation p^B and cost p^S. We know that a profit maximizing intermediary will always set $p^B > p^S$. Combining this with the first order conditions we get $\Phi(p^B) = p^S < p^B = \Gamma(p^S)$. This implies by Lemma 1 (i) that no trade occurs with the optimal mechanism for valuation p^B and cost p^S.

Trade with the Myerson-Satterthwaite optimal mechanism, no trade with intermediary price setting. Take the lowest cost seller with cost $\underline{c}$ and a buyer with valuation v' such that trade just occurs with the optimal mechanism, i.e. $\Phi(v') = \Gamma(\underline{c})$. As $p^S > \underline{c}$ must hold for positive probabilities of trade with price setting, we have $\Phi(v') = \Gamma(\underline{c}) = \underline{c} < p^S = \Phi(p^B)$. This implies $v' < p^B$ and hence no trade with price setting. $\square$

Proposition 4 implies that for generalized power distributions of the seller's cost, fee setting with a linear fee is strictly better for the intermediary than price setting.[10] This

[10] Actually, if we allow for arbitrary fee functions, fee setting is strictly better than price setting for arbitrary G.

result hinges on the assumption that the intermediary does not know the seller's cost c. If the intermediary knows c because, say, the intermediary owns the good (i.e. the seller and the intermediary are a vertically integrated firm), then price setting will be optimal for the intermediary.[11] Not surprisingly, the profit of the vertically integrated seller-intermediary will exceed the joint profits of the stand alone seller and stand alone intermediary. This is broadly consistent with the empirical findings of Levitt and Syverson (2005) and Rutherford, Springer, and Yavas (2005), who show that houses owned by real estate brokers are sold more profitably than houses where the intermediary is not the owner.

However, because the fee setting mechanism that is optimal for the non-integrated intermediary imposes an upward distortion in the seller's effective cost, the price set by the non-integrated seller should be larger than the ask price p^B set by the integrated intermediary-seller. Accordingly, the welfare of the buyer with an integrated intermediary-seller should be larger than when the seller and the intermediary are independent. The prediction of lower prices under vertical integration contrasts with the empirical findings of Levitt and Syverson (2005) and Rutherford, Springer, and Yavas (2005) and reinforces their argument that there is an additional agency problem in real estate brokerage.

4.2 Many Buyers

Our results easily generalize to setups with more than one buyer if we restrict ourselves to a certain class of mechanisms. Define "non-discriminating mechanisms" as mechanisms that cannot distinguish between buyers. Note that both fee setting and price setting as described above fall in this category. An intermediary restricted to non-discriminating mechanisms basically treats all buyers as one "representative buyer" whose valuation is drawn from a distribution F, F being the distribution of the highest order statistic of the buyers.[12] In such a setup all of our results hold. This implies in particular that whenever

[11] If the intermediary and the seller are independent agents, then $p^S = c$ and the seller's profit is zero. Whether the intermediary and the seller are vertically integrated or not, the optimal ask price will be $p^B = \Phi^{-1}(c)$. That price setting is optimal with one buyer follows from the theory of optimal selling mechanisms; see e.g. Myerson (1981).

[12] One can interpret $F(p)$ as the probability that no buyer is willing to buy at price p.

linear fees are an optimal non-discriminating mechanism, they are independent of the distributions of the buyers' valuations and, therefore, also of the number of buyers.

However, we can say more than that for intermediary optimal mechanisms with many buyers. The first result relates to optimal mechanisms when there are finitely many buyers and one seller. As a preliminary, we first derive the intermediary optimal mechanism with many buyers and possibly many sellers. Since this is a generalization of the Myerson-Satterthwaite results on intermediary optimal mechanisms summarized in Lemma 1 above, it is of some interest on its own.[13]

Let N_B and N_S, respectively, be the number of buyers and sellers, whose valuations v_b and costs c_s are independent draws from the distribution F_b with density f_b and support $[\underline{v}_b, \overline{v}_b]$ and the distribution G_s with density g_s and support $[\underline{c}_s, \overline{c}_s]$. As before, we assume that the virtual valuations $\Phi_b(v_b)$ and the virtual costs $\Gamma_s(c_s)$ are strictly increasing and we use b (s) exclusively to indicate a buyer (seller). Order and relabel the realized virtual valuations in decreasing and virtual costs in increasing order, i.e. $\Phi_1 > \Phi_2 > .. > \Phi_{N_B}$ and $\Gamma_1 < \Gamma_2 < .. < \Gamma_{N_S}$. Let K be the integer such that $\Phi_K \geq \Gamma_K$ and $\Phi_{K+1} < \Gamma_{K+1}$ provided such a $K < \min\{N_B, N_S\}$ exists. Otherwise, let $K = \min\{N_B, N_S\}$. This quantity is naturally called Quasi-Walrasian and we call an allocation rule Quasi-Walrasian if all buyers and sellers with $b, s \leq K$ trade and all others do not.

Lemma 2. *The intermediary optimal mechanism that respects individual rationality and incentive compatibility of buyers and sellers has a Quasi-Walrasian allocation rule and gives zero expected utility to buyers with $v_b = \underline{v}_b$ and sellers with $c_s = \overline{c}_s$.*

The proof is in the appendix. For $N_B = N_S = 1$ the Quasi-Walrasian allocation rule reduces to the intermediary optimal allocation rule of Myerson and Satterthwaite (1983). With many buyers and one seller, the intermediary optimal allocation rule requires the good to go to the buyer with the largest virtual valuation, provided this virtual valuation exceeds the seller's virtual cost.

We now assume that there is one seller (i.e. $N_S = 1$) and that the $N_B > 1$ buyers'

[13]See also Baliga and Vohra (2003).

valuations are independently drawn from the identical distribution F with support [0,1].

Proposition 5. *If the seller's cost c is drawn from the distribution $G(c) = c^\beta$ for $c \in [0,1]$, then any standard auction with a reserve price and a linear fee $\kappa(p) = p\beta/(\beta+1)$ levied on the final sale price p is an intermediary optimal mechanism.*

Proof. We first prove the statement for a second price auction and then invoke the payoff equivalence theorem (see e.g. Myerson, 1981, Lemma 3) to prove it for arbitrary standard auctions. So consider a second price auction where the seller faces a fee $\kappa(p) = (1-\mu)p$ and has cost c and sets a reserve price r. This seller's expected profit is

$$(1-\mu)N_B\left\{r(1-F(r))F(r)^{N_B-1} + \int_r^1 y(1-F(y))(N_B-1)F(y)^{N_B-2}f(y)dy\right\} + cF(r)^{N_B},$$

which follows from Krishna (2002, p.25) by multiplying the brackets with $(1-\mu)$ and adding the expected value $cF(r)^{N_B}$ of not selling the good. Note that the good is sold to the buyer with the largest virtual valuation, provided this is larger than the reserve r. The optimal reserve price $r(c,\mu)$ satisfies $\Phi(r(c,\mu)) = c/(1-\mu)$. Observe that the effective cost $c/(1-\mu)$ on the right hand side becomes equal to $\Gamma(c) = c(\beta+1)/\beta$ by choosing $1/(1-\mu) = (\beta+1)/\beta$. Thus, the mechanism implements the intermediary optimal allocation rule if c is drawn from a power distribution. The proof is completed for the second price auction by observing that a buyer with $v_b = 0$ and a seller with $c = 1$ both net zero expected profits.

It follows from Lemma 3 in Myerson (1981) that all standard auction formats will have the same expected revenue and indeed the same reserve price.[14] This completes the proof. □

The second result concerns the equivalence between, and optimality of, price setting by the intermediary and linear fee setting when the number of buyers N_B approaches infinity while the number of sellers N_S is kept fixed at one. For simplicity, assume that buyers' valuations are identically and, as before, independently distributed according to the distribution F on $[\underline{v}, \overline{v}]$. The seller's cost is distributed according to G on $[\underline{c}, \overline{c}]$ with

[14]See also Milgrom (2004, Ch.3) and Jehle and Reny (1999, Th.9.9 and Ex.9.20).

$\underline{c} < \overline{v}$ and as in Proposition 4, we assume that $G(c)/g(c)$ is increasing. Moreover, as in the rest of the paper, we assume that F and G satisfy Assumption 1.

Proposition 6. *As N_B approaches infinity, intermediary price setting and fee setting are equivalent. Moreover, if F has an increasing virtual valuation, both mechanisms are intermediary optimal.*

Proof. Observe that with $N_B \to \infty$ the optimal (ask) price, be it set by a seller or the intermediary, is $\overline{v}$. Therefore, a price setting intermediary's maximization problem reduces to the choice of p^S such that $(\overline{v} - p^S)G(p^S)$ is maximized, the unique solution to which is $\overline{v} - p^{S*}_{\infty} = G(p^{S*}_{\infty})/g(p^{S*}_{\infty})$, where uniqueness follows from the assumption that G/g is increasing. Observe that another way of expressing p^{S*}_{∞} is $p^{S*}_{\infty} = \Gamma^{-1}(\overline{v})$. On the other hand, when using fee setting, the fee $\kappa(p)$ is just the constant $\kappa(\overline{v})$ because all types of sellers who sell set the price equal to $\overline{v}$. Thus, under fee setting, the intermediary's problem is to choose a constant $\kappa(\overline{v})$ that maximizes $(\overline{v} - \kappa(\overline{v}))G(\kappa(\overline{v}))$, whose solution κ^*_{∞} is, obviously identical to p^{S*}_{∞}.

To see that the second phrase is true, observe that for $N_B > 1$ and $N_S = 1$ the intermediary's expected profit under the optimal mechanism (derived in the proof of Lemma 2) can be written as

$$\int_{X\setminus s} \int_{\underline{c}}^{\Gamma^{-1}(\max\{\max_b[\Phi(v_b)],\underline{c}\})} \left\{ \max_b \Phi(v_b) - \Gamma(c) \right\} g(c)dcf(\boldsymbol{v})d\boldsymbol{v},$$

where X denotes the product set containing all $(\boldsymbol{v}, \boldsymbol{c})$ and $X \setminus s$ denotes the product set containing all valuations $\boldsymbol{v}$. As N_B approaches infinity, $\max_b\{\Phi(v_b)\} = \Phi(\overline{v}) = \overline{v}$ with probability one. Therefore, the intermediary's expected profit becomes

$$\int_{\underline{c}}^{\Gamma^{-1}(\overline{v})} [\overline{v} - \Gamma(c)]g(c)dc = \overline{v}G(\Gamma^{-1}(\overline{v})) - \int_{\underline{c}}^{\Gamma^{-1}(\overline{v})} \Gamma(c)g(c)dc.$$

Notice that $\int_{\underline{c}}^{\Gamma^{-1}(\overline{v})} \Gamma(c)g(c)dc = \int_{\underline{c}}^{\Gamma^{-1}(\overline{v})} [cg(c) + G(c)]dc$, which after integrating by parts becomes $\Gamma^{-1}(\overline{v})G(\Gamma^{-1}(\overline{v}))$. Therefore, the intermediary's expected profit when using the optimal direct mechanism is $[\overline{v} - \Gamma^{-1}(\overline{v})]G(\Gamma^{-1}(\overline{v}))$, which is the same as the profit when the intermediary sets either prices or fees. This completes the proof. $\square$

Note that for an infinite number of buyers and fee setting any κ satisfying $\kappa(\bar{v}) = \Gamma^{-1}(\bar{v})$ and incentive compatibility is profit maximizing for the intermediary. For instance the intermediary could just as well charge a fixed fee $\bar{v} - \Gamma^{-1}(\bar{v})$.

5 Conclusions

The widespread use of linear fee setting mechanisms is empirically well documented and so is the fact that these fees exhibit very little variance across different regional markets (Hsieh and Moretti, 2003; Hendel, Nevo, and Ortala-Magné, 2007). In the present paper, we have shown that linear fee setting is optimal for the intermediary, i.e. satisfies the intermediary optimality conditions of Myerson and Satterthwaite (1983), if and only if the seller's cost is drawn from a generalized power distribution. Whenever such a mechanism is optimal, the fee structure is also independent of the distribution of buyers' valuations. Moreover, we have generalized the intermediary optimal mechanism of Myerson and Satterthwaite (1983) to a setup with many buyers and possibly many sellers. We have further shown that any standard auction format with a reserve price set by the seller and a linear fee is intermediary optimal if the seller's cost is drawn from a power distribution under the assumption that the buyers' valuations are identically and independently distributed.

While focussing on direct mechanisms is, of course, extremely fruitful for theoretical research, it renders difficult recognizing when and where such direct mechanisms are used in practice. Further research on real world mechanisms is likely to uncover other instances of applications of well-known direct mechanisms.

Appendix

A Proof of Lemma 2

Proof. A direct mechanism asks buyers and sellers to report their valuations and costs. Denoting by $(\boldsymbol{v}, \boldsymbol{c})$ a collection of such reports with $\boldsymbol{v} = (v_1, .., v_{N_B})$ and $\boldsymbol{c} = (c_1, .., c_{N_S})$, the direct mechanism is then characterized by the probability $Q_b(\boldsymbol{v}, \boldsymbol{c})$ that b gets a unit of the good and $Q_s(\boldsymbol{c}, \boldsymbol{v})$ that s produces a unit of the good for $b = 1, .., N_B$ and

$s = 1, .., N_S$ and by the payments $M_b(\boldsymbol{v}, \boldsymbol{c})$ it asks from buyers and the payments $M_s(\boldsymbol{c}, \boldsymbol{v})$ it makes to sellers. Clearly, a mechanism is only feasible if for all $(\boldsymbol{v}, \boldsymbol{c})$, $\sum_{b=1}^{N_B} Q_b(\boldsymbol{v}, \boldsymbol{c}) \leq \sum_{s=1}^{N_S} Q_s(\boldsymbol{c}, \boldsymbol{v})$. Let $\boldsymbol{Q}$ be the collection of these probabilities. We refer to $\boldsymbol{Q}$ as the allocation rule of the mechanism.

We only sketch the proof, a fully detailed version of which is available upon request. Lengthy, though completely standard arguments (see e.g. Krishna, 2002) can be applied to show that a revenue (or payoff) equivalence theorem holds. Formally, $m_b(v_b) = m_b(\underline{v}_b) + q_b(v_b)v_b - \int_{\underline{v}_b}^{v_b} q_b(t)dt$ and $m_s(c_s) = m_s(\underline{c}_s) + q_s(c_s)c_s - \int_{c_s}^{\bar{c}_s} q_s(t)dt$ for all c, v, lower case functions standing for expectations about all others' valuations and costs (e.g. $m_b(v_b) := E_{\boldsymbol{v}_{-b},\boldsymbol{c}}[M_b(\boldsymbol{v}, \boldsymbol{c})]$). Again, by standard arguments, this implies $E[m_b(v_b)] = m_b(\underline{v}_b) + E[\Phi_b(v_b)q_b(v_b)]$ and $E[m_s(c_s)] = m_s(\bar{c}_s) + E[\Gamma_s(c_s)q_s(c_s)]$. A profit maximizing intermediary will make the individual rationality constraint just binding, therefore, his expected profit $\sum_{b=1}^{N_B} E[m_b(v_b)] - \sum_{s=1}^{N_S} E[m_s(c_s)]$ is

$$\int_X \left\{ \sum_{b=1}^{N_B} \Phi_b(v_b) Q_b(\boldsymbol{v}, \boldsymbol{c}) - \sum_{s=1}^{N_S} \Gamma_s(c_s) Q_s(\boldsymbol{c}, \boldsymbol{v}) \right\} f(\boldsymbol{v}) g(\boldsymbol{c}) d\boldsymbol{v} d\boldsymbol{c}, \tag{10}$$

where $f(\boldsymbol{v})$ and $g(\boldsymbol{c})$ are the joint densities of all buyers and sellers, respectively, and X is the product set containing all $(\boldsymbol{v}, \boldsymbol{c})$. Inspection of the term in curly brackets reveals that the profit can be maximized point by point by implementing the Quasi-Walrasian allocation for each realization $(\boldsymbol{v}, \boldsymbol{c})$. □

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