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Abstract

Data for measuring poverty and income inequality are frequently available in a summary form that describes the proportion of income or expenditure for each of a number of population proportions. While various discrete measures can be applied directly to data in this limited form, these discrete measures typically ignore inequality within each group. This problem can be overcome by fitting a parametric income distribution to the grouped data and computing required quantities from the estimated parameters of this distribution. In this paper we show how to calculate several poverty measures from parameters of the generalized beta distribution of the second kind, and its popular special cases. An analysis of poverty changes in ten countries from South and Southeast Asia is used to illustrate the methodology.

I Introduction

Despite recent success in reducing the incidence of poverty in some parts of the world, particularly in East and South Asia, the extent and persistence of poverty in many other regions, such as Africa, continue to be of global concern. Eradication of global poverty has been at the centre of millennium goals and a universal commitment to human rights protection. Given this prominent international commitment to eliminate global poverty, measuring the incidence of poverty and how it has changed over time is of vital importance[1]. Although it is crucially necessary, measuring poverty globally is a challenging task. Among the challenges are various definitions of poverty, scarcity of data, issues related to converting country specific data to a common currency, and finding suitable poverty lines. Poverty arguably has many dimensions including monetary (measured by Chen and Ravallion, 2004; 2007; 2010 for example), health, social, environmental and cultural (Dreze and Sen, 1989; Wisor, 2012). Methods of poverty calculation therefore vary according to ones definition of poverty. In this paper we examine the calculation of poverty in monetary terms, using consumption data which is generally used as a welfare indicator for poor countries (Ravallion and Chen, 2009), and as a basis for measuring deprivation.

To assess global poverty, internationally comparable data is required; its collection is a mammoth task. The World Bank has a large ongoing research project documented on its Povcal website[2] where data, compiled from household surveys for numerous countries and several years, are provided in the form of population proportions and income or expenditure proportions. The Bank uses these grouped data to estimate Lorenz curves which are in turn used to calculate poverty incidence for designated countries and years. The Lorenz curves

[1] Overviews of the importance of measuring poverty and the World Bank's findings on the extent of poverty and how it has changed over time can be found in Kakwani (1999), Ravallion (2010) and Chen and Ravallion (2004, 2007, 2010).

[2] http://iresearch.worldbank.org/PovcalNet/jsp/index.jsp

estimated by the World Bank are the general quadratic (Villasenor and Arnold, 1989) and the beta Lorenz curve (Kakwani, 1980).

An alternative to computing values of poverty measures based on estimated Lorenz curves is to compute them directly from estimated distributions of income or expenditure. The same income[3] and population proportions that are used to estimate a Lorenz curve can be used to estimate a probability distribution without first estimating a Lorenz curve. Estimating an income distribution has the advantage that it can be used to compute a variety of characteristics of that distribution, including the Lorenz curve, but, conversely, it is not always possible to retrieve an underlying income distribution and its characteristics from a Lorenz curve. Moreover, Lorenz curves derived from income distributions will always satisfy the necessary properties of a Lorenz curve, whereas some functional forms for Lorenz curves require the parameter restrictions to satisfy these properties. Both approaches – the direct estimation of Lorenz curves and the direct estimation of income distributions – have been widely used in the literature for measuring income inequality and other characteristics of income distributions (Chotikapanich, 2008), but the same degree of attention does not seem to have been paid to deriving poverty measures from estimated distributions. To partially fill this gap, in this paper we describe how to compute a number of poverty measures from the generalized beta distribution of the second kind (GB2). This distribution is a popular and flexible one for modeling income distributions. Parker (1999) shows how it arises from a neoclassical model of optimal firm behaviour under uncertainty. It nests many popular income distributions as special or limiting cases, including the beta-2 (B2), Singh-Maddala, and Dagum distributions. Its characteristics and its relationship within a hierarchy of distributions are described by, for example, Mcdonald (1984), McDonald and Xu (1995), and

[3] Henceforth we refer to "income" proportions and "income" distributions with the understanding that in many cases data will be in the form of expenditure proportions which are used to estimate expenditure distributions.

Kleiber and Kotz (2003). Chotikapanich *et al* (2007) show how method-of-moments estimates of the parameters of one of its special cases – the beta-2 (B2) distribution –can be obtained from income and population proportions. Their work is extended in Chotikapanich *et al* (2012) where B2 distributions are estimated for 91 countries, and where methodology for estimating the GB2 distribution and two of its other special cases, the Dagum and Singh-Maddala distributions, is described. This methodology was refined by Harjagasht *et al*. (2012) who develop an optimal GMM estimator for any income distribution estimated from population and income proportions. McDonald and Ransom (2008) and Jenkins (2009) derived expressions for various inequality measures in terms of the parameters of the GB2 distribution. We extend this work by expressing poverty measures in terms of the parameters of the GB2 distribution so that they can be computed from an estimated GB2 income distribution or one of its special cases.

Commonly used poverty measures are all expressed relative to some poverty line. Mathematically, they are given by the expectation of a function that describes the intensity of poverty suffered by those whose income is below the pre-defined poverty line, with the expectation taken with respect to the income distribution. Poverty measurement is, therefore, sensitive to how the poverty line is defined. There have been two main principles used to select a poverty line. One leads to an absolute poverty line which is intended to measure the cost of certain basic needs and to have a constant real value. The other is a relative concept, not designed to represent physiological minima, but typically expressed as a fraction of a central measure, such as the mean or median. Absolute poverty lines are commonly used in developing countries whereas relative poverty lines are popular in advanced economies. In our illustrative examples, we examine the sensitivity of poverty measures to a range of poverty lines and compare outcomes across countries and over years.

The article is organized as follows. In the next section we give details of the generalized beta distribution followed by a comprehensive discussion of how various poverty measures can be expressed in terms of the parameters of that distribution. In most cases two types of expressions are given: one in terms of beta moments and distribution functions and one in terms of decompositions into intuitive appealing components. In Section III we show how poverty measures for a region can be computed from those for its component countries. Data and estimation for some illustrative examples involving countries from South and Southeast Asia are discussed in Section IV. Results from estimating the poverty measures and an analysis of poverty changes in South and Southeast Asia are given in Section V. Concluding remarks complete the paper in Section VI.

II The GB2 Distribution and Poverty Measures

(i) The GB2 Distribution and Some General Results

The probability density function (pdf) for the generalized beta distribution of the second kind (GB2) is given by[4]

$$f(y) = \frac{ay^{ap-1}}{b^{ap}B(p,q)\left(1+\left(\frac{y}{b}\right)^{a}\right)^{p+q}} \qquad y > 0 \tag{1}$$

where $a > 0,\ b > 0,\ p > 0$ and $q > 0$ are its parameters and $B(p,q)=\int_0^1 t^{p-1}(1-t)^{q-1}dt$ is the beta function. The corresponding cumulative distribution function (cdf) is given by

$$F(y) = \frac{1}{B(p,q)}\int_0^w t^{p-1}(1-t)^{q-1}dt = B_w\left(p,q\right) \tag{2}$$

where $w=\left(y/b\right)^a\Big/\left[1+\left(y/b\right)^a\right]$. The function $B_w(p,q)$ is the cdf for the normalized beta distribution defined on the (0,1) interval. It is a convenient representation because it is

[4] See Kleiber and Kotz (2003, Ch.6) for details. Important special cases of the GB2 are the B2 distribution where $a=1$, the Singh-Maddala distribution where $p=1$ and the Dagum distribution where $q=1$.

commonly included as a readily-computed function in statistical software. The poverty-measure expressions that we present are written in terms of this function. Also required are the first and second moments of $f(y)$. The k-th moment exists if $-ap < k < aq$. It is given by

$$\mu^{(k)} = E\left(y^k\right) = \int_0^\infty y^k f\left(y\right)dy = \frac{b^k B\left(p+k/a, q-k/a\right)}{B\left(p,q\right)} \tag{3}$$

A poverty index measures the intensity of poverty suffered by those whose income is below a pre-defined poverty line. If the function $D(z,y)$ describes the level of deprivation suffered by an individual whose income y is less than the poverty line z, then most poverty measures (and those that we consider) can be written as the average deprivation suffered by the poor. That is,

$$P = E_y\left[D(z,y)I(y<z)\right] = \int_0^z D\left(z,y\right) f(y)dy \tag{4}$$

where $I(y<z)$ is an indicator function equal to 1 when $y<z$, and 0 otherwise. Alternative poverty measures correspond to alternative specifications for the function $D(z,y)$. Before turning to these alternative specifications, we provide some preliminary results useful for evaluating P when $f(y)$ is the GB2 distribution or one of its special cases. The first of these results are the mean and the variance of income for the poor. For their calculation, we note that the k-th incomplete moment for the GB2 distribution is given by

$$\mu_z^{(k)} = E\left(y^k \mid y<z\right) = \frac{\int_0^z y^k f\left(y\right)dy}{F\left(z\right)} = \frac{\mu^{(k)} B_v\left(p+k/a,\, q-k/a\right)}{B_v\left(p,q\right)} \tag{5}$$

where $v = \left(z/b\right)^a \Big/ \left[1+\left(z/b\right)^a\right]$. Mean income for the poor is given by $\mu_z = \mu_z^{(1)}$, and variance of income of the poor is given by $\sigma_z^2 = \mu_z^{(2)} - \mu_z^2$.

Another component of many poverty measures is the aggregate income gap ratio defined as $g_z = (z - \mu_z)/z$. It represents the amount of money one must give to each of the poor to bring them up to poverty line, expressed relative to the poverty line. Poverty is greater the larger the difference between the poverty line and the mean income of the poor.

(ii) Poverty Measures

The most common poverty measure is the **head-count ratio**. It measures the proportion of population in poverty. By definition it is written as $H_z = F(z)$ or by setting $D(z, y) = 1$ in (4). Based on (2), the head-count ratio can be expressed in terms of the GB2 parameters as:

$$H_z = F(z) = B_v(p, q) \tag{6}$$

Setting $D(z, y) = 1$ means that we give equal weight to each individual whose income is below the poverty line no matter whether s/he is just below or far below the poverty line. As a result, the head-count ratio measures the incidence of poverty, but does not take into account other information such as the degree of poverty or the intensity of deprivation experienced by the poor. It provides useful information in its own right and, as will be seen later, it is also a component of other measures.

The other poverty measures that we consider are the poverty gap, and the Foster-Greer-Thorbecke, Atkinson, Watts, and Sen indices. In what follows we define each of these indices and then provide expressions that can be used to calculate them for the GB2 distribution and its special cases. Where possible, two alternative expressions are provided. In the first the poverty index can be computed directly from beta cdfs, the moments in (3), and the partial moments given in (5). In the second, we follow Kakwani (1999), and write each poverty measure as a function of the head count ratio H_z, the aggregate income gap ratio g_z, and a measure of inequality of the poor. In this latter case a poverty measure can be computed indirectly by first computing H_z, g_z, and the relevant inequality measure. The first approach is more direct, but the second approach has some intuitive appeal. For current and future reference, the various expressions are summarized in Table 1 for the GB2, B2, Singh-Maddala and Dagum distributions.

A third way to compute or estimate a poverty measure is to use simulated draws from the estimated distribution. Since a poverty measure is the expectation of a deprivation function with the expectation taken with respect to the income density, it can be estimated using a sample average of the deprivation function. Thus, given estimates of the parameters of the GB2 distribution, one way to compute (estimate) a poverty measure is to generate a large number of observations $y_1, y_2, \ldots, y_M$ from the GB2 distribution, compute a value of the deprivation function for each observation, and then average those values. That is,

$$\hat{P} = \frac{1}{M}\sum_{i=1}^{M} D(z, y_i) I(y_i < z) \tag{7}$$

Where possible, we prefer to derive exact expressions that are readily computable by software, but, in those instances where derivations are not possible, we can rely on a sample average from generated observations. Also, for instances where derivations of exact expressions are lengthy, computing the corresponding sample average provides a check on the validity of the derivations.

To obtain a measure that accommodates not just the proportion of poor, but also the magnitude of poverty among those defined as poor, a number of alternatives to the head-count ratio have been suggested. The **poverty gap ratio** PG_z is the simplest extension, obtained by setting $D(z, y)$ equal to the income shortfall $\left((z-y)/z\right)$. In this case we can show that

$$PG_z = \int_0^z \left(\frac{z-y}{z} \right) f(y)dy = H_z g_z$$

$$= B_v\left(p,q\right) - \left(\mu/z\right) B_v\left(p+1/a,\, q-1/a\right) \qquad (8)$$

The first expression $PG_z = H_z g_z$ shows how the poverty gap depends on the head count ratio and the aggregate income gap ratio; the second shows how it can be computed from beta cdf's.

A generalization of the poverty gap is the ***FGT* measure** suggested by Foster, Greer and Thorbecke (1984). In this case $D\left(z,y\right) = \left[\left(z-y\right)/z\right]^{\alpha}$ and the measure is defined as

$$FGT_z(\alpha) = \int_0^z \left(\frac{z-y}{z} \right)^{\alpha} f(y)dy \qquad \text{for } \alpha \geq 1 \qquad (9)$$

where α measures the inequality aversion. The larger the value of α the more emphasis is given to the lower end of the income distribution. When $\alpha = 1$, $FGT_z(1) = PG_z$. For $\alpha = 2$, a case commonly used in practice, more progress towards an exact expression can be made. In this case, we can show

$$FGT_z(2) = \int_0^z \left(\frac{z-y}{z} \right)^{2} f(y)dy$$

$$\begin{aligned} &= B_v\left(p,q\right) - \left(2\mu/z\right) B_v\left(p+1/a,\, q-1/a\right) \\ &\qquad + \left(\mu^{(2)}/z^2\right) B_v\left(p+2/a,\, q-2/a\right) \end{aligned} \qquad (10)$$

$$= H_z \left[g_z^2 + (1-g_z)^2 \frac{\sigma_z^2}{\mu_z^2} \right]$$

The middle line in (10) shows how $FGT_z(2)$ can be computed directly from beta moments and cdf's. The last line shows how it depends on the head count ratio, the aggregate income gap ratio and the mean and variance of the poor.

For integer values of α greater than 2, $\left((z-y)/z\right)^{\alpha}$ can be expanded and expressed in terms of further incomplete moments. For example, when $\alpha = 3$, $FGT_z(3)$ depends on $\mu_z^{(3)}$, $\mu_z^{(2)}$ and μ_z. For non-integer values of α, we suggest proceeding in one of three possible ways. (i) Use a series expansion of $(1-y/z)^{\alpha}$. See, for example, Jeffrey (2000, p.68). (ii) Estimate via simulation as depicted in equation (7). (iii) Use a Taylor-series expansion as an approximation. For example, a Taylor series expansion around the point $z/2$, up to the quadratic term, is

$$\left(\frac{z-y}{z}\right)^{\alpha} \approx \left(\frac{1}{2}\right)^{\alpha} - \frac{\alpha}{z}\left(\frac{1}{2}\right)^{\alpha-1}\left(y-\frac{z}{2}\right) + \frac{\alpha(\alpha-1)}{2z^2}\left(\frac{1}{2}\right)^{\alpha-2}\left(y-\frac{z}{2}\right)^2 \tag{11}$$

The integral of this expression up between 0 and *z* can be expressed in terms of μ_z and $\mu_z^{(2)}$.

Another index that takes into account the magnitude of the poverty suffered by the poor is the **Atkinson index** (Atkinson, 1987) defined by setting,

$$D(z,y) = \frac{1}{e}\left[1-\left(\frac{y}{z}\right)^{e}\right] \tag{12}$$

where $0 < e \le 1$ is the degree of inequality aversion. This poverty index can be written as

$$\begin{aligned} A_z(e) &= \frac{1}{e}\int_0^z \left[1-\left(\frac{y}{z}\right)^{e}\right] f(y)dy \\ &= \frac{1}{e}\left[B_v\left(p,q\right) - \frac{\mu^{(e)}}{z^e} B_v\left(p+\frac{e}{a},\, q-\frac{e}{a}\right)\right] \qquad (13) \\ &= \frac{H_z}{e}\left[1-(1-g_z)^e \frac{\mu_z^{(e)}}{\mu_z^e}\right] \end{aligned}$$

The middle line of (13) expresses the index in terms of beta cdf's and the *e*-th moment for the poor; the last line shows how it can be written in terms of the head-count ratio, the aggregate income gap ratio, and the *e*-th moment for the poor relative to the mean of the poor raised to the *e*-th power. When $e=1$, $A_z(1)=PG_z$.

Considering the income shortfall in log format and setting $D(z,y)=(\ln(z)-\ln(y))$, leads to the **Watts index** (Watts, 1968) defined as

$$W_z=\int_0^z(\ln(z)-\ln(y))\,f(y)dy \tag{14}$$

Zheng (1993) shows that this index satisfies a number of axiomatic conditions. It can be viewed as a special case of Atkinson measure as $e\to 0$. Using a result in the Appendix, it can be written as

$$W_z=\ln\left(\frac{z}{b}\right)B_v(p,q)-\frac{1}{a}\left\{D_pB_v(p,q)-D_pB_v(p,q)+B_v(p,q)\left[\psi(p)-\psi(q)\right]\right\} \tag{15}$$

where $D_pB_v(p,q)$ and $D_qB_v(p,q)$ are the derivatives of the beta cdf $B_v(p,q)$ with respect to p and q, respectively, and $\psi(p)=\partial\ln(\Gamma(p))/\partial p$ is the digamma function. We have expressed W_z in terms of these functions because they are readily computed using EViews software.[5] When convenient software is not available, one can resort to the simulation approach given by equation (7). A closely related measure given by $K_z^*=1-\exp(-W_z)$ has been suggested by Kakwani (1999).

Another popular poverty measure is the **Sen index** (Sen, 1976) where the poverty gap is weighted by a person's rank in the ordering of the poor. This index is given by

$$S_z=2\int_0^z\left(\frac{z-y}{z}\right)\left(\frac{F(z)-F(y)}{F(z)}\right)f(y)dy$$

$$=H_z\left(g_z+(1-g_z)G_z\right) \tag{16}$$

[5] Specifically, the commands are given by $D_pB_v(p,q)=@\text{betaincder}(v,p,q,2)$, $D_qB_v(p,q)=@\text{betaincder}(v,p,q,3)$ and $\psi(p)=@\text{digamma}(p)$.

where G_z is the Gini coefficient for the poor given by

$$G_z = -1 + \frac{2}{\mu_z F^2(z)} \int_0^z yF(y)f(y)dy \tag{17}$$

The last line in (16) shows how the index can be written in terms of the head count ratio, the aggregate income gap ratio and the inequality of the poor measures using G_z. Expressing S_z in terms of the parameters of the beta distribution is more difficult than it was for the other indices. In (16) we can use $H_z = B_v(p,q)$ and $g_z = 1 - \mu_z/z$, but evaluation of G_z is more troublesome. If we follow the simulation approach, and draw *M* observations y_i from $f(y)$, it can be estimated using

$$\hat{G}_z = -1 + \frac{2}{\mu_z F^2(z)} \frac{1}{M} \sum_{i=1}^{M} \left[y_i B_{w_i}(p,q) I(y_i \le z) \right] \tag{18}$$

where $w_i = (y_i/b)^a \Big/ \left[1 + (y_i/b)^a\right]$. More progress can be made analytically for the special case where $a = 1$ (the B2 distribution); for this case it is shown in the Appendix that

$$G_z = -1 + \frac{\mu}{\mu_z F^2(z)} \left[G \times B_{z/(b+z)}(2p+1,\, 2q-2) + B^2_{z/(b+z)}(p+1,\, q-1) \right] \tag{19}$$

where $G = 2B(2p, 2q-1)\big/ pB^2(p,q)$ is the Gini coefficient for the whole population.

III Computing Regional or Global Poverty

Once income distributions have been estimated for a number of countries in a given region and their corresponding poverty measures have been calculated, poverty assessment for the region is likely to be of interest. If $f_i(y)$ denotes the income density for the *i*-th country and λ_i is the population share for the *i*-th country in the region, then the income density for the region is the mixture $f_R(y) = \sum_{i=1}^{n} \lambda_i f_i(y)$. If a poverty measure's deprivation function is such that $D_i(z,y)=D_R(z,y)$, where D_i and D_R are deprivation functions for the i-th country and the region, respectively, then the poverty measure is additively decomposable in the sense that $P_R=\sum_{i=1}^{n}\lambda_i P_i$ where P_i and P_R are the poverty indices for i-th country and the region (Kakwani, 1999). To see this result, note that

$$\begin{aligned} P_R &= \int_0^z D_R(z,y)\, f_R(y)dy = \int_0^z D_R(z,y)\sum_{i=1}^{n}\lambda_i f_i(y)dy \\ &= \sum_{i=1}^{n}\lambda_i \int_0^z D_R(z,y)\, f_i(y)dy = \sum_{i=1}^{n}\lambda_i \int_0^z D_i(z,y)\, f_i(y)dy \\ &= \sum_{i=1}^{n}\lambda_i P_i \end{aligned}$$

All poverty measure considered above have this property with the exception of the Sen index where $D_i(z,y)\neq D_R(z,y)$.

IV Data and Estimation

To illustrate computation of the poverty measures we used grouped data from the World Bank Povcal web site[6] on several countries from South Asia and Southeast Asia for years as close as possible to 1992, 2000, 2005 and 2010. The data are available in the form of population shares and corresponding expenditure shares for a number of expenditure classes[7], together with the reported mean monthly expenditure from surveys converted using a 2005 purchasing-power-parity exchange rate. The B2 special case of the GB2 distribution was estimated for all countries and all years using the methodology in Chotikapanich et al. (2007). The countries and their corresponding parameter estimates are given in Table 2. India, China and Indonesia are broken into rural and urban. Because of its relative importance, China is listed separately; the other countries appear in groupings for South Asia or Southeast Asia. Standard errors of the estimates are not reported because the sample sizes used to create the

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[6] The latest version of the data was downloaded on 24 July 2012 at http://iresearch.worldbank.org/PovcalNet/index.htm
[7] Sometimes income shares are reported.

grouped data are not available. Also included are mean monthly expenditure and the Gini coefficients. Two estimates of the Gini coefficients are given, one from the Lorenz curves estimated by the World Bank and one from the expression for the B2 distribution, namely $G = 2B(2p, 2q-1) \big/ pB^2(p,q)$. In general, the two sets of Gini estimates are very similar. A check of the mean expenditures implied by the B2 distribution estimates, $\mu = bp/(q-1)$, also yields mean expenditures close to those used by the World Bank.

V Results for Poverty Measures

We present results from our poverty calculations in two different formats. In Table 3 we report the results for all poverty indices for all countries in all years using a poverty line of $1.25/day ($38/month), a value proposed by the World Bank to measure extreme poverty[8]. We use $\alpha = 2$ for the FGT measure and $e = 0.5$ for the Atkinson measure. For consistency, the South Asia and Southeast Asia results in Table 3 are reported for the years 1992, 2000, 2005 and 2010, although the actual years (given in Table 2) may differ slightly from these benchmarks. Because there has been debate about the use of absolute versus relative poverty lines (Osberg and Xu, 2008), and some argue that absolute poverty lines should be updated over the time to reflect changes in standards of living and expectations (Madden, 2000), in Figures 1 and 2 we use graphs to examine the sensitivity of the head count and poverty gap ratios to changes in the poverty line up to $3.20/day ($97/month). These graphs are presented for a selection of countries with the highest populations. Figure 1 contains rural and urban India and China. Figure 2 displays combined rural and urban for Indonesia, and combined Southeast Asia with Indonesia excluded, all calculated using the result in Section III.

From Table 3 we can compare the degree of poverty in different countries, observe how poverty incidence has changed over time, and examine whether relative poverty

[8] See Ravallion *et al.* (2009) and Ravallion (2010) for a detailed explanation of how the $1.25 poverty line was set and for discussion about alternative poverty lines for different countries.

assessments are robust to choice of poverty index. Although the magnitudes of the various poverty indices vary considerably, reflecting their different definitions, poverty comparisons over time and countries are generally not sensitive to choice of index. The following observations can be made from any one of them:

- In 2010, poverty was greatest in Bangladesh, rural India and urban India; it was least in urban China (2008), Thailand and Malaysia. Nearly 20 years earlier in 1992, rural China, Bangladesh, Pakistan and Vietnam were the poorest countries. Malaysia, Thailand and urban China had the least poverty.
- Countries which have made the greatest progress towards eliminating poverty, and the periods in which the major poverty reductions took place, are rural China (1992-2005), rural and urban Indonesia (2000-2010), Vietnam (1992-2006), urban China (1992-2008), and Pakistan (1992-2005). India (rural and urban) and Bangladesh have made some progress, but the incidence of poverty still remains extremely high.
- Comparing the combined figures for South Asia and Southeast Asia shows that Southeast Asia has less poverty, and has been more successful in reducing poverty, than South Asia. China has been the most successful.

In Figure 1 we contrast the level of poverty and the changes that have occurred in the poverty level in rural and urban India from 1994 to 2010 to those in rural and urban China from 1992 to 2008, for poverty lines from $1.25/day to $3.20/day. Both the poverty gap and head count ratios show the enormous progress in poverty reduction that has been made in rural China relative to rural India. The graph for rural China has moved from completely above both of those for rural India to completely below them. However, despite making substantial progress, rural China still has more than 70% of people whose daily expenditure is less than $3.20/day. The relative changes are similarly dramatic for urban India and China, although, in contrast to the rural areas, urban China in 1992 was better off than urban India in 2010, when measured by the poverty gap ratio or the head count ratio for poverty lines less than $80/month. A steep line in Figure 1 suggests there are large numbers of poor between $1.25/day and $3.20/day, whereas a flat line suggests small numbers within this range. In this respect, China urban is better off than the other regions.

The upper two diagrams in Figure 2 contain similar graphs for South Asia, Southeast Asia and China as a whole. China is referred to as CHINA10, in line with the other regions, although the year is actually 2008. The location of the lines for China again emphasizes their large advances in reducing poverty. Also evident are the substantial improvements in Southeast Asia relative to the more modest improvements in South Asia. Changes in Southeast Asia are investigated further in the lower two diagrams where Indonesia has been separated from the remaining countries. Indonesia is the most populous country in the region and has reduced poverty considerably. This has led to a relatively large drop in the curves for Indonesia and a lower drop in the curves for the remainder of the region.

The observations made from Table 3 and Figures 1 and 2 are reinforced by examining the estimated expenditure densities graphed in Figures 3 and 4. Comparing the top two diagrams in Figure 3, we see that rural China's expenditure density shifted substantially from 1992 to 2005, and slightly after that, whereas the shift in rural India's density has only been minor. Similarly, the bottom two diagrams show that urban China's density has shifted dramatically compared to urban India's density. A comparison of rural and urban China (the two left diagrams) shows, as expected, that urban China is considerably wealthier than rural China, and it continues to become wealthier at a faster rate.

In Figure 4 the expenditure densities for the three regions are graphed for 1992 and 2010, revealing again the extent to which the density for China has changed relative to those for South Asia and Southeast Asia. The change in Indonesia and in Southeast Asia without Indonesia is given in the lower right diagram. South Asia has lagged behind Southeast Asia in terms of improvement, and general level of income. In Southeast Asia, Indonesia has improved greatly, but still has not caught up to the remainder of Southeast Asia.

VI Concluding Remarks

When income or expenditure data are available in grouped form, estimation of a parametric income distribution facilitates calculation of quantities of interest such as Lorenz curves, inequality measures and poverty measures. The GB2 distribution and its various special cases are popular functions for estimating income distributions. We have derived expressions for several poverty measures in terms of the parameters of the GB2 distribution. These are convenient expressions that can be readily calculated with standard software. The advantage of using these expressions rather than the raw form of grouped data is that they allow for the distribution of income or expenditure within groups. We illustrated how the expressions could be used to compute poverty measures for several countries in South and Southeast Asia.

APPENDIX : SOME DERIVATIONS

Watts Poverty Index

We begin by establishing some useful results.

Let $J_v\left(p,q\right) = B_v\left(p,q\right) B\left(p,q\right) = \int_0^v t^{p-1} \left(1-t\right)^{q-1} dt$, then

$$\frac{\partial J_v\left(p,q\right)}{\partial p} = \int_0^v \ln(t)\, t^{p-1}\left(1-t\right)^{q-1} dt$$

$$\frac{\partial J_v\left(p,q\right)}{\partial q} = \int_0^v \ln(1-t)\, t^{p-1}\left(1-t\right)^{q-1} dt$$

To express the Watts index in terms of derivatives of the beta cdf we consider

$$D_p B_v(p,q) \equiv \frac{\partial B_v(p,q)}{\partial p}$$

$$= J_v(p,q)\frac{\partial}{\partial p}\left(\frac{1}{B(p,q)}\right) + \frac{1}{B(p,q)}\frac{\partial J_v(p,q)}{\partial p}$$

Rearranging, and taking the derivative $\partial\left[B(p,q)\right]^{-1}\big/\partial p$ we have

$$\frac{1}{B(p,q)}\frac{\partial J_v(p,q)}{\partial p} = D_p B_v(p,q) - B_v(p,q)\left[\psi(p+q)-\psi(p)\right]$$

Similarly,

$$\frac{1}{B(p,q)}\frac{\partial J_v(p,q)}{\partial q} = D_q B_v(p,q) - B_v(p,q)\left[\psi(p+q)-\psi(q)\right]$$

We use these results to get a convenient expression for the following integral.

$$I = \int_0^z \ln(y)\ f(y)\,dy$$

$$= \frac{1}{B(p,q)}\int_0^v \ln\left[b\left(\frac{t}{1-t}\right)^{1/a}\right] t^{p-1}(1-t)^{q-1}\ dt$$

$$= \ln(b)\ B_v(p,q) + \frac{1}{a\,B(p,q)}\left[\int_0^v \ln(t)\ t^{p-1}(1-t)^{q-1}\,dt - \int_0^v \ln(1-t)\,t^{p-1}(1-t)^{q-1}\,dt\right]$$

$$= \ln(b)\ B_v(p,q) + a^{-1}\left\{D_p B_v(p,q) - D_q B_v(p,q) + B_v(p,q)\left[\psi(p)-\psi(q)\right]\right\}$$

Gini Coefficient for the Poor

To obtain an expression for the Gini coefficient for the poor in terms of the parameters of the B2 distribution where $a=1$, we begin by considering the distribution function

$$F(y) = \frac{1}{B(p,q)}\int_0^{y/(b+y)} t^{p-1}(1-t)^{q-1}\,dt$$

Using integration by parts, we have

$$F\left(y\right)=\frac{1}{B\left(p,q\right)}\left[\frac{1}{p}\left(\frac{y}{b+y}\right)^{p}\left(1-\frac{y}{b+y}\right)^{q-1}+\left(\frac{q-1}{p}\right)\int_{0}^{y/(b+y)}t^{p}(1-t)^{q-2}dt\right]$$

The beauty of this expression is that it allow us to express $F(y)$ in terms of the distribution function $F^{*}(y)$ for a B2 distribution with parameters $\left[b,(p+1),(q-1)\right]$. Specifically, let,

$$F^{*}\left(y\right)=\frac{1}{B\left(p+1,q-1\right)}\int_{0}^{y/(b+y)}t^{p}\left(1-t\right)^{q-2}dt$$

and note that $(q-1)/p=B\left(p,q\right)/B\left(p+1,q-1\right)$, implying that

$$F\left(y\right)=\frac{1}{pB\left(p,q\right)}\left(\frac{y}{b+y}\right)^{p}\left(1-\frac{y}{b+y}\right)^{q-1}+F^{*}\left(y\right)$$

Armed with this expression we can consider the integral

$$\begin{aligned}I_{z}&=\int_{0}^{z}yF\left(y\right)f\left(y\right)dy\\&=\int_{0}^{z}\frac{1}{pB\left(p,q\right)}\left(\frac{y}{b+y}\right)^{p}\left(1-\frac{y}{b+y}\right)^{q-1}yf\left(y\right)dy+\int_{0}^{z}F^{*}\left(y\right)y\,f\left(y\right)dy\\&=\frac{1}{pb^{p}B^{2}\left(p,q\right)}\int_{0}^{z}\frac{y^{p}}{\left(1+y/b\right)^{p+q}}\left(\frac{y}{b+y}\right)^{p}\left(1-\frac{y}{b+y}\right)^{q-1}dy+\mu\int_{0}^{z}F^{*}\left(y\right)f^{*}\left(y\right)dy\end{aligned}$$

where $yf\left(y\right)=\mu f^{*}\left(y\right)$ with $f^{*}\left(y\right)$ being the beta density function with parameters $\left[b,(p+1),(q-1)\right]$. Setting $y=bt/\left(1-t\right)$ and simplifying yields

$$\begin{aligned}I_{z}&=\frac{b}{pB^{2}\left(p,q\right)}\int_{0}^{z/(b+z)}t^{2p}\left(1-t\right)^{2q-3}dt+\frac{\mu\left[F^{*}\left(z\right)\right]^{2}}{2}\\&=\frac{bB\left(2p+1,2q-2\right)B_{z/(b+z)}\left(2p+1,2q-2\right)}{pB^{2}\left(p,q\right)}+\frac{\mu}{2}B_{z/(b+z)}^{2}\left(p+1,q-1\right)\\&=\frac{\mu}{2}\left[\frac{2B\left(2p,2q-1\right)B_{z/(b+z)}(2p+1,2q-2)}{pB^{2}\left(p,q\right)}+B_{z/(b+z)}^{2}\left(p+1,q-1\right)\right]\\&=\frac{\mu}{2}\left[G\times B_{z/(b+z)}(2p+1,2q-2)+B_{z/(b+z)}^{2}\left(p+1,q-1\right)\right]\end{aligned}$$

The next to last equality uses the result

$$B\left(2p+1,2q-2\right)=\frac{\mu}{b}B\left(2p,2q-1\right)$$

while the last equality uses the expression for the Gini coefficient of the whole distribution, $G = 2B(2p,2q-1)\big/ pB^2(p,q)$. For readers who have lasted the distance, the Gini coefficient for the poor can now be written as

$$\begin{aligned} G_z &= -1+\frac{2}{\mu_z F^2\left(z\right)} I_z \\ &= -1+\frac{\mu}{\mu_z H_z^2}\Big(G\times B_{z/(b+z)}(2p+1,2q-2)+B^2_{z/(b+z)}(p+1,q-1)\Big) \end{aligned}$$

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Table 1: Components for Computing Poverty Measures from the GB2 distribution and its Special Cases

	General form	GB2	Beta-2 ($a=1$)	Singh-Maddala ($p=1$)	Dagum ($q=1$)
The mean μ	$\mu=\int_0^\infty y\,f(y)dy$	$\mu=\frac{bB(p+1/a,q-1/a)}{B(p,q)}$	$\mu=\frac{bp}{q-1}$	$\mu=bqB(1+1/a,q-1/a)$	$\mu=bpB(p+1/a,1-1/a)$
The *k*-th moment	$\mu^{(k)}=\int_0^\infty y^k f(y)dy$	$\mu^{(k)}=\frac{b^k B(p+k/a,q-k/a)}{B(p,q)}$	$\mu^{(k)}=\frac{b^k B(p+k,q-k)}{B(p,q)}$	$\mu^{(k)}=b^k qB(1+k/a,q-k/a)$	$\mu^{(k)}=b^k pB(p+k/a,1-k/a)$
Head count ratio	$H_z=B_v(p,q)$	$v=\frac{(z/b)^a}{1+(z/b)^a}$	$v=\frac{z}{b+z}$	$v=\frac{(z/b)^a}{1+(z/b)^a}$	$v=\frac{(z/b)^a}{1+(z/b)^a}$
Mean income of the poor	$\mu_z=\frac{\int_0^z y\,f(y)dy}{F(z)}$	$\mu_z=\mu\frac{B_v(p+1/a,\,q-1/a)}{B_v(p,q)}$	$\mu_z=\mu\frac{B_v(p+1,q-1)}{B_v(p,q)}$	$\mu_z=\mu\frac{B_v(1+1/a,\,q-1/a)}{B_v(1,q)}$	$\mu_z=\mu\frac{B_v(p+1/a,\,1-1/a)}{B_v(p,1)}$
k-th moment of the poor	$\mu_z=\frac{\int_0^z y^k\,f(y)dy}{F(z)}$	$\mu_z^{(k)}=\mu^k\frac{B_v(p+k/a,\,q-k/a)}{B_v(p,q)}$	$\mu_z^{(k)}=\mu^k\frac{B_v(p+k,q-k)}{B_v(p,q)}$	$\mu_z^{(k)}=\mu^k\frac{B_v(1+k/a,\,q-k/a)}{B_v(1,q)}$	$\mu_z^{(k)}=\mu^k\frac{B_v(p+k/a,\,1-k/a)}{B_v(p,1)}$

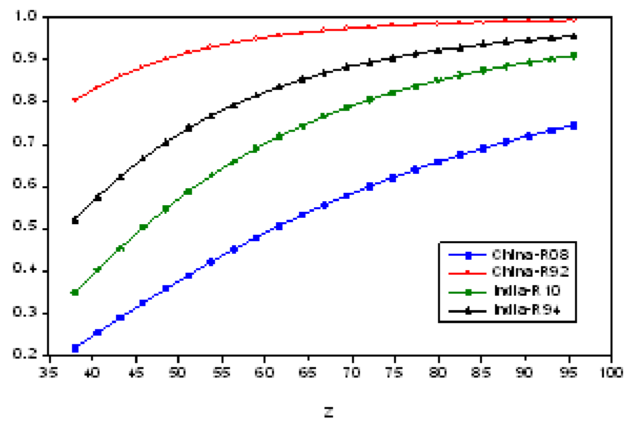
Table 2: Parameter Estimates, Mean Expenditure and Gini Coefficients from B2 Distribution

Country	Year	b	p	q	Estimated Monthly Mean Expenditure	World Bank Monthly Mean Expenditure	Estimated Gini	World Bank Gini
South Asia								
Bangladesh	2010	0.016	8973	3.847	51.61	51.67	0.320	0.321
	2005	0.015	9168	3.750	48.34	48.27	0.325	0.332
	2000	0.013	8980	3.660	43.36	43.27	0.330	0.335
	1992	1.230	112.2	5.008	34.43	34.49	0.278	0.276
India Rural	2010	0.021	8970	4.434	54.65	54.96	0.294	0.300
	2005	0.017	9801	4.282	49.85	49.85	0.300	0.305
	2000	0.033	5411	4.705	48.78	N/A	0.281	N/A
	1994	0.020	7838	4.626	43.88	43.76	0.286	0.286
India Urban	2010	0.067	1968	2.803	72.98	73.01	0.393	0.393
	2005	0.312	399.4	2.997	62.39	62.43	0.376	0.376
	2000	9.625	17.04	3.777	59.05	N/A	0.349	N/A
	1994	1.505	91.77	3.514	54.93	54.91	0.343	0.343
Pakistan	2008	0.019	11407	4.323	65.47	65.77	0.298	0.300
	2005	0.022	9037	4.047	65.49	65.76	0.310	0.327
	2002	0.019	9520	4.253	54.24	54.66	0.301	0.304
	1991	1.753	59.46	3.728	38.21	38.30	0.334	0.332
Sri Lanka	2007	0.022	9344	2.712	118.9	119.0	0.401	0.403
	2002	0.021	7794	2.626	99.89	100.1	0.410	0.411
	1991	0.022	9564	3.791	75.87	76.30	0.323	0.325
Southeast Asia								
Indonesia Rural	2010	2.920	79.90	4.098	75.32	75.36	0.314	0.315
	2006	1.874	107.8	4.675	54.99	55.13	0.289	0.287
	1999	0.272	744.2	5.919	41.17	41.24	0.249	0.247
	1993	0.036	4934	5.440	39.78	39.78	0.260	0.260
Indonesia Urban	2010	12.01	16.71	3.225	90.19	90.21	0.380	0.381
	2006	0.055	2869	3.227	71.20	71.35	0.358	0.357
	1999	0.013	9867	3.353	56.27	56.85	0.349	0.353
	1993	0.039	2996	3.287	51.04	51.07	0.353	0.353
Malaysia	2009	188.1	4.043	2.910	398.3	399.8	0.460	0.462
	2004	99.17	6.228	4.024	204.2	204.3	0.377	0.379
	1997	98.25	4.774	2.431	327.9	328.2	0.491	0.492
	1992	80.34	4.897	2.556	252.9	253.1	0.477	0.477
Philippines	2009	15.17	11.78	2.723	103.7	103.7	0.428	0.430
	2006	22.41	7.761	2.757	98.99	98.99	0.438	0.440
	2000	12.65	11.55	2.415	103.2	103.2	0.460	0.461
	1991	8.321	15.25	2.567	80.95	80.88	0.437	0.438
Thailand	2009	29.24	14.80	3.017	214.5	214.6	0.398	0.400
	2006	43.71	9.105	2.902	209.3	209.1	0.420	0.424
	2000	25.17	11.18	2.787	157.6	157.6	0.424	0.428
	1992	19.45	9.976	2.355	143.2	143.4	0.471	0.479
Vietnam	2008	20.95	11.63	3.856	85.35	85.31	0.356	0.356
	2006	26.01	9.237	4.039	79.04	79.05	0.357	0.358
	2002	0.026	5204	3.283	59.80	59.72	0.354	0.376
	1993	0.035	2560	3.248	40.04	40.07	0.356	0.357
China								
China Rural	2008	6.082	26.38	2.926	83.30	83.49	0.395	0.394
	2005	8.582	21.24	3.561	71.18	71.34	0.355	0.359
	1999	8.869	14.89	3.717	48.59	48.07	0.355	0.354
	1992	65.36	3.965	10.66	26.82	26.05	0.325	0.320
China Urban	2008	42.17	13.18	3.823	196.9	197.7	0.353	0.352
	2005	52.55	9.716	4.154	161.9	161.8	0.350	0.348
	1999	47.41	9.226	5.369	100.1	100.1	0.317	0.316
	1992	33.49	15.27	8.550	67.75	67.67	0.246	0.242

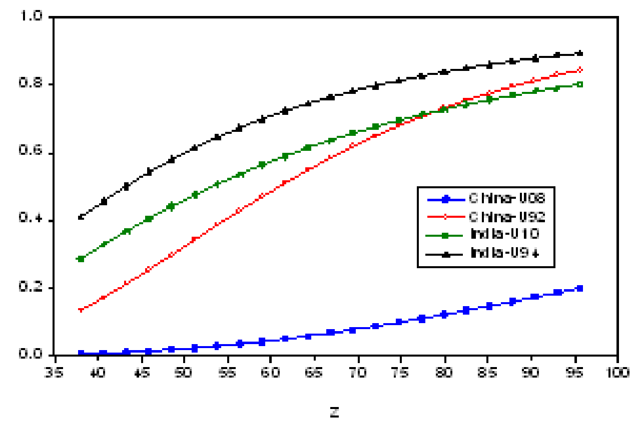
Table 3: Poverty Measure Estimates

Year	Country	HC Ratio	Poverty Gap Ratio	FGT (α=2)	Atkinson (e=0.5)	Watts Index	Sen Index	Population Millions	No of Poor Millions
South Asia									
2010	Bangladesh	0.427	0.115	0.043	0.129	0.150	0.156	148.7	63.50
	India Rural	0.349	0.081	0.027	0.090	0.102	0.111	810.8	282.8
	India Urban	0.287	0.073	0.026	0.081	0.082	0.099	329.1	94.30
	Pakistan	0.221	0.043	0.013	0.047	0.048	0.060	167.4	36.95
	Sri Lanka*	0.073	0.013	0.003	0.014	0.013	0.018	20.27	1.48
	Combined	0.324	0.077	0.026	0.086	0.095	n/a	1476	479.0
2005	Bangladesh	0.482	0.140	0.055	0.159	0.176	0.188	153.3	73.92
	India Rural	0.431	0.112	0.040	0.125	0.143	0.152	664.1	286.3
	India Urban	0.364	0.100	0.038	0.113	0.135	0.136	307.7	112.0
	Pakistan	0.239	0.049	0.015	0.054	0.061	0.069	158.7	37.86
	Sri Lanka	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
	Combined	0.397	0.105	0.038	0.118	0.135	n/a	1283	510.2
2000	Bangladesh	0.567	0.184	0.078	0.211	0.249	0.243	131.1	74.35
	India Rural	0.457	0.118	0.042	0.131	0.153	0.159	724.5	330.9
	India Urban	0.361	0.101	0.040	0.114	0.122	0.137	436.1	157.5
	Pakistan	0.365	0.088	0.030	0.097	0.104	0.120	150.4	54.90
	Sri Lanka*	0.145	0.030	0.009	0.033	0.029	0.042	18.75	2.72
	Combined	0.425	0.114	0.043	0.129	0.145	n/a	1460.81	620.4
1992	Bangladesh	0.700	0.242	0.107	0.279	0.320	0.315	112.0	78.43
	India Rural	0.522	0.147	0.056	0.165	0.198	0.197	664.1	346.6
	India Urban	0.409	0.114	0.044	0.128	0.148	0.154	235.3	96.17
	Pakistan	0.650	0.237	0.111	0.277	0.321	0.309	115.0	74.78
	Sri Lanka	0.167	0.032	0.009	0.034	0.036	0.044	17.74	2.956
	Combined	0.524	0.157	0.063	0.178	0.210	n/a	1144	598.9
Southeast Asia									
2010	Indonesia Rural	0.160	0.030	0.009	0.033	0.033	0.042	111.1	17.75
	Indonesia Urban	0.164	0.037	0.013	0.041	0.041	0.052	128.8	21.08
	Malaysia	0.008	0.002	0.001	0.002	0.001	0.003	27.95	0.22
	Philippines	0.163	0.041	0.015	0.045	0.053	0.056	91.70	14.98
	Thailand	0.006	0.001	0.000	0.001	0.000	0.001	68.71	0.45
	Vietnam	0.163	0.037	0.013	0.042	0.048	0.052	85.12	13.85
	Combined	0.133	0.030	0.010	0.033	0.035	n/a	513.4	68.32
2006	Indonesia Rural	0.337	0.077	0.025	0.085	0.093	0.106	116.8	39.32
	Indonesia Urban	0.253	0.058	0.019	0.064	0.084	0.080	113.2	28.65
	Malaysia	0.014	0.003	0.001	0.003	0.007	0.004	25.59	0.357
	Philippines	0.200	0.056	0.022	0.063	0.078	0.077	87.12	17.40
	Thailand	0.016	0.003	0.001	0.003	0.006	0.004	67.28	1.089
	Vietnam	0.202	0.051	0.019	0.057	0.065	0.070	83.31	16.85
	Combined	0.210	0.051	0.018	0.056	0.067	n/a	493.2	103.7
2000	Indonesia Rural	0.545	0.144	0.051	0.160	0.171	0.193	124.9	68.04
	Indonesia Urban	0.400	0.110	0.042	0.124	0.149	0.148	85.76	34.30
	Malaysia	0.016	0.004	0.001	0.004	0.006	0.005	21.78	0.34
	Philippines	0.203	0.055	0.021	0.062	0.067	0.076	77.31	15.71
	Thailand	0.045	0.009	0.003	0.010	0.009	0.013	63.16	2.86
	Vietnam	0.364	0.096	0.036	0.108	0.120	0.131	79.54	28.97
	Combined	0.332	0.088	0.032	0.099	0.109	n/a	452.4	150.2
1992	Indonesia Rural	0.584	0.166	0.063	0.187	0.220	0.221	128.5	75.05
	Indonesia Urban	0.474	0.143	0.058	0.163	0.183	0.192	65.02	30.83
	Malaysia	0.029	0.007	0.003	0.008	0.008	0.010	19.20	0.558
	Philippines	0.289	0.084	0.034	0.095	0.111	0.114	63.15	18.27
	Thailand	0.102	0.024	0.009	0.027	0.032	0.034	58.23	5.925
	Vietnam	0.636	0.235	0.110	0.274	0.336	0.306	68.45	43.54
	Combined	0.433	0.133	0.055	0.152	0.179	n/a	402.6	174.2

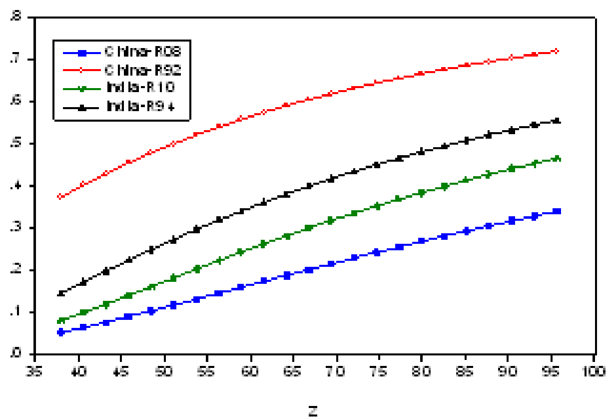
Year	Country	HC Ratio	Poverty Gap Ratio	FGT (α=2)	Atkinson (e=0.5)	Watts Index	Sen Index	Population Millions	No of Poor Millions
China									
2008	China Rural	0.217	0.053	0.019	0.059	0.057	0.073	753.7	163.8
	China Urban	0.005	0.001	0.000	0.001	0.001	0.001	570.9	2.615
	Combined	0.126	0.030	0.011	0.034	0.033	n/a	1324	166.4
2005	China Rural	0.252	0.062	0.022	0.069	0.077	0.085	759.7	191.4
	China Urban	0.016	0.003	0.001	0.003	0.003	0.004	544.8	8.607
	Combined	0.153	0.037	0.013	0.041	0.046	n/a	1304	200.0
2000	China Rural	0.499	0.164	0.072	0.190	0.231	0.219	815.9	406.9
	China Urban	0.072	0.014	0.004	0.016	0.019	0.020	437.8	31.61
	Combined	0.350	0.112	0.049	0.129	0.157	n/a	1253	438.5
1992	China Rural	0.805	0.374	0.215	0.461	0.585	0.479	827.3	666.0
	China Urban	0.134	0.024	0.007	0.026	0.027	0.034	351.2	46.88
	Combined	0.605	0.270	0.153	0.332	0.418	n/a	1178	712.9



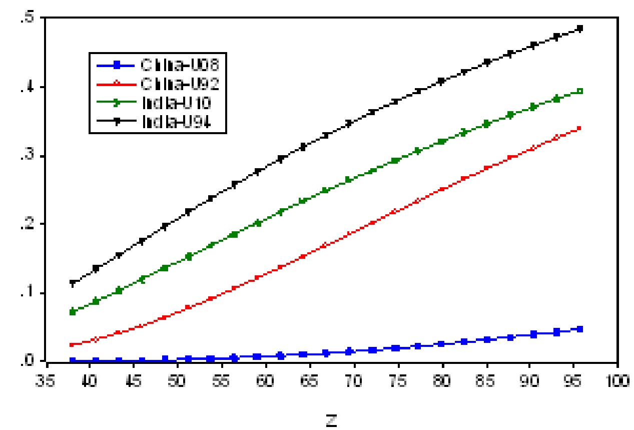
Head Count Ratio: Rural China and Rural India



Head Count Ratio: Urban China and Urban India

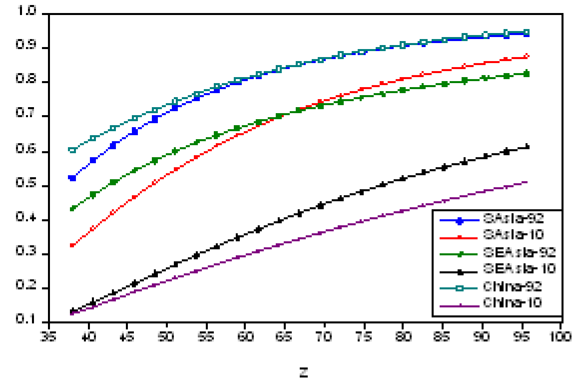


Poverty Gap Ratio: Rural China and Rural India

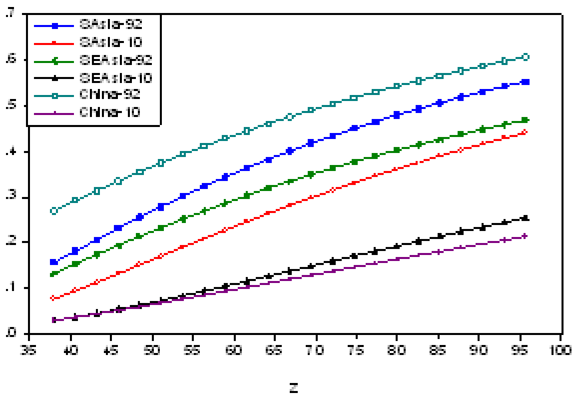


Poverty Gap Ratio: Urban China and Urban India

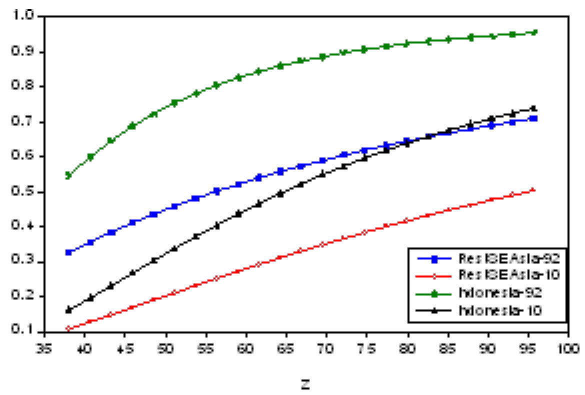
Figure 1: Sensitivity of Head Count and Poverty Gap Ratios to Poverty Line for China and India



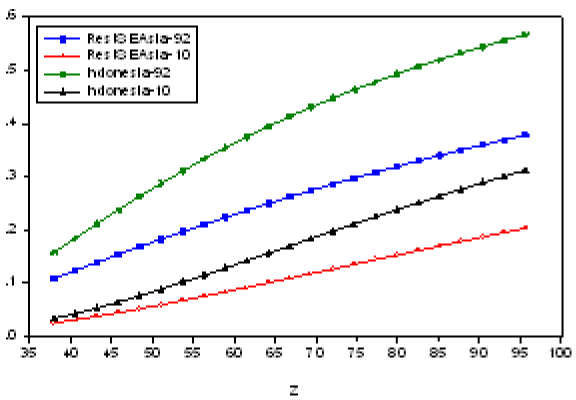
Combined Head Count Ratio: 1992 and 2010



Combined Poverty Gap Ratio: 1992 and 2010



Head Count Ratio: Indonesia and Rest of Southeast Asia 92-10



Poverty Gap Ratio: Indonesia and Rest of Southeast Asia 92-10

Figure 2: Sensitivity of Head Count and Poverty Gap Ratios to Poverty Line for South Asia and Southeast Asia

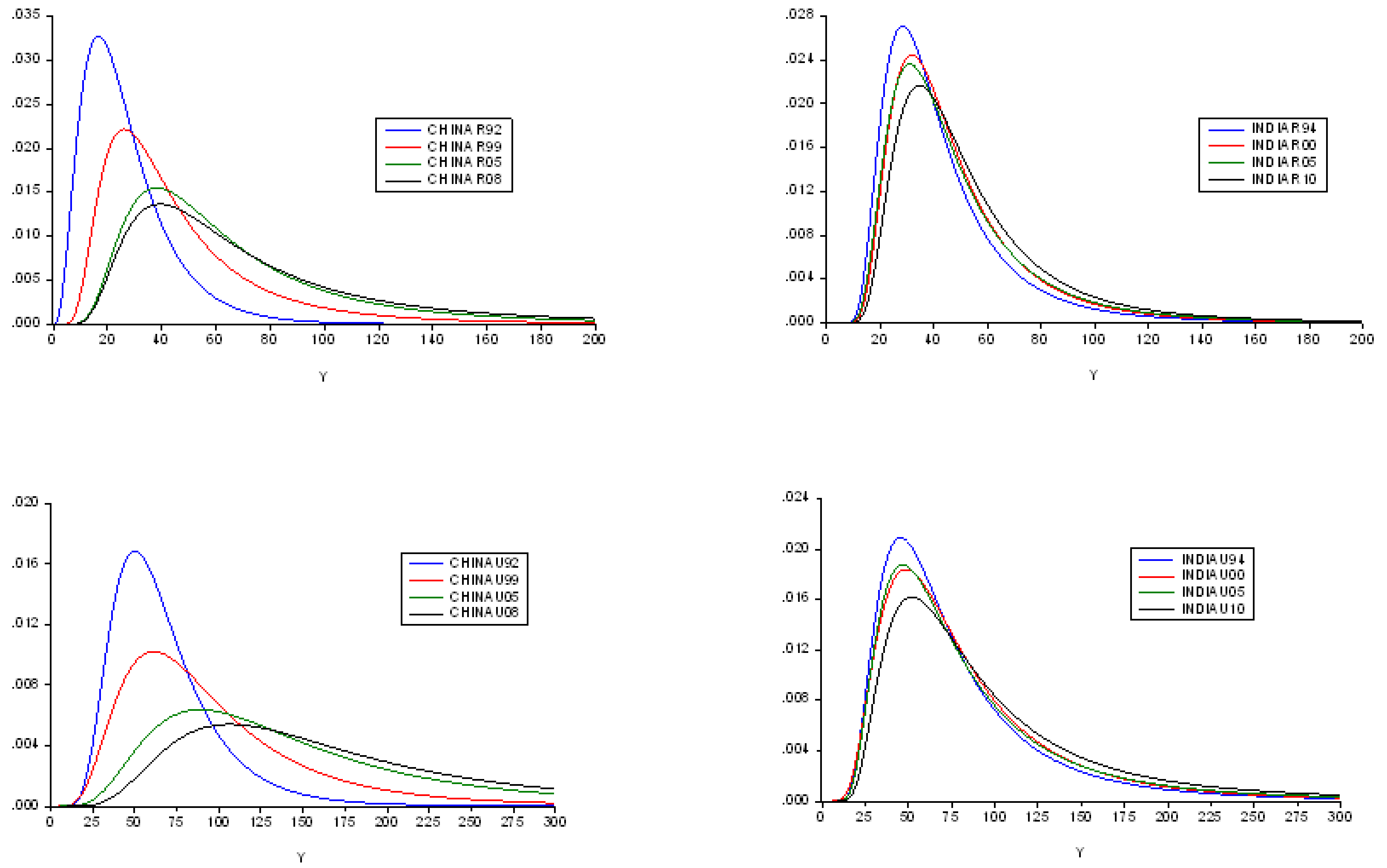


Figure 3: Expenditure Densities for China and India

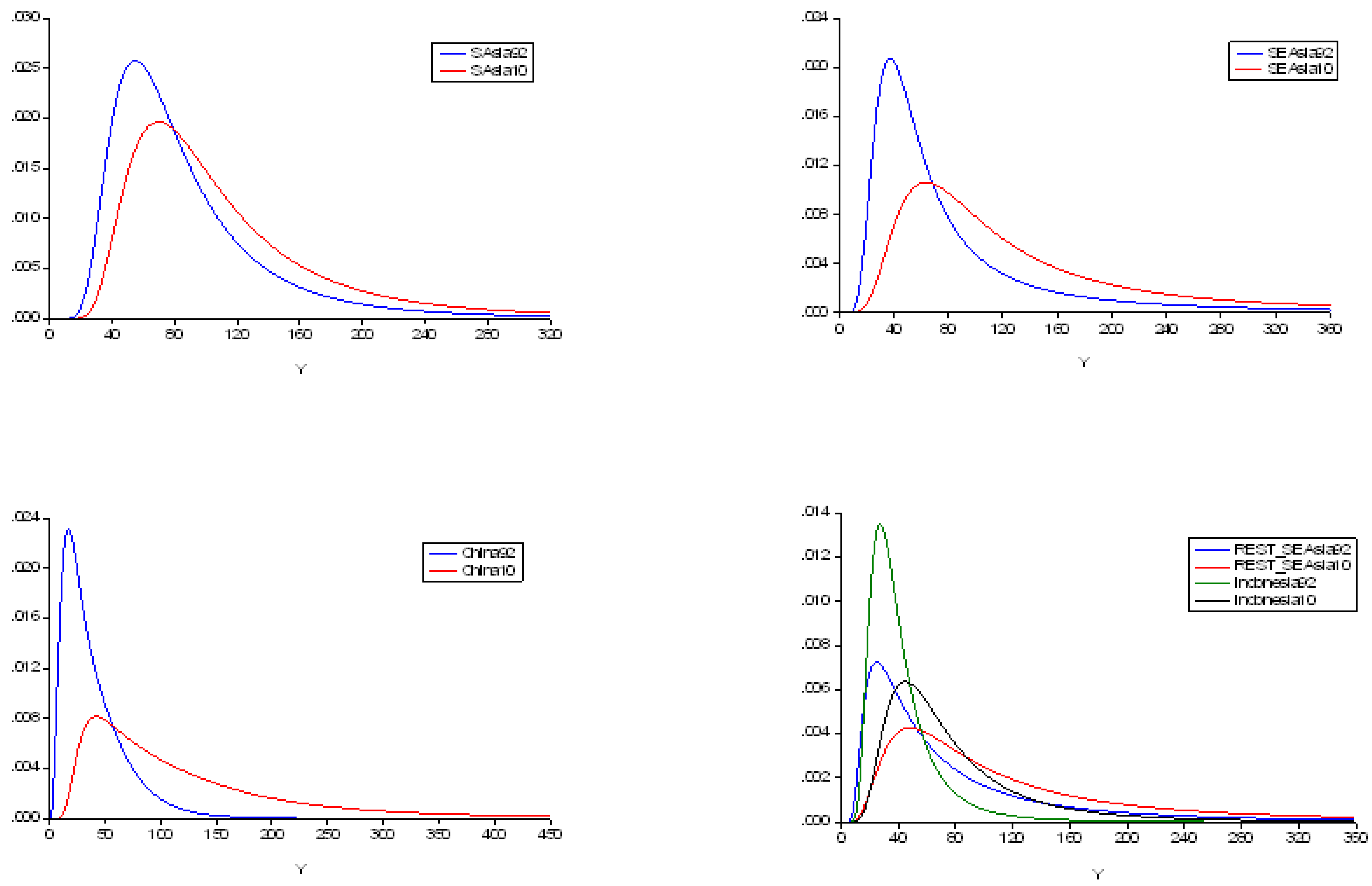


Figure 4: Expenditure Densities for South Asia, Southeast Asia and China