**Labor’s Share, the firm’s market power and TFP**

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**Abstract**

We investigate the relationship between labor’s share, firm’s market power and the elasticity of output with respect to labor input using an approach based on an unobserved components model. The approach yields time-varying estimates of market power and the elasticity. Evidence on the market power of firms (which we find to be rising since 2000) gives a deeper understanding of movements in labor’s share and the labor wedge. The generated values of the elasticity yield revised estimates of TFP growth which is informative about the extent of the downwards bias inherent in traditional estimates which use labor’s share as a proxy for the elasticity.

**JEL codes**: O47, C32, E25

**Keywords**: labor’s share, market power, TFP growth, labor wedge, state-space modelling

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Abstract
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I. INTRODUCTION
Recently there has been an upsurge of interest in the functional distribution of income (see for example Autor et al (2017a; 2017b), Barkai (2016), Caballero et al (2017), Elsby et al (2013), Karabarbounis & Neiman (2014), Piketty (2014), Rognlie (2015) and Shao & Silos (2014)). While Atkinson (2009) has argued that understanding the functional distribution of income remains one of the most important questions for political economy, it remains the case that “little consensus exists on the causes of the decline in the labor share … since the 1980s [and] particularly in the 2000s” (Autor et al, 2017b, p 180).1

In this paper we investigate the nexus between labor’s share, the market power of firms and the elasticity of output with respect to labor input. Our approach utilises the Kmenta-Nelson approximation to the CES function and state-space modelling to estimate a neoclassical model of the time-varying relationship between labor’s share, market power and the elasticity. Importantly, our approach allows us to avoid assuming either perfect

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1 We note also Robert Solow’s remark when commenting on Rognlie (2015) that: “the degree of monopoly in U.S. industry remains an open question and needs more research, both microeconomic and macroeconomic” (Solow, 2015, p 64).
competition in all markets or that the deviation from perfect competition is solely to be found in the market for products and not in the labor market. Further, the framework does not assume that either the elasticity or market power are constant over time.

Our empirical study is for the US business sector over the sample period 1947:2-2016:3. Applying our econometric framework, we generate a time series for the firm’s market power and the associated evolution of the elasticity of output with respect to labor input. Critically, these two series allow us (inter alia) to: identify the proximate source of the marked decline in labor’s share since the early 2000s; examine the cyclicality of the firm’s contribution to the labour wedge (also known as the inefficiency gap), and; use our time series for the elasticity of output with respect to labor input to generate a revised series for TFP growth in the US over the period (and thus to study the bias which results from using labor’s share as a proxy for the elasticity in growth accounting exercises).

This paper contributes to the literature concerned with variations in the wage share by offering an empirical framework which relates the wage share, monopoly power, the elasticity of output with respect to labor input and TFP growth. Specifically, our state-space estimation approach is different from the extant literature, in that it provides an estimate of the variation over time in firms’ market power. We then show how changes in market power contributes to observed changes in labor’s share. We also delve deeper into the separate contributions of variations over time in both product and labor market markups to variations in firms’ market power, and we investigate the cyclicality of both labor’s share and the wedge (and their determinants). Our results suggest that the fall in the wage-share commencing in the early 2000’s is associated with a marked rise in the market power of firms (a result also found in Barkai (2016), De Loecker & Eeckhout (2017), Gutierrez (2017) and Kurz (2017)) and also that variations in the capital/labor ratio do not explain the observed movements of labor’s share (a finding consistent with Elsby et al (2013), Glover & Short (2017), Oberfield & Raval (2014) and Rognlie (2015)). Importantly our approach allows us to generate a time series for the elasticity of output with respect to labour input which in turn yields estimates of the bias in estimating TFP using the traditional wage-share approach.
The paper is structured as follows:

In section II we outline the approach which consists of three parts. First, we show that under the maximisation assumption, labor’s share (also known as the wage share) can be decomposed into two components – the firm’s market power and the elasticity of output with respect to labor input. Second, we utilise a linear approximation to the CES production function to incorporate the role of the capital-labor ratio and third, we use a state-space modelling framework to yield estimates of the evolution of latent (unobservable) variables.

In section III, we present our empirical analysis which is based on the quarterly time series data given in Fernald (2014) over the sample period 1947:2-2016:3. Our econometric method yields estimates of the evolution of the firm’s market power and the elasticity of output with respect to labor input. To anticipate the results in this section of the paper: We find that the level of the firm’s market power is (weakly) pro-cyclical and not countercyclical - it begins to fall late in expansions, to trough in contractions and then rise, reaching a maximum during expansions. We also find that there is a marked rise in the market power of firms commencing in the early 2000s.

Our estimate of the behaviour of the firm’s market power over time is informative about the behaviour of the firm’s contribution to the labor wedge (or inefficiency gap). Amongst other things, we find that the use of the (logarithm of the inverse) of labor’s share to estimate the firms contribution to the labor wedge overstates the firm’s contribution and that the firms contribution to the inefficiency gap is pro-cyclical.

In relation to the behaviour of the wage share over the period, we find there is a very close co-movement between the wage share and the firm’s market power. Also our estimates for the firm’s market power and the elasticity of output with respect to labor input imply that for the period up to the early 2000’s, changes in the elasticity have been offset by changes in the firm’s market power and this explains why there is no marked trend in the wage share over that period. In other words, the constancy of labor’s share over the second half

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2 In order for us to interpret the latent variable this way it is necessary for us to assume that technological change is Hicks neutral and that the factor intensity parameter is constant.
of the 20th Century appears to have been the result of a combination of two offsetting forces. We also find that the fall in the wage-share commencing in the early 2000’s is associated with a marked rise in the market power of firms.

In section IV we use our estimates of the elasticity of output with respect to labor input to generate a revised series for Total Factor Productivity (TFP) growth for the US Business sector. We then compare our measure of TFP growth with the conventional measure of TFP growth based on using labor’s share (profit share) as a proxy for the elasticity of output with respect to labor (capital) input. The conventional measure of TFP growth will be biased if there is imperfect competition and the extent of the bias will be related (inter alia) to the degree of market power.

Estimates of the persistence and behaviour of the bias (including its sign and size) is provided for different time periods, such as contractions (peaks to troughs), expansions (troughs to peaks) and the whole of the business cycles (measured peak to peak and trough to trough). To anticipate the results in this section of the paper: our finding that the firm’s possess market power implies that the ‘true’ elasticity of output with respect to labor input will be greater than the wage share. It follows that the use of the wage share as a proxy for the elasticity will usually result in an underestimate of the rate of TFP growth.3 The intuition is that our elasticity-weighted TFP growth calculation puts more weight on labor input - the slower growing factor in most periods - while the wage share weighted TFP growth applies a correspondingly larger weight to the faster growing factor which is capital in most periods. The upshot of this is that in most periods the elasticity-based calculation will be attributing less of GDP growth to the growth in inputs and more to technological change than the conventional measure.

Over the whole of our sample period we estimate that TFP has grown at an average of 1.45 percent per annum, while the wage share-weighted estimate of TFP growth is, on average, 1.21 percent per annum. In other words, the conventional measure of TFP growth (ie using

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3 De Loecker & Eeckhout (2017) and Kurz (2017) also consider the impact of departures from perfect competition on estimates of TFP. Both papers find (as do we) that productivity growth is under-estimated if perfect competition is assumed.
labor’s share) is, on average, 0.24 percent lower than the estimate of TFP growth using the elasticity.

We also estimate TFP growth to be 0.15 percent per annum higher than that given by the wage share weighted approach in the period 2007-2015. Over the business cycle the underestimation of TFP growth is, on average, as much as a quarter of one percent (ranging from -0.13 to -0.43 percent). This is not negligible when compared to an average labor share weighted measure of TFP growth of about 1.27 percent per annum. Importantly, we also find that there are periods when the two approaches give opposite signs for the rate of TFP growth. Finally, the bias is pro-cyclical and, given that the firm’s market power is pro-cyclical, we find that allowing for departures from competition reduces the pro-cyclicality of TFP growth, consistent with the argument in Hall (1987).

Concluding remarks are in section V of the paper.

II. THE APPROACH

A. Labor’s share, market power and the elasticity of output with respect to labor input

While the actual calculation of the share of total income accruing to labor input can sometimes prove problematic (see for example Gollin (2002), Elsby et al (2013), Giandrea & Sprague (2017) and the references cited therein), the concept is easy to state. Define the wage share \( S \) and expand it as follows:

\[
S = \frac{WL}{PY} = \frac{W/P}{Y/L} = \left( \frac{W/P}{\partial Y/\partial L} \right) \left( \frac{\partial Y/\partial L}{Y/L} \right)
\]

where \( W \) is the nominal wage, \( Y \) is real output, \( P \) is the output price and \( L \) is labor input.

This expression shows that the wage share can be decomposed into a term capturing the relationship between the real wage and the marginal product of labor and another term capturing the relationship between the marginal product of labor and the average product of labor. The former will reflect the presence of any market power in either or both the goods and labor markets while the latter will reflect the characteristics of the production function and the technique being used. As Solow (1958, p 620) has remarked, “Between production functions and factor-ratios on the one hand, and aggregate distributive shares on the other lies a whole string of intermediate variables: elasticities of substitution,
commodity-demand and factor-supply conditions, markets of different degrees of competitiveness and monopoly …”.

Consider the case of profit maximisation involving a single, representative, firm producing output with the aid of two inputs, labor and capital:

\[ \pi = PY - WL - RK \]

where total profit (\( \pi \)), is equal to total revenue (\( PY \)) less labor cost (\( WL \)) and the cost of using capital (\( RK \), where \( R \) is the nominal rental and \( K \) is the number of units of capital). Differentiating with respect to \( L \) and rearranging terms gives:

\[ \frac{\partial Y}{\partial L} P \left( 1 + \frac{Y}{P} \frac{\partial P}{\partial Y} \right) = W \left( 1 + \frac{L}{W} \frac{\partial W}{\partial L} \right) \]

This can be re-expressed in terms of elasticities

\[ S = \frac{WL}{PY} = \left( \frac{\varepsilon_{YP} + 1}{\varepsilon_{WP}} \right) \left( \frac{\partial Y}{\partial L} / Y \right) = \left( 1 + \frac{1}{\varepsilon_{YP}} \right) \left( \frac{\partial Y}{\partial L} / Y \right) \]

(2)

where \( \varepsilon_{YP} = (dY/Y)/(dP/P) \) is the demand elasticity\(^4\) of output with respect to (own) price and \( \varepsilon_{LP} = (dL/L)/(dW/W) \) is the supply elasticity of the labor input with respect to the wage.

The above is a neoclassical model of the wage share which explicitly incorporates market power and can be traced back to Chapter 27 of the first edition of Joan Robinson’s *The Economics of Imperfect Competition* which was published in 1933.\(^5\) A more recent derivation can be found in Wickens (2011, pp 220-2) which shows exactly the same relationship as that given above between the wage share, the elasticity of output with respect to labour input, the (own price) elasticity of demand for the product and the supply elasticity of the labor input.

Notice that (2) above may be rewritten as:

\(^4\) If \( \varepsilon_{YP} \) is defined as a negative number, the numerator would be written as \( (1 - (1/\varepsilon_{YP})) \).

The two terms in the denominator of (3) relate to easily recognised concepts. The firm’s price ‘mark-up’ in the product market is the ratio of the price set by the firm to its marginal cost (also known as ‘monopoly power’) and is denoted here as $M_p = (1/(1 + (1/\varepsilon_{YP})))$. The firm’s wage “mark-down” (Depew and Sorensen, 2013, p 198) is a measure of the firm’s monopsony power in the labor market and is denoted here as $M_L = (1 + (1/\varepsilon_{LY}))$ – this will also be equal to the ratio of the marginal wage to the average wage.

For convenience of notation, rewrite (3) as:

$$S = \frac{WL}{PY} = \left(\frac{1}{M_L \times M_p} \right) \left( \frac{\partial Y}{\partial L} / \frac{Y}{L} \right) = \left( \frac{1}{M} \right) E$$

where the firm’s market power reflects its combined monopoly ($M_p$) and monopsony ($M_L$) powers. Also, for convenience of exposition in subsequent sections, we have defined $M = M_L \times M_p$ and $E$ as the elasticity of output with respect to labor input, that is $E = \left( \frac{\partial Y}{Y} \right) / \left( \frac{\partial L}{L} \right)$. (Note: In what follows we will, for the sake of brevity, refer to the elasticity of output with respect to labor input as “the elasticity”.)

The explicit inclusion of monopsony power recognises the firm’s (potential) role in the labor market as well as the product market. As Boal and Ransom note in their survey of Monopsony, “since Robinson, numerous models of buyer market power have been developed that do not assume a single buyer or even a small number of buyers. Today the term “labor monopsony” is applied more broadly to any model where individual firms face upward-sloping labor supply” (Boal and Ransom, 1997, p 86). Indeed, the modern theory of monopsony encompasses “wage-setting power in labour markets consisting of many competing firms. Potential reasons include search frictions, mobility costs, or job differentiation, all of which impede workers’ responsiveness to wages. This causes the labour supply curve to the single firm to be upward-sloping, rather than being horizontal as under perfect competition” (Hirsch et al, 2017, p3).
In the case where $M_L = 1$, (which occurs when $e_{lw} = \infty$), the real wage will be related to the marginal product of labour such that $M_p = (\partial Y/\partial L)/(W/P)$. In other words, $M_p$ informs us about the extent to which the marginal product of labour is above the real wage as a result of the exercise of market power in the product market by the firm. In the case where $M_p = 1$, ie when the firm has no market power in the product market, we obtain $M_L = (\partial Y/\partial L)/(W/P)$ and in this case $M_L$ informs us about the extent to which the firm is able to mark-down the real wage below the marginal product of labour.\(^6\)

Thus, the term $M_L$ is informative about the extent to which the marginal product of labour is above the real wage as a result of the exercise of market power by the firm in both product and labor markets. Given that $M_L$ is likely to be greater than 1, equation (4) shows that the elasticity of output with respect to labor input, $E$, will be larger than the wage share.

Equation (4) shows that the wage share can be decomposed into two unobservable variables. In the following sub-sections, we show how we utilise information about an observable series – the capital-labor ratio – to facilitate the estimation of both the elasticity of output with respect to labor input and the evolution of the firm’s market power over time.

**B. The CES production function and the elasticity of output with respect to labor input**

There are many ways to model the ratio of the marginal product to the average product $(\partial Y/\partial L)/(Y/L)$. The simplest and most common approach is to assume a Cobb-Douglas production function where the ratio of the marginal product to the average product is taken to be a parameter and constant over time. If this were the case variations in the wage share would simply reflect variations in the firm’s market power. The more general CES production function allows the elasticity of output with respect to labor input to (potentially at least) be time-varying.

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\(^6\) Some of the literature in this area (eg Gali et al, 2007, p 45) assumes that when setting prices firms behave ‘as if’ they are wage-takers. If that were the case $M_L$ would be an estimate of the ‘price-marginal cost’ markup.
In logarithms, the CES production function with constant returns to scale and with Hick’s neutral (and disembodied) technological progress may be written as:

$$\ln Y = \ln(\gamma) - \frac{1}{\theta} \ln\left[\delta K^{-\theta} + (1-\delta)L^{\theta}\right]$$

where $\gamma$ is the ‘efficiency parameter ($\gamma > 0$), $\delta$ is the distribution or ‘factor-intensity’ parameter ($0 < \delta < 1$) and $\theta$ is the substitution parameter ($\theta \geq -1$) – it determines the size of the elasticity of substitution which is equal to $1/(1 + \theta)$. If $\theta$ equals zero, the CES function has an elasticity of substitution of unity, the Cobb-Douglas case; however, in general, in a CES function the elasticity of substitution is not limited to be unity but may take any value between zero and infinity.

Following Nelson (1965) and Kmenta (1967) we take a Taylor series expansion around $\theta = 0$ which gives (disregarding higher order terms):

$$\ln Y = \ln \gamma + \delta \ln K + (1-\delta)\ln L - \frac{1}{2} \theta\delta(1-\delta)(\ln K - \ln L)^2$$

Differentiating $\ln Y$ with respect to $\ln L$ gives an estimate of the elasticity of $Y$ with respect to $L$, that is, the ratio of the marginal product to the average product.7

$$\frac{\partial \ln(Y)}{\partial \ln(L)} = E = (1 - \delta) + \theta\delta(1-\delta)\ln(K/L) = (1 - \delta)[1 + \theta\delta\ln(K/L)]$$

which is to say that the elasticity of output with respect to the labor input is a function of the logarithm of the capital-labor ratio.8 The logarithm of (6) may be expressed as:

$$\ln E = \ln(1-\delta) + \ln[1 + \theta\delta\ln(K/L)] \approx \ln(1-\delta) + \theta\delta\ln(K/L))$$

Substituting (7) into $\ln(S) = \ln(1/M) + \ln(E)$ yields:

$$\ln S = \ln(1/\ ) + \ln(1-\delta) + \theta\delta\ln(K/L))$$

Artus (1984) and McCallum (1985) both utilise this approximation to derive an expression for the factor shares and ultimately the profit maximising real wage but, unlike us, they both assume perfect competition in factor and product markets.

Note, that the approximation to the CES function can be re-arranged to yield an expression for the growth in technical change: $\Delta \ln Y = \Delta \ln Y - (\Phi\Delta \ln L + (1-\Phi)\Delta \ln K)$; where $(1-\Phi) = 0.5[(1+\delta) - E]$. In the Cobb-Douglas case, when $E = (1-\delta)$, $\Phi = E$. 

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Taking first differences of the above gives:

\[ \Delta \ln S = \Delta \ln \left( \frac{1}{M} \right) + \beta \Delta \ln (K/L) \]  

where \( \beta = \theta \delta \). Equation (9) is a relationship between observables (the rate of growth in the wage share \( \Delta \ln S \) and the capital-labor ratio \( \Delta \ln (K/L) \)), and an unobservable (the rate of change in (the inverse of) market power \( \Delta \ln \left( \frac{1}{M} \right) \)). In the next sub-section we show that we can derive an estimated series for the change in (the inverse of) market power (\( \Delta \ln \left( \frac{1}{M} \right) \)), using state-space estimation methods. However, in order for us to interpret the latent variable this way it is necessary for us to assume that technological change is Hicks neutral and that the factor intensity parameter (\( \delta \)) is constant. Some researchers – for example Acemoglu & Restrepo (2016), Koh et al (2016), Lawrence (2015), Martinez (2017) and Oberfield & Ravel (2014) – take an approach which is the ‘polar’ opposite of ours and argue that the movement in labor’s share is due to biased technological change (broadly defined to include IT, automation and the creation of new tasks) and/or a change in the factor intensity distribution parameter (perhaps as a result of automation) and not the result of changes in firms’ market power. Caballero et al (2017) explicitly consider two “polar hypotheses” (ibid, p 616): one case involving only a role for monopoly power (rents) and the other involving capital-biased technological change and/or a change in the factor intensity parameter (referred to as “automation” by them) but with no role for monopoly power. They show that any one of these or any combination of these is consistent with the observed movements in labor’s share. In passing, we note also that recent papers by Kurz (2017), Bessen (2017), De Loecker & Eeckhout (2017, p 32) and Grullon et al (2017) point out that variations in monopoly power and technical change ought not be seen as alternative explanations. This is because, in their view, technical change (again, broadly defined) is

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9 Although Martinez (2017, p 29) notes that “the model fails to match most of the steep post-2000 drop in the aggregate labor share”.

10 The papers listed above all assume that either perfect competition prevails throughout or that firms can be characterised as monopolistically competitive but where the markups do not vary over time. In other words, they all assume variations in market power are not an explanation of the variation in labor’s share.

11 Caballero et al (2017, p 619f) conclude (and we concur) that “[d]isentangling the relative importance of the different mechanisms behind the increase in rents …[and] technical change … defines an important research agenda.”
the source of variations in industry concentration and market power which, in turn, results in a change in labor’s share.

C. A State-space estimation model

The econometric model is written as:

\[
\Delta \text{ln}(S) = m_t + b_t \Delta \text{ln}(K_t/L_t)
\]

\[
m_t = \rho m_{t-1} + \eta_{1t}, \quad \eta_{1t} \sim N(0, \sigma^2_{\eta_{1}})
\]

\[
b_t = \rho_2 b_{t-1} + \eta_{2t}, \quad \eta_{2t} \sim N(0, \sigma^2_{\eta_{2}})
\]

where \(\Delta \text{ln}(S)\) and \(\Delta \text{ln}(K/L)\) are the measured rates of change in the wage share and the capital-labor ratio respectively over the sample period. The term \(m_t = \Delta \text{ln}(1/M_t)\) is the rate of change in the (inverse of) the firm’s market power in the labor and product markets. It is also time-varying but it is a latent variable modelled as an autoregressive process. We have allowed \(b_t\) - the estimate of the coefficient on \(\Delta \text{ln}(K_t/L_t)\), in equation (8) - to also be time varying.

The first equation is the observation/measurement equation which allows for the decomposition of the wage share, while the second and third equations are the state/transition equations in this state-space model. The errors are modelled as normally distributed and serially uncorrelated (the possibility of correlated errors will be tested). The model will be estimated using the maximum likelihood approach including the application of a Kalman filter.

Thus, application of the approach will yield a time-varying measure of the firm’s market power. This series is of interest in its own right, and we shall also use it to derive an estimate of the elasticity of output with respect to labor input. Collectively, the results will provide a deeper understanding of behaviour of the labor’s share (especially the fall since the 2000’s) as well as a revised measure of TFP growth in the US including an understanding of the size and behaviour over time of the bias inherent in TFP estimates which use the wage share as a proxy for the elasticity.
III. EMPIRICAL ANALYSIS

A. Data

Our empirical analysis is based on John Fernald’s quarterly data set\(^\text{12}\) for the US Business sector. The sample period is 1947:1–2016:3. Capital’s share of income (\(\text{alpha}\) in Fernald’s spreadsheet) is based primarily on NIPA data for the corporate sector and is measured such that the corresponding wage share includes an adjustment for ‘proprietor’s income’. The data on output growth (\(dy\)) is a ‘composite’ series which is the simple average of the growth rates of Business Gross value added and Business output, measured from the income side. The data on growth in capital input (\(dk\)) are based on disaggregated quarterly NIPA investment data. Fernald’s data set also includes estimates of the rate of growth of hours worked (\(d\text{hours}\)) and a variable to capture changes in labor composition/quality (\(dLQ\)). Fernald shows (2014, p 13) that his quarterly data for output, capital and labor input growth when converted to their implied annual rates yield a series for TFP growth that resembles closely the BLS TFP series.\(^\text{13}\)

Plots of the annualised growth in the wage share (where \(S_t = (1 – \text{alpha}_t)\)) and the capital-labor ratios\(^\text{14}\) \(\Delta\ln(K_t/L_t) = dk_t – d\text{hours}_t\), are shown in Figure 1.\(^\text{15}\) The shaded regions show contractions as designated by the NBER (peaks to troughs).

\[\text{FIGURE 1 NEAR HERE}\]

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\(^\text{13}\) Fernald’s wage share series when converted to annual averages is highly correlated (\(r = 0.95\)) with BLS estimates of labor’s share for the Private Business Sector as reported in their MFP Tables and Charts: “Historical multifactor productivity measures (SIC 1948-87 linked to NAICS 1987-2015)” spreadsheet. Downloaded February 12\(^\text{th}\) 2017 from the BLS website https://www.bls.gov/mfp/tables.htm The BLS series has been adjusted for non-corporate income. See Bureau of Labor Statistics (2007, n9) for details. On the importance of allowing for non-corporate income (such as the income of the self-employed) see Gollin (2002) and Mućk et al (2015).

\(^\text{14}\) In the Fernald data set the rate of under-utilisation of labor and capital is assumed to be the same. This means that \((dk – d\text{hours})\) is also a measure of the ‘utilisation adjusted’ rate of growth in the capital-labor ratio.

\(^\text{15}\) Since \(dLQ\) is taken into account when calculating TFP growth, we also checked the contribution of labor quality \(dLQ\) to the growth in the capital-labor ratio. The contribution is negligible; the correlation of the two series - capital-labor ratios with and without adjustment for labor quality - is very high (0.974).
We see in the Figure that the growth in the capital-labor ratio peaks during periods of contractions and troughs in expansions, whereas the growth in the wage-share peaks late in recoveries. Both series are counter-cyclical with GDP growth (correlation coefficients are -0.69 and -0.20 respectively) and positively related with each other (correlation coefficient equals 0.16).

B. Estimated Market-Power and Elasticity

The results from estimating the system of equations (10) are shown in Table 1. The results for Model 1 showed that \( \hat{\rho}_2 \) may be unity and that \( \hat{b}_t \) may not be time-varying (according to the conventional 10% level of significance). The model was re-estimated imposing \( \hat{\rho}_2 = 1 \) (Model 2) and also with a constant \( b \) (Model 3).\(^{16}\) Model 4 assumes that both latent variables are random walks (\( \rho_1 = \rho_2 = 1 \)). Overall, the preferred model (Model 1) is one with time-variation in both \( m_t \) and \( b_t \), with \( b_t \) being more persistent and less variable than \( m_t \).

\[ \text{[TABLE 1 NEAR HERE]} \]

The econometric method yields an estimated time series for the rate of growth in the inverse of the firm’s market power, \( \Delta \ln \left( \frac{1}{M_t} \right) \). Subtracting this from \( \Delta \ln(S_t) \) generates an estimate of the growth in the elasticity of output with respect to labor input, i.e. \( \Delta \ln(\hat{E}_t) \).

To obtain the levels of \( \hat{E}_t \), we generate a series for \( \hat{M}_t \) and then obtain the implied values for the elasticity as: \( \hat{E}_t = S_t \times \hat{M}_t \). We have calibrated our estimate of the firm’s market power \( M_t = M_{L,t} \times M_{P,t} \) to have a mean of (approximately) 1.2.\(^{17}\) However we have not

\(^{16}\) Notice that the estimated value of a constant \( b \) in Model 3 (\( b \) is time-varying in the other models) implies that the substitution parameter in the CES function is, on average, positive, given that \( \hat{\delta} \) is positive. This in turn implies that the elasticity of substitution (which is equal to \( 1/(1 + \hat{\delta}) \), is less than 1 and that the appropriate production function is not Cobb-Douglas. Model 3 thus supports the findings of Chirinko (2008), Lawrence (2015) and Oberfield & Raval (2014), amongst others, who argue that the elasticity of substitution in the US is below 1.

\(^{17}\) Specifically, a value of 1.177 (and by implication a value for \( 1/M \) of 0.85) has been arrived at by multiplying 1.06 (the mean value of the ratio of the marginal wage to the average wage (average for \( M_{L} \)) given in Nekada and Ramey (2013)) with 1.11 (a conservative estimate of the price-marginal cost markup).
restricted its value \textit{apriori} to always be above one and nor have we estimated the model with pre-filtered time-series data.

The Evolution of the firm’s market power. Figure 2 shows that the estimated values for market power, $\hat{M}_t$, over the period 1947:1–2016:3 are all above 1. The level of firm’s market power tends to trough at or near the mid-point of contractions and then rise, reaching a maximum near the mid-point of expansions. The contemporaneous correlation between the rate of growth in $\hat{M}_t$ and the GDP growth rate is 0.222 while the correlation between the (HP filtered) deviations from the logarithmic trend in $\hat{M}_t$ and deviations from the logarithmic trend in GDP is 0.403. The Harding-Pagan measure of the degree of concordance (I)\textsuperscript{19} between the two growth rates is 0.534. All of the measures indicate that $\hat{M}_t$ is (weakly) procyclical – it is clearly not counter-cyclical.

Given equations (3) and (4), the evolution of $\hat{M}_t$ over time will reflect the behaviour of its two components, $\hat{M}_L$ and $\hat{M}_p$. Now $\hat{M}_L$ will be equal to the ratio of the marginal wage to the average wage and since Nekarda & Ramey (2013) provide estimates of the ratio of

Recall that $1 + (1 / \varepsilon_{1,0}) = \hat{M}_L$ is also equal to the ratio of the marginal wage to the average wage. The literature abounds with estimates of the price-marginal cost markup, we have chosen to work with a relatively conservative estimate of 1.11, a value which has been utilised by Born & Pfeifer (2014), Born et al (2013), and Leduc & Liu (2016) and is in the range suggested by Basu & Fernald (1997) and Altig et al (2011), amongst others. Robert Hall (2014, p 6) uses a value of 1.2 as his base case - see also his remarks in the General Discussion of Rognlie (2015) and also the remarks by Solow (2015), p 64.

\textsuperscript{18} A referee has pointed out that if the production function is Cobb-Douglas the implied level of market power would be identical to the inverse of the observed labor share. Since our measure of market power is highly correlated with labor’s share it’s evolution over time is not dissimilar to what one would find if a Cobb-Douglas production function were used.

\textsuperscript{19} See Harding and Pagan (2002). The index of concordance (I) is such that it must lie between 0 and 1. An index value above 0.5 indicates pro-cyclicality while a value below 0.5 indicates counter-cyclicality.
the marginal wage to the average wage over the period 1976:1 – 2012:4, we can infer the levels of the firm’s price-marginal cost markup over that period as \( M_p = \frac{M}{M_L} \). The results are shown in Figure 3.

[FIGURE 3 NEAR HERE]

The time series for ratio of the marginal wage to the average wage (\( \hat{M}_L \)) shows very little variation although it tends to rise in expansions and fall in contractions and is pro-cyclical\(^{20}\) as reported by Bils (1987) and Nekarda & Ramey (2013). This evidence is consistent with the notion that it is plausible to assume “that the elasticity of labour supply decreases as the hours hired by the firm increase” (Rotemberg and Woodford (1999 p 1070)). Further, Nekarda and Ramey (2013) note that the variation in the ratio of the marginal wage to the average wage “is so small that it is unlikely to change the cyclicity of the [price] markup” (ibid, p 21). Thus the main source of the cyclicity of \( \hat{M} \) over the period is the cyclicity in the price-marginal cost markup (\( \hat{M}_P \)) and it is hence weakly pro-cyclical.

Figure 3 also shows the evolution of the implied value of the firm’s price-marginal cost mark-up (\( \hat{M}_P \)) over the period (1976–2012). It peaks during expansions, reaching a minimum at peaks or near the mid-point of contractions. As is the case over the longer period (1947 – 2016), the level of firm’s market power (\( \hat{M} \)) is weakly pro-cyclical.

While some (eg Bils (1987) and Rotemberg and Woodford (1999)) have argued that the price-marginal cost markup is counter-cyclical others have tended to find procyclical or acyclical markups. For example, Nekarda and Ramey (2013) present evidence that the markup is not counter-cyclical but is instead “procyclical or acyclical” (ibid, p 2). The contemporaneous correlation between the rate of growth in \( \hat{M}_P \) and the GDP growth rate is -0.064 while the correlation between the (HP filtered) deviations from the logarithmic trend in \( \hat{M}_P \) and deviations from the logarithmic trend in GDP is -0.064. The Harding-

\(^{20}\) The contemporaneous correlation between the rate of growth in \( M_L \) and the GDP growth rate is 0.413 while the correlation between the (HP filtered) deviations from the logarithmic trend in \( M_L \) and deviations from the logarithmic trend in GDP is 0.786. The Harding-Pagan measure of the degree of concordance (I) between the two growth rates is (I =) 0.544. All three measures indicate that \( M_L \) is pro-cyclical.
Pagan (2002) measure of the degree of concordance (I) between the two growth rates is \( I = 0.551 \). All of which is to say that some measures indicate that \( \hat{M}_p \) is essentially a-cyclical while the Harding-Pagan measure suggests that it is (weakly) procyclical. Having said that, all of the measures indicate it is not counter-cyclical and that it might be described as weakly procyclical or a-cyclical.

Turning to the behaviour of the trend in market power (\( \hat{M} \)) over the period, we see that the level of market power was tending to rise slowly over the period until the early 2000’s when it begins to rise sharply. We conjecture that the firm’s markups would likely be rising, at least since the 1980’s as “industrial concentration in the US began to increase in the early 1980s in most sectors” (see Pryor (2001 p 317) and Council of Economic Advisers (2016)) possibly due to a relaxation of anti-trust activity and the associated boom in mergers (see Pryor (2002, p 185); Krugman (2016) and Stiglitz (2016, 2017)). Recent papers by Autor et al (2017a; 2017b), Barkai (2016), Bessen (2017), Döttling et al (2017) and Grullon et al (2017), amongst others, report rising concentration ratios and rising market power for the US. Autor et al find that there is a clear upward trend in industry concentration over the period 1982 – 2012 “across the vast bulk of the US private sector” (Autor et al (2017a p 3)). Furthermore, Autor et al (2017b, Figure 1) indicates that much of the increase occurs after the mid-90s while Barkai (2016, Table 2) shows concentration rising markedly over the period 1997–2012. Döttling et al (2017) find that “the increase in concentration in the US is evident in the average Herfindahl, which has been trending upwards since the early 2000s” (ibid, p 150). Grullon et al (2017) find that “since the beginning of the 21st century … [US] product markets have undergone a structural change that is transforming the nature of competition. Markets have become more concentrated … and the increase in industry concentration levels correlates with remaining firms generating higher profit margins (p 31).

The firm’s market power and the ‘Labor Wedge’. There is a close association between the concept of the firm’s market power and equivalent concepts in the literature on the labor wedge. More specifically, the labor wedge or ‘inefficiency gap’ (see Gali et al (2007, p 44)), is defined as “the gap between the firm’s marginal product of labor and the households marginal rate of substitution” (ibid). The focus of the literature on the labor wedge has
been on its size and cyclicality and the relative contributions of households and firms to the total size of the wedge or ‘inefficiency gap’.  

While our measure of the firm’s market power is not identical to the total amount of the labor wedge or inefficiency gap, we can provide estimates of the firm’s contribution to the gap. We will adopt the approach in Karabarbounis (2014) and Shimer (2009) and measure the firm’s contribution to the gap as: $\ln\left(\partial Y/\partial L\right) - \ln\left(W/P\right)$.  

If we multiply both sides of (4) above by $(Y/L)$, we have $(W/P) = (1/M)\left(\partial Y/\partial L\right)$, indicating that we may estimate the firm’s contribution to the gap as the logarithm of $M$. Some authors (eg Gali et al (2007)) use the logarithm of the inverse of labor’s share as a measure of the firm’s contribution to the gap. However since $(1/S) = (M/E)$, and $E$ is less than 1, it follows that $(1/S)$ will be larger than $M$. Thus the firm’s contribution to the gap will be smaller if it is measured using the logarithm of $M$ than if it is measured by the (logarithm of) the inverse of labor’s share. All of which is to say that labor’s share is a biased measure of the firm’s contribution to the gap and that the bias is such that the use of labor’s share will tend to overstate the contribution of the firms to the gap, the total size of the gap and the proportion of the total gap attributable to firms.  

There also appears to be general agreement that the total size of the gap (ie the labor wedge including the contributions of firms and households) is counter-cyclical. For example: there is “a sharp increase in the labor wedge during every recession” (Shimer, 2009, p 287); “business cycles may involve significant efficiency costs” (Gali et al, 2007, p 57) and this is because markups are countercyclical, while Karabarbounis (2014, p 207) writes “[t]he labor wedge is volatile over the business cycle and countercyclical”. An obvious question is to ask if the firm’s contribution is also countercyclical? The series for the logarithm of $M$ suggests that it is not counter-cyclical as it rises near the mid-point or late in recessions and falls near the mid-point of expansions. The simple (contemporaneous) correlation between the growth rate of $M$ and the GDP growth rate is s 0.222 while the correlation


22 Note that Gali et al (2007) define the firms contribution to the gap in negative terms as: $\ln\left(W/P\right) - \ln\left(\partial Y/\partial L\right)$.
between the (HP filtered) deviations from the logarithmic trend in $\dot{M}$ and deviations from the logarithmic trend in GDP is 0.403. The Harding-Pagan (2002) measure of concordance for the two growth rates is (I =) 0.534. All of the measures indicate that $\dot{M}$ is (weakly) procyclical.

**Market Power, Elasticities and the Wage Share over time.** Drawing upon our decomposition of the wage share into a component reflecting the firm’s market power and a component reflecting the elasticity of output with respect to labor input (see equation (4) above), we are able to make inferences about the relative roles of market power and the elasticity in the determination of the wage share. We discuss the cycles and the trend in the wage share separately.

In relation to cyclical features of labor’s share, it is clear from Figure 2 that the share “tends to rise late in expansions and to fall late in recessions” (Rotemberg and Woodford, 1999, p 1061)). The fluctuations in labor’s share clearly mirror (i.e are the inverse of) the fluctuations in market power ($\dot{M}$) shown in the top panel where we see that $\dot{M}$ tends to fall in expansions and rise during recessions. The simple correlation between the growth rate of labor’s share and the growth rate of $\dot{M}$ over the whole of the period (1947–2016) is -0.991 while the correlation between the (HP filtered) deviations from the logarithmic trend in labor’s share and deviations from the logarithmic trend in $\dot{M}$ is -0.995. Clearly, the fluctuations in labor’s share are very closely associated (inversely) with fluctuations in the firm’s market power.

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23 The simple (contemporaneous) correlation between the growth rate of labor’s share and the GDP growth rate over the whole of the period is -0.177 while the correlation between the (HP filtered) deviations from the logarithmic trend in labor’s share and deviations for the logarithmic trend in real GDP is -0.123. The Harding-Pagan measure of concordance for the growth rates is (I =) 0.458. All of the measures indicate that the wage share is (weakly) counter-cyclical.

24 The Harding-Pagan measure of concordance for the growth rates is (I =) 0.040. All of the measures indicate a close inverse relationship between the firm’s market power and labor’s share and this close inverse relationship holds for the cycles as well as the trends in the two variables.

25 The reason why market power and labor’s share is highly correlated is because the elasticity shows relatively little variation and this is because the variable that influences the determination of the elasticity, namely, the observed variation in the growth of the capital-labor ratio, is small compared with the observed
If we turn now to the trends in labor’s share it would appear that there are two distinct periods. For the period up until the early 2000’s, the series appears to be cyclical around a constant, but after 2000 the series appears to be cyclical around a negative trend. Again, drawing upon our decomposition of the wage share into a component reflecting the firm’s market power and a component reflecting the elasticity (see equation (4) above), it would appear that prior to 2000 the very slight trend increase in the elasticity of output with respect to labor input (E) which, ceteris paribus, would tend to raise labor’s share appears to have been offset by the very slight trend decrease in the firm’s market power which, ceteris paribus, would tend to lower labor’s share. As a consequence there is no marked trend in the wage share prior between 1947 and 2000. In other words, the constancy of labor’s share over the second half of the 20th Century appears to have been the result (accidental or otherwise) of two offsetting forces. However, since then there has been a marked fall in the wage-share, a rise in the firm’s market power, but a relatively constant elasticity. This suggests that the fall in the wage share since the early 2000’s is likely due to a rise in the market power of firms, a finding which is consistent with Barkai (2016), De Loecker & Eeckhout (2017), Gutierrez (2017) and Kurz (2017), amongst others, who report rising market power (rising markups) and see this as the explanation for the falling wage share over the period.

26 The Perron unit root break test statistics (-4.177) shows that the null of unit root with a break in trend (at 1999:3) could not be rejected.

27 This also explains the apparent ability of the aggregate Cobb-Douglas production function to appear to perform well over the period. As Franklin Fisher (and others) have pointed out: “… in economies in which labor’s share happens to be roughly constant, even though the true relationships are far from yielding an aggregate Cobb-Douglas, such an aggregate production function will yield a good explanation of wages”. “The point … is that an aggregate Cobb-Douglas will continue to work well so long as labor’s share continues to be roughly constant, even though that rough constancy is not itself a consequence of the economy having a technology that is truly summarized by an aggregate Cobb-Douglas”. (Fisher, 1971, p 306f). The point we are making is similar to that of Fisher. It is ironic that the Cobb-Douglas would appear to work well simply because of a failure of two of its assumptions (perfect competition and a constant elasticity) to apply in practice.

28 Our finding that variations in the capital/labor ratio do not explain the observed movements of labor’s share is consistent with Elsby et al (2013), Glover & Short (2017), Oberfield & Raval (2014) and Rognlie (2015), amongst others. A common argument is that one cannot reconcile a falling wage share with a rising capital-labor ratio unless the elasticity of substitution is implausibly high.
IV. ESTIMATES OF TFP GROWTH

A. Growth Accounting and the Bias

Our estimate of the elasticity of output with respect to labour input can also be used to provide estimates of TFP growth. The growth accounting approach to the measurement of TFP starts with a general production function \( Y = AF(L, K) \) where \( Y \) is output, \( A \) is total factor productivity, and \((L, K)\) are the factor inputs (labor and capital). Note that time subscripts have been dropped to reduce the amount of notation. Expressing the function in logs and taking derivatives yields the discrete form as:

\[
\Delta \ln Y = \Delta \ln A + \frac{\partial Y}{Y} \Delta \ln K + \frac{\partial Y}{Y} \Delta \ln L
\]

TFP growth \( \Delta \ln(A) \) is then derived by subtracting the weighted sum of the rates of growth in capital and labor from the rate of growth in output, where the weights are the relevant production function elasticities, that is (assuming constant returns to scale):

\[
\Delta \ln A = \Delta \ln Y - ((1 - E) \Delta \ln K + E \Delta \ln L)
\]

where again, \( E = (\partial Y/\partial L)/(Y/L) \).

Since estimates of the elasticities are not readily available, it is common practice to use the measured wage share \((S)\) as an estimate of the elasticity of output with respect to labor input \((E)\), giving an estimate of TFP growth as:

\[
\Delta \ln \hat{A} = \Delta \ln Y - ((1 - S) \Delta \ln K + S \Delta \ln L)
\]

Subtracting (11) from (12) shows that the bias due to differences between the wage share and the elasticity will equal:

\[
(\Delta \ln \hat{A} - \Delta \ln A) = (S - E)(\Delta \ln K - \Delta \ln L)
\]

Equation (13) shows that the extent of the bias in any period depends on the difference between the measured wage share and the elasticity of output with respect to labor input, as well as on the rate of change in capital relative to the rate of change in labor input over the period of interest.
Given the decomposition of the wage share \( S = \frac{1}{M} (E) \) we may re-write the expression for the bias (13) as:

\[
\left( \Delta \ln \hat{A} - \Delta \ln A \right) = E \left( \frac{1}{M} - 1 \right) \left( \Delta \ln K - \Delta \ln L \right)
\]

Equation (14) identifies the source of the potential bias which results from the common practice of computing TFP using the wage share. The first point to note is that the bias will be zero when \( M = 1 \) (the case when the wage share is equal to the elasticity of output with respect to labor input - this will hold in the absence of market power) and/or the case when the rates of growth of capital and labour are identical. In general though, the underlying determinants of the sign, size and dynamics of the bias depends on the size of the elasticity \((E)\), on the firm’s market power \( (M) \) and also on the relative growth rates of capital and labor over the period. Given that the elasticity of output with respect to labor input \( E \), is always positive while, as we have seen above, the estimated value of \( M \) is always greater than one, the term \( E \left( \frac{1}{M} - 1 \right) \) in equation (14) will always be negative. Consequently, the sign of the bias in (14) will be negative if \( K \) grows faster than \( L \) and positive if \( L \) grows faster than \( K \), while the size of the bias will depend on the values of \( E, M \) and the relative growth rates of capital and labor.

A few propositions follow. First, even without generating new estimates of TFP growth we can infer that bias exists as the estimated values of the elasticity of output with respect to labor input \( E \) are different to (and higher than) the wage share in every period (since we find that the estimated measure of the firm’s market power \( \hat{M} \) is greater than one throughout the sample period).

Second, over long periods, the trend of \( \Delta \ln K - \Delta \ln L \) is positive and as a consequence over long periods the bias on the LHS of (14) ie \( \left( \Delta \ln \hat{A} - \Delta \ln A \right) \) will be negative. We thus hypothesize that the use of the wage share instead of the elasticity will most likely
result in estimates which understate the true rate of TFP growth given that there will be a trend increase in the capital-labor ratio over time.\textsuperscript{29}

Third, over the cycle, depending upon the dynamic behaviour of M, the wage share and the elasticity might conceivably move in different directions. The bottom panel of Figure 2 shows that the elasticity and the wage share can move in different directions for lengthy periods (the simple correlation between the two series is very low, at \( r = -0.073 \)). Thus the reported direction of change in TFP using the conventional measure might be misleading so that TFP could conceivably appear to be (say) falling when it is in fact rising or appear to be rising faster (or slower) than it is. This implies in turn that conventional estimates of TFP in the US over the period 1948–2016 are not only possibly biased but are possibly biased over long periods. Clearly, it is of interest to not only generate a revised series for TFP growth but also to estimate the bias so as to ascertain its size, persistence and cyclicality.

\textbf{B. The behaviour of the Bias in the Estimates of TFP growth over time}

Figure 4 shows the extent of the bias in the conventional measure of TFP growth for each quarter over the period 1947:2 – 2016:3. As before, the shaded regions in the Figure show contractions as designated by the NBER. We see that the bias is (as foreshadowed above) predominantly negative – i.e., the value of TFP weighted by the wage share is less than TFP weighted by the elasticity - with sharp downward spikes in contractions, while sharp upwards spikes occur mainly during periods of recoveries.\textsuperscript{30} The degree of persistence in the bias is not high (the first-order auto-correlation coefficient for the bias is 0.55).

\textsuperscript{29} De Loecker & Eeckhout (2017) and Kurz (2017) also find that productivity growth is under-estimated if perfect competition is assumed.

\textsuperscript{30} Kurz (2017) has estimated the bias for the Non-Financial Business Sector over the period 1990-2015 using a different method to ours (note that our estimates are for the Business Sector as a whole). Kurz sets out his results on page 35 of the paper. For the period 2000-2015 his estimates of the bias are in close agreement with ours, once we allow for the difference in sign of the reported bias (what we record as a negative value he records as a positive value). Over the period 2000-2015 Kurz finds the mean error was 0.225, our estimates put it at -0.294; he finds that the largest error is for 2009 when it is 0.876, it is also our largest error, we estimate it to be -0.926; and; he finds that the error exceeds 0.2% in nine of the sixteen years, our estimate of the error ‘exceeds’-0.2% in eight of the sixteen years. Whilst the two sets of estimates are similar for the period 2000-2015, our estimates for the bias in the 1990s (which average -0.303) are greater than Kurz’s estimates (which average 0.088).
The bias in the TFP growth series is clearly pro-cyclical vis a vis the GDP growth rate (the contemporaneous correlation coefficient is 0.640). Over the cycle the bias tends to be negative (and large) in contractions (when L is falling relative to K) and positive in recovery episodes (when L is rising relative to K). This effect tends be strengthened by the pro-cyclical movement in the firm’s market power mentioned above as this leads to the term \((1/M − 1)\) in equation (14) becoming more negative in recoveries and becoming less negative in contractions.

Table 2 shows estimates of the average values of the wage-share weighted measure of TFP growth, the elasticity-weighted measure of TFP growth and the average bias for NBER contractions (peak to trough) and expansions (trough to peak). The sample period is 1947:2 –2016:3, the first NBER dated peak is 1948:4 and the latest trough is 2009:2. Consequently the table reports averages for each of the contractions and expansions over the period 1948:4–2009:2. The average biases are consistently negative and ‘greater’ in contractions (peaks to troughs) than in expansions (-0.64 percent p.a. compared to -0.19 percent p.a. respectively).

Note too that during expansions, both approaches showed that TFP growth was positive, but during contractions, there were times when the two approaches give opposite signs for the rate of TFP growth (1948:4–1949:4, 1957:3–1958:2 and 2001:1–2001:4). To understand what is happening, consider the results for 2001, when the wage-share weighted approach showed that TFP growth was negative (-0.10 percent p.a.), whilst the elasticity-weighted approach showed that TFP growth was positive (+0.57 percent p.a.). In 2001, output fell, labor input also fell but capital grew. In that year, the larger elasticity weight on the positive effect associated with the growth in the capital-labor ratio out-weighed the negative effect associated with a fall in the growth of the output-capital ratio (see equation (11) rewritten as \(Δ\ln A = (Δ\ln Y − Δ\ln K) + E(Δ\ln K − Δ\ln L)\)).

To abstract from variations in the bias over different phases of the business cycle, we show in Table 3, average values of the wage-share weighted and elasticity weighted measures of
TFP growth together with the bias measured for the periods covering NBER dated peak-to-peak and for trough-to-trough.

[TABLE 3 NEAR HERE]

Table 3 highlights two results. First that the underestimation of TFP growth persisted over the business cycle by, on average, as much as a quarter of one percent p.a. (ranging from -0.13 to -0.43 percent p.a.). This is not negligible when compared to an average labor-income weighted measure of TFP growth of about 1.27 percent. Second, over the cycle 1980:3–1982:4, the traditional approach estimated a fall in TFP (growth of, on average, negative 0.39 percent p.a.) unlike the elasticity-weighted approach which estimated a rise in TFP (growth of, on average, 0.05 percent p.a.). This can again be understood by looking at the rates of growth of output, capital and labor in that period. The average growth rate of output, while positive, is low (an average of 0.74 of 1 percent p.a.) while at the same time the capital stock was growing much faster than labor (3.91 percent p.a. compared with the average growth rate of labor input which was 0.018 percent p.a.). Since the weight on capital growth using the wage share approach is higher than that using the estimated elasticity approach (0.317 compared to 0.203), it follows that the contribution to output growth of the two factor inputs will be larger in the conventional approach; conversely the attribution to TFP growth will be lower.

Table 4 presents wage-share weighted TFP growth estimates and elasticity weighted estimates over longer periods of time. The selected years reported are based on the long periods used by the BLS in reporting average rates of TFP growth.

[TABLE 4 NEAR HERE]

Over the whole of our sample period (1948–2015) we estimate that TFP has grown at an average of 1.45 percent p.a. while the wage share-weighted estimate of TFP growth was, on average, 1.21 percent p.a. In other words, we estimate TFP growth to have been, on average, 0.24 percent p.a. more than average TFP growth as measured using labor’s share over the period.

Fernald’s data shows that, since the beginning of the 21st century, the average growth of US GDP, capital and labor was 2.17 percent p.a., 1.58 percent p.a. and 0.90 percent p.a.,
respectively. Over this same period, TFP growth averaged 0.75 percent p.a. according to the traditional measure, whereas we estimate it to have averaged 0.99 percent p.a. The underestimation of TFP growth (ie the bias) is on average, about 0.24 percent p.a., this being close to one-third of the average TFP growth as measured using labor’s share over the period.

Our estimates, like those in Fernald (2014) and Cette et al (2016), show that TFP growth was slowing before the Great Recession. However, for the period 2007–2015, we estimate TFP growth to be 0.15 percent p.a. higher than the estimate of 0.34 percent p.a. arrived at using the wage share weighted approach.

C. Market Power and the Procyclicality of TFP growth

Our estimated series for TFP growth, like the traditional TFP growth measure, is procyclical with GDP growth. It tends to be low in contractions, and high in recoveries, especially early in recoveries. The correlation coefficient between Fernald’s TFP growth rate and the GDP growth rate is 0.826 while the correlation of the TFP growth rate computed using our estimates of the elasticity and GDP is lower (that is, less procyclical), at 0.772. This result comes about because our estimated the firm’s market power is greater than one and is pro-cyclical; in particular \( M \) rises in recoveries (alternatively the term \( (1/M – 1) \) becomes more negative) thus increasing the size of the bias. Hall (1987) also found that when productivity shifts are measured taking market power into account the correlation falls, although the resultant series “are still quite cyclical” (1987, p 422). In other words, allowing for departures from competition (and in particular allowing for a rise in firms’ markups during recoveries) has reduced the pro-cyclicality of TFP growth.

Since the beginning of 2000, the correlation between growth in GDP and growth in TFP growth has fallen to 0.756 and 0.678 according to the wage-weighted and elasticity-weighted measures of TFP growth respectively (from their respective correlations of 0.841 and 0.792 for the period 1947:2–1999:4). This has important implications as a presumed strong correlation between TFP and GDP have underpinned studies about the propagation
of real business cycles. Our result highlights the significance of other factors/shocks as propagators of GDP.\textsuperscript{31}

V. CONCLUDING REMARKS

In this paper we have proposed an approach to investigate the nexus between labor’s share, the market power of firms and the elasticity of output with respect to labor input. Our approach utilised the Kmenta-Nelson approximation to the CES function and state-space modelling to estimate a neoclassical model of the time-varying relationship between these three variables. Application of the approach for the US business sector over the period 1947:1–2016:3, yielded time-varying estimates of markups and elasticities.\textsuperscript{32} We have used these estimates to draw inferences about the nature of the declining labour share since the early 2000s, the contribution of the firm’s market-power to the labour wedge and the bias inherent in current estimates of TFP growth.

First, in relation to the behaviour of the wage share and its two components - firm’s market power and the elasticity of output with respect to labor input. Up to the early 2000’s, our estimates suggest that changes in the elasticity have been offset by changes in the firm’s market power with the result that there was no marked trend change in the wage share over that period. However post 2000, we find that the marked fall in the wage-share is associated with a marked rise in the market power of firms, as the elasticity is relatively constant.

Second, our results show that the level of the firm’s market power is (weakly) pro-cyclical and not counter-cyclical. We also find that the use of the (logarithm of the inverse) of labor’s share to estimate the firm’s contribution to the labor wedge is a biased measure of the firm’s contribution to the gap and that the bias is such that the use of labor’s share will

\textsuperscript{31} See Cardarelli and Lusinyan (2015) for a recent study which considers the relative roles played by information technology, efficiency or market dynamism, educational attainment and R&D spending.

\textsuperscript{32} Some caveats: In arriving at the results we have used a simple model where technological change is (Hick’s) neutral and dis-embodied. We further assume a CES production function with two inputs (labor and capital) and following Fernald, that both labour and capital are over (or under) utilised in the same proportion. Relaxation of these assumptions may affect the results, but they are topics for future research.
tend to *overstate* the contribution of the firms to the gap, the total size of the gap and the proportion of the total gap attributable to firms.

Third, we have used our estimates of the elasticity to generate a revised series for Total Factor Productivity growth for the USA. Our elasticity weighted TFP growth calculation puts a larger weight on (the usually slower growing) labor input than if the labor share were used as the weight and, as a result, attributes less of GDP growth to the growth in inputs and more to technological change. It would appear that use of the measured wage share as a proxy for elasticity results in underestimation of TFP growth\(^{33}\) by as much as a quarter of 1% per annum over the NBER business cycles with the bias being much larger in contractions than in expansions (-0.64 percent p.a. on average compared to -0.19 percent p.a. respectively). We show that conventionally measured TFP growth is pro-cyclical and that the bias due to market power is also pro-cyclical. As a result, allowing for departures from competition (and in particular allowing for a rise in firms’ markups during recoveries) reduces the pro-cyclicality of TFP growth. The presence of a consistent negative bias over the NBER business cycles implies that the conventional measure of TFP growth is understating the ‘true’ rate of TFP growth. It also suggests that TFP shocks explain less of the cyclicality in the GDP than previously thought.

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\(^{33}\) De Loecker & Eeckhout (2017) and Kurz (2017) also find that productivity growth is under-estimated if perfect competition is assumed.
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Rognlie, M. “Deciphering the fall and rise in the net capital share: accumulation or scarcity?” *Brookings Papers on Economic Activity*, 2015 (Spring), 1-54.


Solow, R. “Comment on Deciphering the Fall and Rise in the Net Capital Share: Accumulation or Scarcity?” *Brookings papers on economic activity*, 2015(Spring), 59-65.
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FIGURE 1


Wage Share: annualised growth rate (per cent)

Capital Labour Ratios: annualised growth rate (per cent)
FIGURE 2
Estimates of the Firm’s Market Power ($M$), the Elasticity of Output with Respect to Labor Input ($E$) and Actual Wage Share ($S$)
FIGURE 3

Note: The estimates of monopsony power are the ratios of marginal to average wage provided in Nerkada and Ramey (2013).

FIGURE 4
The Bias in the Estimate of TFP: 1947:2 – 2016:3
### TABLE 1

Estimated coefficients

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>0.873 (0.028)</td>
<td>0.875 (0.028)</td>
<td>0.871 (0.026)</td>
<td>1</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>0.931 (0.082)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.816 (0.038)</td>
<td>0.827 (0.038)</td>
<td>0.846 (0.036)</td>
<td>0.826 (0.038)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.024 (0.014)</td>
<td>0.014 (0.004)</td>
<td>0.014 (0.005)</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td></td>
<td></td>
<td></td>
<td>0.030 (0.014)</td>
</tr>
</tbody>
</table>

Log-likelihood

-346.280   -364.472   -348.872   -373.429

* p-values in parentheses
<table>
<thead>
<tr>
<th>Peaks-troughs</th>
<th>Duration</th>
<th>TFP (weighted by the wage share)</th>
<th>TFP (weighted by the elasticity)</th>
<th>Bias*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1948:4-1949:4</td>
<td>5</td>
<td>-0.51</td>
<td>0.24</td>
<td>-0.75</td>
</tr>
<tr>
<td>1953:2-1954:2</td>
<td>5</td>
<td>-1.05</td>
<td>-0.57</td>
<td>-0.48</td>
</tr>
<tr>
<td>1957:3-1958:2</td>
<td>4</td>
<td>-0.72</td>
<td>0.11</td>
<td>-0.83</td>
</tr>
<tr>
<td>1960:2-1961:1</td>
<td>4</td>
<td>-2.50</td>
<td>-2.20</td>
<td>-0.30</td>
</tr>
<tr>
<td>1969:4-1970:4</td>
<td>5</td>
<td>-0.62</td>
<td>-0.01</td>
<td>-0.62</td>
</tr>
<tr>
<td>1973:4-1975:1</td>
<td>6</td>
<td>-2.62</td>
<td>-1.98</td>
<td>-0.64</td>
</tr>
<tr>
<td>1980:1-1980:3</td>
<td>3</td>
<td>-2.84</td>
<td>-2.19</td>
<td>-0.65</td>
</tr>
<tr>
<td>1981:3-1982:4</td>
<td>6</td>
<td>-1.87</td>
<td>-1.31</td>
<td>-0.56</td>
</tr>
<tr>
<td>1990:3-1991:1</td>
<td>3</td>
<td>-2.38</td>
<td>-1.89</td>
<td>-0.49</td>
</tr>
<tr>
<td>2001:1-2001:4</td>
<td>4</td>
<td>-0.10</td>
<td>0.57</td>
<td>-0.67</td>
</tr>
<tr>
<td>2007:4-2009:2</td>
<td>7</td>
<td>-1.62</td>
<td>-0.76</td>
<td>-0.86</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td><strong>-1.50</strong></td>
<td><strong>-0.87</strong></td>
<td><strong>-0.64</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Troughs-peaks</th>
<th>Duration</th>
<th>TFP (weighted by the wage share)</th>
<th>TFP (weighted by the elasticity)</th>
<th>Bias*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1949:4-1953:2</td>
<td>15</td>
<td>3.49</td>
<td>3.59</td>
<td>-0.10</td>
</tr>
<tr>
<td>1954:2-1957:3</td>
<td>14</td>
<td>1.98</td>
<td>2.18</td>
<td>-0.20</td>
</tr>
<tr>
<td>1958:2-1960:2</td>
<td>9</td>
<td>2.80</td>
<td>2.83</td>
<td>-0.03</td>
</tr>
<tr>
<td>1961:1-1969:4</td>
<td>36</td>
<td>2.28</td>
<td>2.55</td>
<td>-0.27</td>
</tr>
<tr>
<td>1970:4-1973:4</td>
<td>13</td>
<td>2.25</td>
<td>2.43</td>
<td>-0.18</td>
</tr>
<tr>
<td>1975:1-1980:1</td>
<td>21</td>
<td>1.37</td>
<td>1.50</td>
<td>-0.13</td>
</tr>
<tr>
<td>1980:3-1981:3</td>
<td>5</td>
<td>2.30</td>
<td>2.59</td>
<td>-0.29</td>
</tr>
<tr>
<td>1982:4-1990:3</td>
<td>32</td>
<td>1.25</td>
<td>1.37</td>
<td>-0.12</td>
</tr>
<tr>
<td>1991:1-2001:1</td>
<td>41</td>
<td>1.25</td>
<td>1.48</td>
<td>-0.23</td>
</tr>
<tr>
<td>2001:4-2007:4</td>
<td>25</td>
<td>1.39</td>
<td>1.66</td>
<td>-0.27</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td><strong>1.81</strong></td>
<td><strong>2.01</strong></td>
<td><strong>-0.19</strong></td>
</tr>
</tbody>
</table>

*The Bias is the difference between Columns 2 and 3. It is calculated by comparing the measure of TFP growth weighted by the wage share (computed using Fernald’s data as: \(dy-(\alpha)\times dk-(1-\alpha)\times(dhours+dLQ)\)) with the measure of TFP growth weighted by our estimated elasticity of output with respect to labor input (computed as: \(dy-(1-\text{elasticity})\times dk-(\text{elasticity})\times(dhours+dLQ)\)).
### TABLE 3
Average Rates of TFP growth % p.a. Peak-to-Peak and Trough-to-Trough Cycles

<table>
<thead>
<tr>
<th>Peaks-Peaks</th>
<th>Duration</th>
<th>TFP (weighted by the wage share)</th>
<th>TFP (weighted by the elasticity)</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>1948:4-1953:2</td>
<td>19</td>
<td>2.78</td>
<td>3.02</td>
<td>-0.24</td>
</tr>
<tr>
<td>1953:2-1957:3</td>
<td>18</td>
<td>1.09</td>
<td>1.35</td>
<td>-0.26</td>
</tr>
<tr>
<td>1957:3-1960:2</td>
<td>12</td>
<td>1.63</td>
<td>1.88</td>
<td>-0.25</td>
</tr>
<tr>
<td>1960:2-1969:4</td>
<td>39</td>
<td>1.80</td>
<td>2.08</td>
<td>-0.27</td>
</tr>
<tr>
<td>1969:4-1973:4</td>
<td>17</td>
<td>1.79</td>
<td>2.06</td>
<td>-0.27</td>
</tr>
<tr>
<td>1973:4-1980:1</td>
<td>26</td>
<td>0.48</td>
<td>0.67</td>
<td>-0.19</td>
</tr>
<tr>
<td>1980:1-1981:3</td>
<td>7</td>
<td>0.39</td>
<td>0.82</td>
<td>-0.42</td>
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<tr>
<td>1981:3-1990:3</td>
<td>37</td>
<td>0.76</td>
<td>0.94</td>
<td>-0.18</td>
</tr>
<tr>
<td>1990:3-2001:1</td>
<td>43</td>
<td>1.07</td>
<td>1.31</td>
<td>-0.24</td>
</tr>
<tr>
<td>2001:1-2007:4</td>
<td>28</td>
<td>1.19</td>
<td>1.49</td>
<td>-0.31</td>
</tr>
<tr>
<td><strong>AVERAGE</strong></td>
<td></td>
<td>1.28</td>
<td>1.53</td>
<td>-0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Troughs-Troughs</th>
<th>Duration</th>
<th>TFP (weighted by the wage share)</th>
<th>TFP (weighted by the elasticity)</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>1949:4-1954:2</td>
<td>19</td>
<td>2.41</td>
<td>2.60</td>
<td>-0.19</td>
</tr>
<tr>
<td>1954:2-1958:2</td>
<td>17</td>
<td>1.39</td>
<td>1.73</td>
<td>-0.34</td>
</tr>
<tr>
<td>1958:2-1961:1</td>
<td>12</td>
<td>1.85</td>
<td>1.98</td>
<td>-0.13</td>
</tr>
<tr>
<td>1961:1-1970:4</td>
<td>40</td>
<td>2.04</td>
<td>2.35</td>
<td>-0.31</td>
</tr>
<tr>
<td>1970:4-1975:1</td>
<td>18</td>
<td>0.73</td>
<td>1.06</td>
<td>-0.33</td>
</tr>
<tr>
<td>1975:1-1980:3</td>
<td>23</td>
<td>0.89</td>
<td>1.07</td>
<td>-0.18</td>
</tr>
<tr>
<td>1980:3-1982:4</td>
<td>10</td>
<td>-0.39</td>
<td>0.05</td>
<td>-0.43</td>
</tr>
<tr>
<td>1982:4-1991:1</td>
<td>34</td>
<td>1.00</td>
<td>1.14</td>
<td>-0.14</td>
</tr>
<tr>
<td>1991:1-2001:4</td>
<td>44</td>
<td>1.19</td>
<td>1.45</td>
<td>-0.27</td>
</tr>
<tr>
<td>2001:4-2009:2</td>
<td>31</td>
<td>0.76</td>
<td>1.15</td>
<td>-0.39</td>
</tr>
<tr>
<td><strong>AVERAGE</strong></td>
<td></td>
<td>1.26</td>
<td>1.53</td>
<td>-0.27</td>
</tr>
<tr>
<td>BLS selected years</td>
<td>Duration (years)</td>
<td>TFP (weighted by the wage share)</td>
<td>TFP (weighted by the elasticity)</td>
<td>Bias</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------</td>
<td>----------------------------------</td>
<td>----------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1948-1973</td>
<td>26</td>
<td>2.05</td>
<td>2.32</td>
<td>-0.27</td>
</tr>
<tr>
<td>1973-1990</td>
<td>18</td>
<td>0.50</td>
<td>0.70</td>
<td>-0.20</td>
</tr>
<tr>
<td>1990-1995</td>
<td>6</td>
<td>0.61</td>
<td>0.75</td>
<td>-0.14</td>
</tr>
<tr>
<td>1995-2000</td>
<td>6</td>
<td>1.57</td>
<td>1.90</td>
<td>-0.33</td>
</tr>
<tr>
<td>2000-2007</td>
<td>8</td>
<td>1.21</td>
<td>1.55</td>
<td>-0.34</td>
</tr>
<tr>
<td>2007-2015</td>
<td>9</td>
<td>0.34</td>
<td>0.49</td>
<td>-0.15</td>
</tr>
<tr>
<td>1948-2015</td>
<td>68</td>
<td>1.21</td>
<td>1.45</td>
<td>-0.24</td>
</tr>
</tbody>
</table>