ISBN 0 7325 1211 5

THE UNIVERSITY OF MELBOURNE

THE EFFECT OF INTEREST ON NEGATIVE SURPLUS

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RESEARCH PAPER NUMBER 30
March 1996

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ABSTRACT

In the classical continuous time surplus process, we allow the process to continue if the surplus falls below zero. When the surplus is below zero, we assume that the insurer borrows any sums of money required to pay claims, and pays interest on this borrowing. We use simulation to study moments and distributions of three quantities: the time to recovery to surplus level zero, the number of claims that occur when the surplus is below zero, and the maximum absolute value of the surplus process when it is below zero. We also show how simulation can be used to estimate the probability of absolute ruin.

KEYWORDS

Ruin theory; severity of ruin; negative surplus; interest; absolute ruin; simulation.

1. Introduction and Notation

In the classical surplus process the insurer's surplus at time t is denoted $U(t)$ and given by

$$U(t) = u + ct - S(t)$$

where u is the insurer's surplus at time 0, c is the insurer's premium income per unit time, assumed to be received continuously, and $S(t)$ is the aggregate claim amount up to time t. $\{S(t)\}_{t\geq 0}$ is a compound Poisson process with Poisson parameter λ per unit time, and individual claim amounts have distribution function $P(x)$, with $k-th$ moment p_k, and we will assume that $P(0) = 0$. We will write $c = (1+\theta)\lambda p_1$ where $\theta > 0$ is the insurer's premium loading factor.

For this process, the time to ruin, denoted T, is defined as

$$T = \begin{cases} \inf\{t : U(t) < 0\} \\ \infty \text{ if } U(t) \geq 0 \text{ for all } t > 0 \end{cases}$$

The probability of ultimate ruin given initial surplus u is denoted $\psi(u)$, and defined by $\psi(u) = \Pr(T < \infty)$. The probability of survival is denoted $\delta(u)$ and defined as $\delta(u) = 1 - \psi(u)$. The probability that ruin occurs and that the deficit at the time of ruin is less than y is denoted $G(u, y)$ and defined as

$$G(u, y) = \Pr(T < \infty \text{ and } U(T) > -y)$$

We denote by $g(u, y)$ the (defective) density associated with $G(u, y)$ and define $\tilde{G}(u, y) = G(u, y)/\psi(u)$ and $\tilde{g}(u, y) = g(u, y)/\psi(u)$. We use the notation $Y(u)$ to denote the conditional distribution of the deficit at ruin, so that $Y(u)$ has distribution function $\tilde{G}(u, y)$.

In the following sections we shall let the surplus process continue if it falls below zero. We shall be interested in what happens during the time the surplus process takes to recover to zero.

We shall introduce a modification to the surplus process if the surplus falls below zero. We shall assume that if the surplus process falls below zero, the insurer has to borrow any sums of money required to pay claims. The insurer may borrow any sums required at force of interest δ per unit time (equivalent to an effective rate i per unit time where $\delta = \log(1 + i)$), and may repay amounts owed continuously over time. These repayments will be made using premium income. The introduction of interest into the model means that unlike in the case when $\delta = 0$, recovery to surplus level zero is no longer certain. The reason for this is that if the surplus falls below $-c/\delta$ at any time, the insurer would be unable to repay its borrowings since the rate of repayment would exceed the rate of premium income. If the surplus falls below $-c/\delta$ absolute ruin is said to occur. We will use the notation $\psi_A(u)$ to denote the probability of absolute ruin, and $\tilde{\psi}_A(u) = \psi_A(u)/\psi(u)$ denotes the probability of absolute ruin given that ruin occurs. We define the survival probability for this modified process as $\delta_A(u) = 1 - \psi_A(u)$.

Dassios and Embrechts (1989) consider this model and describe it in the following way. Let τ_i be the time of the $i - th$ claim. Then for $\tau_i \leq s < \tau_{i+1}$,

$$U(s) = U(\tau_i) + c(s - \tau_i)$$

if $U(\tau_i) \geq 0$ and

$$U(s) = \begin{cases} (U(\tau_i) + c/\delta)\exp\{\delta(s - \tau_i)\} - c/\delta & \text{for } s < s^* \\ c(s - \tau_i) & \text{for } s \geq s^* \end{cases}$$

if $-c/\delta < U(\tau_i) < 0$, where $s^* = \tau_i + (1/\delta)\log((c/\delta)/((c/\delta) + U(\tau_i))$. The definition of s^* is such that if no claims occur between times τ_i and s^* then the surplus at time $\tau_i + s^*$ will be zero.

Throughout this paper we will consider conditional random variables. We will condition on ruin occurring from initial surplus u and recovery to surplus level zero occurring (i.e. absolute ruin not occurring). We define the following conditional random variables:

- $T(u)$ denotes the time to recovery to surplus level zero.
- $N(u)$ denotes the number of claims that occur during the time to recovery to surplus level zero (not counting the claim that causes ruin).
- $L(u)$ denotes the maximum absolute value of the surplus process during the time to recovery to surplus level zero.

Our approach to studying these variables is to use simulation. We will illustrate some of their properties through the use of examples. We use two single claim amount distributions - exponential and Pareto - each scaled to have mean 1. For each of the examples in this paper we simulated 100,000 realisations of the surplus process given that ruin occurs, so that the starting point for each realisation was simulation of the insurer's deficit at the time of ruin given that the deficit is less than c/δ. This requires simulation from a distribution with distribution function $\tilde{G}(u,y)/\tilde{G}(u,c/\delta)$ for $0 \leq y \leq c/\delta$. In general this poses no problem when $u = 0$ since

$$\tilde{G}(0,y) = \int_0^y \frac{1-P(x)}{p_1} dx$$

Similarly, there is no problem when the individual claim amount distribution is exponential since $\tilde{G}(u,y)$ is independent of u in this case - see, for example, Bowers *et al* (1986) - and hence all quantities of interest are independent of u. When the individual claim amount distribution is Pareto and $u > 0$ we simulated values of $G(u,y)$ by first calculating this distribution numerically for $y = 0, 0.05, 0.1, ...$ using the method of averaging lower and upper bounds for $G(u,y)$ as described by Dickson *et al* (1995, Section 4). We then assumed that $\tilde{G}(u,y)/\tilde{G}(u,c/\delta)$ (which is the same as $G(u,y)/G(u,c/\delta)$) was a piecewise linear function over the intervals $[0.05k, 0.05(k+1))$ for $k = 0, 1, 2, ...$ allowing simulation from this distribution by standard techniques.

2. Preliminary Remarks

In the following sections we will study $T(u)$, $N(u)$ and $L(u)$ as functions of i. In Sections 3, 4 and 6 we will set $\lambda = 1$ and allow i to vary. In Section 5, we will study a relationship which depends on λ. In that section only, we fix i and let λ vary. We will study quantities such as $E(T(u))$ as a function of i. As a mathematical problem, it is of interest to see how such quantities behave as i varies. However, some of our observations will be of limited practical value. For example, if $\lambda = 1$ and $i = 0.1$, then we are saying that we expect one claim per unit of time and the effective rate of interest for that unit of time is 0.1. If the insurer expects, say, three claims a day, then the effective annual rate of interest given by our model is clearly not a realistic one! However, as we shall see in

the following sections, the features of interest in the functions we study occur for small values of i which would translate into realistic effective annual rates of interest for most insurance portfolios.

In the following sections we will make remarks such as "$E(T(u))$ is a decreasing function of i". What we really mean when we describe the behaviour of a function in such a way is that this is how the function behaves according to the results of our simulations. Since, with the exception of Example 1, our studies are all based on simulation, we believe that this abuse of language should cause no problems to the reader.

Finally, we remark that the graphs presented in this paper are, with the exception of Figure 1, based on simulation. The values of functions obtained by simulation have all been smoothed. This smoothing has been performed by the statistical package S-PLUS using the "supsmu" function. We have taken somewhat of a black box approach to smoothing. Our aim has been to illustrate the shapes of functions. Optimal smoothing is a side issue and has not been a consideration.

3. The Probability of Absolute Ruin

In this section we shall consider the probability of absolute ruin occurring before the surplus process can recover to zero given that absolute ruin does not occur at the time of ruin. Hence we are interested in the probability of some period of negative surplus which results in absolute ruin rather than recovery to surplus level zero. Let us denote this probability by $\phi(u)$. We can find $\phi(u)$ as follows. We have

$$\psi_A(u) = \int_0^{c/\delta} g(u,y)\psi_A(-y)dy + \int_{c/\delta}^{\infty} g(u,y)dy$$

or, equivalently,

$$\tilde{\psi}_A(u) = \int_0^{c/\delta} \tilde{g}(u,y)\psi_A(-y)dy + \int_{c/\delta}^{\infty} \tilde{g}(u,y)dy \tag{3.1}$$

Now let $\gamma(u,y) = \tilde{g}(u,y)/\tilde{G}(u,c/\delta)$ for $0 < y < c/\delta$, so that $\gamma(u,y)$ is the density of the deficit at ruin given that the deficit is less than c/δ. Then we have

$$\int_0^{c/\delta} \gamma(u,y)\psi_A(-y)dy = \frac{\tilde{\psi}_A(u) - 1 + \tilde{G}(u,c/\delta)}{\tilde{G}(u,c/\delta)}$$

and the integral is the probability of absolute ruin given that ruin has occurred with a deficit at ruin less than c/δ. Since absolute ruin can occur with or without an upcrossing through surplus level zero we have

$$\int_0^{c/\delta} \gamma(u,y)\psi_A(-y)dy = \phi(u) + (1-\phi(u))\psi_A(0)$$

and so

$$\phi(u) = 1 - \frac{1 - \tilde{\psi}_A(u)}{(1 - \psi_A(0))\tilde{G}(u, c/\delta)} \tag{3.2}$$

In Examples 1 and 2 we will study the probability $\phi(u)$. In Example 1 we calculate it from a formula while in Example 2 we estimate it by simulation. An important consequence of estimating $\phi(u)$ by simulation is that it allows us to estimate $\psi_A(u)$ by simulation as follows. From (3.2) we have

$$\delta_A(0)\tilde{G}(u, c/\delta)(1 - \phi(u)) = 1 - \tilde{\psi}_A(u)$$

and multiplying throughout by $\psi(u)$ we have

$$\delta_A(0)G(u, c/\delta)(1 - \phi(u)) = \psi(u) - \psi_A(u) = \delta_A(u) - \delta(u)$$

and so

$$\delta_A(u) = \delta(u) + \delta_A(0)G(u, c/\delta)(1 - \phi(u))$$

Setting $u = 0$ we find

$$\delta_A(0) = \frac{\delta(0)}{1 - G(0, c/\delta)(1 - \phi(0))}$$

giving

$$\delta_A(u) = \delta(u) + \frac{\delta(0)G(u, c/\delta)(1 - \phi(u))}{1 - G(0, c/\delta)(1 - \phi(0))}$$

Since algorithms exist to calculate the functions $\delta(u)$ and $G(u, y)$ (see, for example, Dickson *et al* (1995)), we can use a mixture of calculation and simulation to estimate $\delta_A(u)$, or equivalently, $\psi_A(u)$. This is a very useful result as we can estimate the probability of absolute ruin in infinite time by calculating $\phi(u)$ by simulation. Since the purpose of this paper is to study what happens during a period of negative surplus we shall not illustrate the use of this result.

Example 1: When $P(x) = 1 - \exp\{-\alpha x\}$, an explicit solution exists for $\psi_A(u)$ and hence for $\tilde{\psi}_A(u)$. Dassios and Embrechts (1989) show that for $u \geq 0$

$$\psi_A(u) = \frac{\psi(u)}{\psi(0)} \left(1 + \frac{c\alpha - \lambda}{c} \int_{-c/\delta}^{0} e^{-\alpha x} \left(1 + \frac{\delta x}{c} \right)^{(\lambda/\delta)-1} dx \right)^{-1}$$

Figure 1 shows values of $\phi(u)$ as a function of the interest rate, i, per unit time for three values of θ, namely 10%, 20% and 30%, when $\alpha = \lambda = 1$. This figure has all the features we would expect: as the interest rate increases, $\phi(u)$ increases, and as the premium loading factor increases, $\phi(u)$ decreases. □

In passing, we note the comment by Dassios and Embrechts (1989) that when $P(x) = 1 - \exp\{-\alpha x\}$, $\psi_A(u) = k\psi(u)$ where $0 < k < 1$. In fact $k = \psi_A(0)/\psi(0)$. It is easy to establish this relationship by writing (3.1) as

$$\psi_A(u) = \psi(u) \left(\int_0^{c/\delta} \tilde{g}(u, y)\psi_A(-y)dy + \int_{c/\delta}^{\infty} \tilde{g}(u, y)dy \right)$$

Since $\tilde{g}(u,y)$ is independent of u it follows that $\psi_A(u)/\psi(u) = \psi_A(0)/\psi(0)$. Although this approach gives us $\psi_A(u)$ in terms of $\psi_A(0)$, it does not appear to yield a solution for $\psi_A(0)$.

Example 2: Figure 2 shows simulated values of $\phi(u)$ as a function of the interest rate when the individual claim amount distribution is Pareto(2,1) for $u = 0, 5$ and 10 and $\theta = 0.1$ when $\lambda = 1$. This figure has the same features as Figure 1. In this case $\phi(u)$ does depend on u, and increases with u. □

4. The Duration of the First Period of Negative Surplus

In the special case when $\delta = 0$, the moments and distribution of $T(u)$ are discussed by Egídio dos Reis (1993) and Dickson and Egídio dos Reis (1995). In this section we will consider the density function, mean and variance of $T(u)$. Before considering examples, there is one important point to mention about the moments.

Egídio dos Reis (1993) shows that when $\delta = 0$,

$$E(T(u)) = \frac{E(Y(u))}{c\delta(0)}$$

and

$$V(T(u)) = \frac{E(Y(u))\lambda p_2}{(c\delta(0))^3} + \frac{V(Y(u))}{(c\delta(0))^2}$$

Thus the mean and variance of $T(u)$ exist when $\delta = 0$ if the mean and variance of $Y(u)$ exist. (We show in the Appendix that the first two moments of $Y(u)$ exist if $p_3 < \infty$.). However, when $\delta > 0$, given that ruin occurs but absolute ruin does not occur at the time of ruin, the moments of the distribution of the deficit at ruin must be finite. Thus it will be possible to study the moments of $T(u)$ when $\delta > 0$ in cases when it was not possible to study these moments when $\delta = 0$.

Example 3: Consider again the case when the individual claim amount distribution is exponential and $\lambda = 1$. Figure 3 shows $E(T(u))$ as a function of the interest rate for the same three values of θ as in Figure 1. For a given value of θ, this figure shows $E(T(u))$ increasing with i then decreasing to a limiting value which will be 0. To see why this happens, let us consider the effect of a small increase in i when i is close to zero. First, there is little change in $\tilde{G}(u,y)/\tilde{G}(u,c/\delta)$. Consequently the distribution of the deficit barely changes. Second, the probability of absolute ruin without recovery to surplus level 0 is also unlikely to change much due to the magnitude of c/δ and the magnitude of claims. The main effect will be an increase in recovery time to zero caused by a higher interest rate. Hence the value of $E(T(u))$ should increase. Intuitively this increase cannot be sustained as the value of i increases. The reason is that as i increases, the distribution of the deficit will change, with the expected deficit decreasing. Although an increase in

i causes an increase in the recovery times, it also causes a decrease in c/δ and hence an increase in the probability of absolute ruin without recovery to surplus level zero. This latter effect seems to outweigh the former and so $E(T(u))$ must at some stage have a turning point and eventually decrease to zero as i becomes very large. Figure 4 shows the standard deviation of $T(u)$, and this figure has the same features as Figure 3. □

Example 4: Let the individual claim amount distribution be Pareto(2,1) and let $\lambda = 1$ and $u = 0$. In this case we know that $Y(0)$ has a Pareto(1,1) distribution, and hence none of the moments of $Y(0)$ exists. Figure 5 shows $E(T(0))$ as a function of the interest rate for the same three values of θ as in previous figures. We can see a different pattern from Figure 3, with $E(T(0))$ being a decreasing function of i. In this case the explanation would appear to be that the deficit at ruin is responsible for the shape of $E(T(0))$. As i increases, the expected deficit decreases and hence the expected recovery time should decrease. (Note that this is different to the situation in Example 3. There, the expected deficit reaches a finite limiting value as i decreases to zero, but this does not occur in this example.) In this example the standard deviation of $T(0)$ is a decreasing function of i, as shown in Figure 6 which has the same features as Figure 5. □

The features of these two examples are reproduced for other forms of $P(x)$. For example, when the individual claim amount distribution is Pareto(4,3) the features of Example 3, rather than Example 4, were reproduced. In general, we found that the graph of $E(T(u))$ as a function of i had the same shape as in Figure 3 whenever $E(Y(u))$ was finite. Otherwise it had the same shape as in Figure 5. Similarly, we found that whenever $E(Y(u)^2)$ was finite, the graph of the standard deviation of $T(u)$ had the same shape as in Figure 4. Otherwise the standard deviation of $T(u)$ was a decreasing function of i.

We can also use simulation to construct the densities of $T(u)$. Figure 7 shows the densities of $T(0)$ for the individual claim amount distributions from Examples 3 and 4 when $\lambda = 1$, $i = 0.001$ and $\theta = 0.1$. Further experiments indicated that the basic shape of each density is the same for different values of i.

5. The Number of Claims in the First Duration of Negative Surplus

When $\delta = 0$ Gerber (1990) considers the number of claims, denoted K, that occur during the interval $(0, \tau)$, where τ is the first passage time of the surplus process, starting from initial surplus 0, through level x, where $x > 0$. Subject to a change of scale in the individual claim amount distribution Gerber proves that

$$E(K) = \lambda x / c\delta(0)$$

and

$$V(K) = \frac{\lambda x}{(c\delta(0))^3}\left(c^2 + \lambda^2(p_2 - p_1^2)\right)$$

By conditioning on the severity of ruin, we can use these results to show that

$$E(N(u)) = \lambda E(T(u)) \tag{5.1}$$

and

$$V(N(u)) = \lambda\left(\frac{1+\psi(0)}{\delta(0)}E(T(u)) + \lambda V(T(u))\right) \tag{5.2}$$

Our main purpose in this section is to investigate whether we can apply these identities to approximate $E(N(u))$ and $V(N(u))$ when $\delta > 0$.

Example 5: Let the individual claim amount distribution be exponential. Figure 8 shows $E(N(u))$ as a function of λ when $i = 0.1$. The solid lines shows values of $E(N(u))$ while the dotted lines shows λ times the values of $E(T(u))$. Figure 9 shows the corresponding quantities for the standard deviation of $N(u)$. We can see from Figure 8 that for all values of λ, and particularly for the larger values, (5.1) gives a reasonable approximation. However, it is quite a different story when we consider the standard deviation of $N(u)$. It is clear from Figure 9 that (5.2) does not give a good approximation to the standard deviation of $N(u)$ for small values of λ, but gives a reasonable approximation for large values. (Note that the scale in Figure 9 is very different to that in Figure 8.) Of course, another way of interpreting these figures is to say that increasing λ has the same effect as decreasing i. Hence, as λ becomes large, i becomes small, and so we would expect the quality of the approximations given by (5.1) and (5.2) to improve. □

Further experiments lead us to the conclusion that formulae (5.1) and (5.2) are inappropriate for calculating the mean and variance of $N(u)$ when $\delta > 0$ unless the value of λ is very large, or, equivalently, if for a given value of λ, the interest rate is small. Otherwise, the most reliable means of obtaining the moments (and distribution) of $N(u)$ is through simulation.

6. The Minimum Value of the Surplus Process

In this section we consider the absolute value of the minimum of the surplus process during the first period of negative surplus, which we denote $L(u)$. Picard (1994) considers the distribution of $L(u)$ and shows that when $\delta = 0$

$$\Pr(L(u) \le z) = \frac{\psi(u) - \psi(u+z)}{\psi(u)(1-\psi(z))}$$

If $P(x)$ is such that an analytical expression for $\psi(u)$ exists then it is possible to derive expressions for the moments of $L(u)$ when $\delta = 0$. Of particular interest to

our study are the results when $P(x) = 1 - \exp\{-x\}$. In this case we can show that

$$E(L(u)) = (1+\theta)\log(1+\theta^{-1})$$

and

$$E(L(u)^2) = \frac{2(1+\theta)^2}{\theta}\sum_{j=1}^{\infty}(1+\theta)^{-j}j^{-2}$$

For the remainder of this section we consider the moments of $L(u)$ when $\delta > 0$.

Example 6: Let the individual claim amount distribution be exponential with mean 1 and let $\lambda = 1$. Figure 10 shows $E(L(u))$ as a function of i for $\theta = 0.1, 0.2$ and 0.3. We can see that for each value of θ, $E(L(u))$ increases from its value when $i = 0$ to a maximum value then decreases to a limiting value which will be zero. The features which explain the shape of $E(T(u))$ also explain the features of $E(L(u))$. Consider a realisation of the surplus process when $i = 0$ for which the surplus falls below the deficit at ruin before recovering to zero. If we increase i by a very small amount, then the corresponding realisation of the surplus process must have a larger absolute minimum value since the rate of increase of the surplus process is reduced. Since absolute ruin is very unlikely for very small i, $E(L(u))$ must increase since, for all other realisations of the surplus process for which absolute ruin does not occur, the minimum level of the surplus process cannot be reduced by increasing the interest rate. However, as i increases, it is clear that the absolute ruin barrier dictates that $E(L(u))$ must eventually decrease as i increases. It is interesting to note that for each value of θ, the value of i that maximises $E(T(u))$ in Figure 3 is not the value that maximises $E(L(u))$ in Figure 10. Figure 11 shows the standard deviation of $L(u)$ as a function of i and this figure exhibits the same features as Figure 10. □

Example 7: Let the individual claim amount distribution be Pareto(2,1). Figure 12 shows $E(L(u))$ as a function of i when $\theta = 0.2$ for $u = 0, 5$ and 10. This figure has the same features as Figure 5 for exactly the same reasons. □

In our calculations for Section 4, we found that the shape of the mean and standard deviation of $T(u)$ as a function of i respectively depended on the existence of the first two moments of $Y(u)$. Our calculations in this section lead us to believe that the same is true of the mean and standard deviation of $L(u)$.

7. Conclusions

Simulation provides a means of estimating the probability of absolute ruin and studying the random variables $T(u)$, $N(u)$ and $L(u)$ when $\delta > 0$. Although it does not enable us to draw precise conclusions about these variables, it does allow us see how the moments and distributions of these variables change as δ increases. Our experiments lead us to conclude that moments of the individual

claim amount distribution are determining factors for the behaviour of moments of these three variables as functions of δ.

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