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Volatility, Beta and Return
Was there ever a meaningful relationship?

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Abstract

This paper deals with the relationship between volatility or, more correctly, β values and return within the equities asset class. The existence of such a relationship has been controversial in actuarial circles, but the assumption that a positive linear relationship exists is now taught as part of the curriculum.

Most of the empirical studies of the relationship between β values and mean return have used arithmetic means of 'discrete' rates of return. By contrast, many of the apparently contradictory studies of the relative performance of shares with low price/earnings ratios used continuous compounding, geometric means or their equivalent.

A great deal of the controversy hinges on the definition of mean return, which has not really been identified as an important issue.

If the predominant interpretation of 'mean return' is misleading or not meaningful for investment or asset modeling, then most of the empirical studies published in support of a positive relationship between β values and return may need to be re-evaluated before use in actuarial practice. Depending on the correct interpretation of the term 'mean rate of return' a meaningful relationship between β and return may have never existed.

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1 Introduction

1.1 The relationship between volatility and return, or, more correctly the relationship between a security's β value and its expected return has been a controversial issue. This relationship is an essential cornerstone of the so-called Capital Asset Pricing Model which has been one of the most dominant ideas in the Theory of Finance for three decades.

In practice, there are three broad areas of potential application of the Capital Asset Pricing Model:-

(a) the cross-section of returns – whether there is any relationship between β values and return within ordinary shares as an asset class,

(b) as a basis for asset allocation – whether there is any relationship between the volatility of an asset class and its average return, and

(c) project evaluation – the appropriate discount rate to use when assessing the viability of a project or new business venture.

1.2 This paper is primarily concerned with the first of these issues – whether there is a meaningful relationship between β and return within ordinary shares as an asset class. Such a relationship is a pre-requisite condition for the existence of an equity premium, or superior long-term return from shares as an asset class as a result of volatility aversion by capital market participants.

If there is no meaningful relationship between β and return within ordinary shares as an asset class there still may an equity premium, but it would not be related to risk aversion by portfolio investors. The assumption that such a premium will always exist for this reason then becomes suspect.

1.3 Within the actuarial profession there have been significant elements of dissent on the existence of any relationship between β values and return and the predictive power of β values. For example, it is apparent that a paper on *'Financial Economics'* by Clarkson (1996), led to a heated debate at the Faculty of Actuaries.

Some years earlier, when the idea of a relationship between β and return was beginning to emerge in Australia, Geddes (1974 p.74) used some strong language to express his reservations, effectively describing this new approach to security analysis as *'150,000% [nonsense]'*.

1.4 Judging by the recent adoption of the new UK 109 subject in *'Financial Economics'* as part of its Part I syllabus by The Institute of Actuaries of Australia, the dissenters views are not shared by those responsible for deciding the curriculum for trainee actuaries either in Australia or the UK. A valuable feature of the UK 100 series subjects is their series of detailed 'Core Readings'. These were not intended to be text books, but rather to demonstrate the depth and breadth of the knowledge required of students.

To the extent that The Institute and Faculty of Actuaries (2000) 'Core Reading' for UK Subject 109 can be assumed to be an official actuarial view of the relationship between volatility and return, it contains the following statement (Unit 4 p.6):

> *'A powerful result that can be derived ... is that a linear relationship exists between the expected return on individual securities and their so-called "β factors".'*

1.5 The UK Subject 109 Core Reading acknowledges some limitations of this theory as does The Institute of Actuaries of Australia (2000) in its reading material for the Australian Part III (Fellowship) Investment Management subject.

However there remains an *a-priori* assumption that a positive relationship between β values and investment is still valid and useful.

Consider, for example, the following commentary which appears in the course notes for Part III Subject 1 (Fellowship Investment Management) of The Institute of Actuaries of Australia. As this forms part of formal reading material, it should be reasonable to assume that the views expressed are consistent with those held by the majority of actuaries who are investment practitioners in Australia:

> *'[Notwithstanding its limitations], CAPM is still the starting point for a wide variety of applications. For instance it gave us the important insight that the risk premium on an asset is dependent on its covariance with the market portfolio rather than its pure variance. This is used heavily in portfolio construction and in monitoring portfolio risk.'*

1.6 In addition to the UK Subject 109 Core Reading and the Australian Part III Fellowship Investment Course, it should be recognised that many Australian actuaries will achieve an exemption from subject 109 through courses taught by non-actuaries.

Hence it should be noted that the existence of a positive relationship between β values and return is apparent in most standard tertiary texts on investment. For example, Brailsford and Heaney (1998) is a recognised Australian undergraduate text in Finance. In relation to this issue the authors wrote:

> *'The Capital Asset Pricing Model ... is important for its simplicity and the far reaching effect it has on the study of finance. It is included in*

virtually all finance courses and is used for many tasks in financial analysis both by practitioners and academics.'

After discussing the conflicting views and evidence, Brailsford and Heaney subsequently conclude:

'The linear relationship predicted in the CAPM between [β values] and expected return generally holds.'

1.7 There are wider issues involved in this topic such as the use of volatility to describe risk. However it is intended in this paper to concentrate on one crucial point, namely the relationship between β value and return within equities as an asset class.

It could be argued that the CAPM relationship between β values and returns is a model of expectations rather than outcomes. However, to be used as a model of expectations a link needed to be demonstrated between the model's expectations and outcomes. This link was provided by academic studies such as Black, Jensen and Scholes (1972) and Ball, Brown and Officer (1976); it is the conclusions of these studies that are challenged by this paper. Had these studies not appeared to confirm the existence of a theoretically derived relationship between β values and returns, it seems unlikely that the use of CAPM and the adoption of its assumptions would have become so widespread as is the case today.

If there is no meaningful relationship between β value and return, then much of the modern Theory of Finance, including significant parts of the actuarial curriculum and the existence of an equity ***risk*** premium, may be relying on empirical evidence that has been misinterpreted.

2. Evolution of the assumed β return relationship.

2.1 The idea of the relationship between volatility, β and return appears to have originated in the 1950s when finance academics in the US were using their (recently acquired) computers to investigate the time series behaviour of stock prices.

The historical context of this idea is important. At the time, there was an emerging view that, to a very good first approximation, stock price movements in successive time periods were statistically independent. The reason that independence became important is because its assumption supported the use of volatility of returns as a measure of risk. Markowitz (1959) is a useful text on these ideas.

2.2 Markowitz demonstrated that if investors know the expected return, volatility of returns from all available investments and correlation of returns between all available investments, then they could use a mathematical optimisation procedure to determine a portfolio which produces the greatest expected return for a given level of volatility. Alternatively, given an acceptable level of volatility the same optimisation procedure could be used to select a portfolio which produced the highest return.

In this context, Markowitz, and most of the finance community, used the word 'risk' rather than volatility. This practice endures today and, to avoid ambiguity, use of the word 'risk' has been avoided wherever possible.

To implement Markowitz's optimisation procedure with N possible investments requires N means and variances and $N(N-1)/2$ correlations. With 100 stocks approximately 5,000 parameters are required. Given the large amount of accurate numerical information needed to implement Markowitz's ideas, this approach may not have achieved any prominence if it were not for two developments that were to follow.

2.3 According to Jensen (1972), Markowitz's work laid the foundation on which Sharpe (1964), Lintner (1965) and others developed equilibrium models of the relationship between expected rates of return on individual assets, the covariance of individual asset returns with those of the 'market portfolio' and the risk free rate of interest. As well as the ideas of Markowitz, the theoretical development of 'equilibrium' models require some assumptions. To quote an editorial of Jensen (1972):

> *'[These models] all involve either explicitly or implicitly the following assumptions:-*
>
> *1 All investors are single period expected utility of terminal wealth maximisers who choose among alternative portfolios on the basis of mean and variance (or standard deviation) of return. .....*
>
> *2 All investors can borrow or lend an unlimited amount at an exogenously driven risk-free rate of interest .., and there are no restrictions on short sales of any asset.*
>
> *3 All investors have identical subjective estimates of the means, variances and covariances of returns among all assets.*
>
> *.....*
>
> *5 There are no taxes*
>
> *....'*

It will be noted that for a market to behave as if it were dominated by such investors, it is not necessary for ***all*** investors to use a volatility return optimisation model. It is sufficient if ***enough*** investors use this procedure to take advantage of any opportunities that arise.

The first assumption has become entangled with a further assumption that the behaviour envisaged is economically rational. An important component of this argument is that, in seeking to minimise the overall variance of return of their portfolios, investors are seeking to minimise risk.

2.4 The models that have emerged from these theoretical arguments state that, in a market which is dominated by such investors, the volatility/expected return characteristics of all securities would be related in a simple way.

There was much debate about the most appropriate equilibrium model. Possibly as a result of the empirical evidence discussed below, it became accepted that there should be a linear relationship between the expected returns of all assets and the covariance of these returns with the 'market portfolio'.

The end result was the generally accepted form of what has become known as the Capital Asset Pricing Model (hereinafter CAPM):

$$R_{j,t} = r_t + \beta_j \left(R_{m,t} - r_t \right) + \varepsilon_{j,t}$$

where $R_{j,t}$ is the return from the asset j in the time interval $t - 1, t$

r_t is the riskless rate of return in the time interval $t - 1, t$

$R_{m,t}$ is return on the market portfolio in the time interval $t - 1, t$

β_j is 'beta factor' of asset j and

$\varepsilon_{j,t}$ is the residual error which has mean zero and is not autocorrelated in any way.

In the 1970s there was some discussion about the most appropriate form of this model. Much of the debate related to a discussion about the use of a 'risk-free' rate of return (generally known as the Sharpe-Lintner model) or replacing the 'risk-free' rate of return with the rate of return on a 'zero-beta' portfolio (a variation known as the Black version of the model).

2.5 The final step in the evolution of the volatility/return model was the emergence of empirical studies of the historical relationship between β values and returns. Perhaps the best known of these studies was that of Black, Jensen and Scholes (1972) who fitted a model of the form:

$$R_{j,t} = r_t + \alpha_j + \beta_j \left(R_{m,t} - r_t \right) + \varepsilon_{j,t}$$

where α_j is a constant relating to asset j.

This study involved more than 500 securities listed on the New York Stock Exchange over a 35 year period ending in December 1965. The number of securities in the sample fluctuated between 500 and 1000 stocks over the 35 year period. Each year the securities were divided into 10 portfolios based on their β values calculated from their price history over the previous five years.

Monthly 'excess' rates of return were then calculated for each portfolio. The excess return was the simple arithmetic difference between the discrete monthly rate of return on a portfolio, including dividends and capital appreciation, less the risk-free rate of return. The risk free rate of return was taken as the yield on 30 day US Treasury bills for the period 1948-1966 and the dealer commercial rate for the period 1926-47.

The average excess monthly rates of return for each of these 10 portfolios over 420 months, as well as the average excess monthly return for the market overall are shown below:

<u>Table 1</u>
Black, Jensen and Scholes study US data 1931-1965

Portfolio #	β	Average Excess Return (% per month)
1	1.56	2.13
2	1.38	1.77
3	1.25	1.71
4	1.16	1.63
5	1.06	1.45
6	0.92	1.37
7	0.85	1.26
8	0.75	1.15
9	0.63	1.09
10	0.50	0.91
Market over all	1.00	1.42

2.6 Although Black, Jensen and Scholes noted that their results did not exactly fit their model, there was nevertheless a clear linear relationship between β values and average rates of return. In this respect, the essential accuracy of this study does not appear to have been seriously challenged until Fama and French (1992).

As Walsh (1976) commented in bringing this theory and the empirical evidence to the attention of Australian actuaries:-

> *'This paper by Black Jensen and Scholes shows that* ***after exhaustive testing*** *a linear relationship does exist between market risk and the investment return of a security.'*

2.7 At around the same time that Walsh (1976) brought this theory to the attention of Australian actuaries, Ball, Brown and Officer (1976) published a study of the relationship between β values and returns for 651 industrial securities listed on Australian stock exchanges for the period February 1958 to December 1970.

Like Black, Jensen and Scholes (1972), Ball, Brown and Officer divided their data into portfolios ranked by covariance with the rate of return on *'aggregate economic wealth'* for which they used, as a surrogate, an equally weighted average of 651 individual-equity returns. As a result of their calculations and statistical testing, they concluded:-

> *' the model asserts a positive, linear relation between the expected values of rates of return on securities and their covariances with the rate of return on aggregate economic wealth. We find evidence of such a relationship in the Australian industrial equity market over the period 1958-1970.'*

Although their conclusions are written in more general language, their empirical tests were based on β values and discrete rates of monthly return. The results of Ball, Brown and Officer showed a similar relationship between β values and average excess monthly rates of return to those identified by Black, Jensen and Scholes. In both cases the difference in rate of return between portfolios with β values of 1.5 and 0.5 was approximately 1.2% per month.

2.8 Given the theoretical arguments and empirical evidence, it is not surprising that the idea of a positive linear relationship between β values and rate of return has become entrenched. To quote Fama and French (1992):

> *'The Asset-pricing model of Sharpe (1964), Lintner (1965) and Black (1972) has long shaped the way academics and practitioners think about average returns and risk. The central prediction of the model is that the market portfolio of invested wealth is mean-variance efficient in the sense of Markowitz (1959). The efficiency of the market portfolio implies that (a) expected returns on securities are a positive linear function of their market βs (the slope in the regression of a security's return on the market's return) and (b) market βs suffice to describe the cross-section of expected returns.'*

Although Fama and French acknowledged the existence of some *'empirical contradictions of the Sharpe-Lintner-Black model'* (and then proceeded to suggest some new ones) it is apparent that the assumption of a positive relationship between β values and expected return has dominated intellectual thought and undergraduate teaching for almost 30 years, based predominantly on the theoretical work of Markowitz, Sharpe, Lintner and Black and the empirical evidence of Black, Jensen and Scholes.

3 The meaning of *'mean return'*

3.1 When considering alternative investment propositions or investigating empirical evidence, there are a number of potential definitions of the expression *'mean return'*:

(a) the arithmetic average of successive discrete rates of return. This has been used extensively in tests of the β return relationship, but is not generally used for purposes such as performance measurement.

(b) the time-weighted rate of return. This is used extensively in measuring and comparing the performance of fund managers because it measures the investment results independently of cash flows. It is essentially a geometric mean rate of return of successive time periods.

(c) the money-weighted rate of return or internal rate of return. Used in project evaluation and in allocating surplus within superannuation funds etc. Sometimes also used for short periods as part of the process of estimating a time-weighted rate of return.

(d) the average continuously compounded rate of return. Used mainly in theoretical work, eg stochastic interest rate models. Actuaries will recognise this as the mean force of return. It is also equal to the natural logarithm of (1 plus) the geometric mean or time weighted rate of return.

3.2 Let us now repeat the first assumption of the Capital Asset Pricing Model

> *All investors are single period expected utility of terminal wealth maximisers who choose among alternative portfolios on the basis of mean and variance (or standard deviation) of return.* .

It is unlikely that investors making direct share purchases make extensive use of an estimated mean return in their decisions. However this is an important part of asset allocation studies or, at the retail level, choices between two managed funds. In these cases the performance figures will normally be compound or time-weighted rates of return and not arithmetic averages of discrete rates of return.

Suppose we are given an initial investment of say $100,000 and a termination date of (say) 20 years. We can calculate the terminal value of this investment (ignoring taxes and assuming re-investment of income) if we know either the time-weighted rate of return, the geometric mean return, the internal rate of return or the mean continuously compounded rate of return.

Knowing the arithmetic mean discrete rate of return will not enable us to calculate the final portfolio with any accuracy. Furthermore the arithmetic mean rate of return will be biased in favour of volatile assets as the following example shows.

3.3 Let us suppose we have two portfolios ***A*** and ***B***. Their successive rates of return in two time periods are ***plus 100%*** followed by ***minus 60%*** for portfolio ***A*** and ***plus 10%*** followed by ***plus 10%*** for portfolio ***B***.

Actuaries will recognise the use of the term 'discrete rate of return' to describe what they understand by the term 'effective rate of (compound) return' per period. Let us also suppose that each portfolio (or security) was worth $100 at time zero and apply these discrete rates of return. Then we have the following results:

Table 2

Portfolio	***A***	***B***
	($)	***($)***
As at time		
0	***100***	***100***
1	***200***	***110***
2	***80***	***121***
Mean discrete rate of return per period	***+20%***	***+10%***

Knowing that the mean discrete rate of return of these two periods was +20% per period does not enable us to accurately forecast the final value. However, knowing that the geometric mean was -10.56% or that the mean continuously compounded return was -0.1116 does enable us to accurately calculate the final portfolio value.

Consequently, when an investor is making decisions on the basis of mean rates of return, ***the only definition of 'mean return' that makes any sense is mean continuously compounded return or something that is equivalent***. The arithmetic mean of discrete rates of return is subject to a considerable degree of error.

3.4 While this extreme example illustrates the point, it might be felt that such an example is so extreme that, in practice, differences between arithmetic means of discrete and continuously compounded returns are so small it does not matter. This is not the case.

Consider the following examples from the Black, Jensen and Scholes (1972) study:

Table 3

Portfolio number	2	6
β	1.38	0.92
Mean excess return (% per month)	1.77	1.37
Standard deviation of excess return (% per month)	12.48	8.36

If $1m were invested in these two portfolios at the start of the Black, Jensen and Scholes study in 1931, which portfolio would have the higher termination value 35 years later?

A superficial response might be to point out that as the mean excess return from portfolio 2 exceeded that of portfolio 6 by 0.51% per month, or more than 6% per annum, the answer was obviously portfolio 2 and by a considerable margin.

3.5 If we add the average riskless rate of return to the mean excess return and use the result as an effective monthly rate over 420 months then we can estimate the following termination values for an initial investment of $1 million. This can be approximated by assuming a constant value for the 'riskless' rate of return of (say) 0.1% per month - the average 'riskless' force of return over the period 1931-1965 calculated using data published by Ibbotson and Sinquefield (1996).

Table 4

Portfolio number	2	6
Termination value after 35 years	$2,396m	$459m

If we were to check these results by considering an investment of $1m invested in the Standard and Poors index, and then over 35 years $1 million would have accumulated to less than $35 million over the same period. There is clearly something about this approach this approach which is either wrong or needs further investigation.

If X_t is the total return from a portfolio in month t, then the accumulation of $1 million 420 months later is $A million where

$$A = (1 + X_1) \times (1 + X_2) \times \cdots \times (1 + X_{420})$$

Without knowing sums of all the terms like $X_1 \times X_2$, $X_1 \times X_2 \times X_3$ etc, we cannot use this formula to estimate the termination value. However by taking logarithms and using the first two terms of the MacLaurin series expansion $log_e(1+x) \approx x - \frac{1}{2}x^2$, we can estimate $log_e(A)$ if we know the mean rates of return and the sum of the squares of the rates of return.

Using this approach we obtain the following estimates of the termination value:

Table 5

Portfolio number	2	6
Termination value after 35 years	$91m	$106m

These estimates are still inconsistent with the Standard and Poors index over the same period. If we use the Ibbottson and Sinquefield (1966) data, two initial investments of $1m in large and small capitalisation stocks would have accumulated to $35m and $141m respectively. As the Black, Jensen and Scholes study was, in effect, based on equally weighted portfolios, it seems likely that the difference between the estimates for portfolios 2 and 6 and the Standard and Poors data is explained by a small-capitalisation effect over this period.

However, ***the higher mean excess return of portfolio 2, does not mean that it was a superior investment over the 35 year period. If anything its return was lower than that of the less 'risky' portfolio.***

3.6 As an alternative to calculating terminal values, and given enough data, it is possible to adjust empirical results which show arithmetic averages of discrete rates of return to estimate geometric means or means of continuously compounded returns as follows.

Let:

$r(i)$	be the discrete rate of return for period i (the effective rate of return per period), $i = 1 \dots N$
$\delta(i)$	be the continuously compounded rate of return for period i (the force of return),
a	be the arithmetic average of $r(i)$.
g	be the geometric mean return (time weighted rate of return) given by
	$(1 + g)^N = \{1 + r(1)\} \times \{1 + r(2)\} \times \dots \times \{1 + r(N)\}$
d	be the average continuously compounded rate of return

Then $\delta(i) = \ln\{1+r(i)\}$. which is approximately $r(i) - \frac{1}{2} r(i)^2$

Hence $d = \ln\{1+g)$ and

$$d = \frac{1}{N}\sum \ln\ \{1+r(i)\} \approx \frac{1}{N}\sum\left\{r(i) - \frac{1}{2}r(i)^2\right\} = a - \frac{1}{2}\left[var\{r(i)\} + a^2\right]$$

3.7 The empirical evidence of Black, Jensen and Scholes (1972) was based on the monthly excess rate of return, being the difference between the monthly discrete rate of return and the 'riskless' rate of return. Consequently there will be variations in the 'riskless' rate of return which are a small source of error in applying the formula set out in para 3.6 to the data shown in para 2.5.

In addition, to make the numbers more meaningful, monthly excess rates of return have been converted to effective annual rates rather than continuously compounded monthly rates of return.

The adjusted results (using the above formula to calculate a geometric mean) with β values correct to two decimal places are as follows:

Table 6

Excess return 1931-1965 (Black, Jensen & Scholes)

Portfolio #	β	Arithmetic Mean (% pa)	Geometric mean (Est % pa)
1	1.56	28.8	13.6
2	1.38	23.4	12.4
3	1.25	22.6	13.6
4	1.16	21.4	13.7
5	1.06	18.9	12.6
6	0.92	17.7	12.9
7	0.85	16.2	12.1
8	0.75	14.7	11.5
9	0.63	13.9	11.6
10	0.50	11.5	9.9
Market over all	1.00	18.4	12.9

3.8 The period covered by this study commenced shortly after the great stock market crash of 1929 and finished in the 1960s, a period which was to become known as the *'go-go'* era. Accordingly the Black, Jensen and Scholes data has atypical start and finish points. Another feature which is relevant to the interpretation of the results is that, judging from the Sharpe and Cooper (1972) study, portfolios 8, 9 and 10 may have had an increasing component of utility stocks such as gas or electricity distributors.

On the basis of these estimates, it appears that the relationship between β and return is not nearly as strong when *'mean return'* means the mean of continuously compounded returns. However, after taking the time period and inclusion of utilities into account, the β return relationship may be even weaker.

3.9 Jensen (1972) attempted to fit a continuously compounded version of the Sharpe-Lintner model to the Black, Jensen and Scholes data. His estimate for the mean continuously compounded rate of return for the market portfolio was 0.0105 per month or 13.4% per annum effective (cf the estimate of 12.9% above, which supports the accuracy of the estimates), however he concluded that *'the [continuously compounded] model does not fit the data'*.

3.10 Using arithmetic means of discrete rates of return, Ball, Brown and Officer (1976), documented a linear relationship between β values and return in the Australian Industrial equities market over the period 1958-1970. Unfortunately their study does not provide enough information to enable their results to be adjusted to a geometric or continuously compounded basis. However a few years later, two of the authors, Ball and Brown (1980) investigated the relative performance of mining shares compared to industrial shares, this time using continuous compounding rather than arithmetic averages of discrete rates of return.

The results of Ball and Brown's study of mining and industrial shares showed little difference between the performance of mining shares and industrials despite a considerable difference in volatility and the authors appeared puzzled by their results:

> *'We do not know what to make of [our conclusion that the comparison of average return against standard deviation for mining investment relative to industrial and commercial investments appeared to be unfavourable.] The mining market (considered by itself) appears to have been substantially riskier than its industrial and commercial counterpart, without earning a commensurate risk premium. There is evidence that this result holds for more than the 10-year period for which a precise measurement was possible; it appears to extend over the 21-year period 1958-1979 and possible back two decades before that.'*

3.11 It is clear from this discussion that the precise definition of mean rate of return is quite important. If investors are to be 'terminal wealth maximisers', this is inconsistent with the use of arithmetic means of discrete rates of return to assess investment alternatives. While the stock ***A*** versus stock ***B*** example may be extreme, a β related difference of 12% per annum (according to the Black, Jensen and Scholes study) almost disappears when the definition of mean return is changed from arithmetic average of discrete returns to continuous compounding. In consequence, many of the empirical studies of β values and return may need to be re-interpreted.

4 Evidence that contradicts a positive relationship between return and β

4.1 At around the same time that mathematicians were investigating the time series nature of stock prices, fundamental analysts such as Nicholson (1968) and McWilliams (1966) were beginning to investigate the predictive power of price/earnings ratios. At the time, the prevailing interest was in 'growth stocks' – an attitude which came to be remembered as the 'go-go' era.

There is a large number of these studies. Typically they involved the division of a sample of stocks into deciles or quintiles, thus forming portfolios in a mechanical fashion on the basis of their price/earnings ratios. Some of these studies were merely interested in price appreciation and others, such as Fama and French (1992) did not give enough information to enable continuously compounded rates of return to be calculated. Some of the less sophisticated studies merely calculated the amount accumulated at the end of the study period from which a continuously compounded return can be calculated.

The 'ASX' data is based on the ASX/Russell All Growth and All Value accumulation indices which are, in effect, two portfolios periodically reallocated on the basis of their price/book-value ratios. The return differences are calculated between the highest or lowest deciles or quintiles (as the case may be) and converted to a continuously compounded figure.

Table 7

Study	Basis	Groups	Return diff (%pa)
Basu (1977) US 1957/71	p/e	quintiles	7.0
McWilliams (1966) US 1953/64	p/e	deciles	6.7
Dreman (1982) US 68/77	p/e	deciles	10.9
ASX stocks 1990/99	p/b	halves	5.2

4.2 Except for Basu (1977), who was also interested in β values, most of the US studies were conducted solely for the purpose of investigating the predictive power of price/earnings ratios rather than testing the Capital Asset Pricing Model. When they were advanced as evidence that there were variables, other than β, which were useful in estimating future returns thereby contradicting the semi-strong form of the 'efficient' market hypotheses, the calculations were dismissed on the grounds that they had not correctly allowed for 'risk'.

For security analysts the price/earnings ratio is often regarded as an indicator of risk because the market valuation of low price/earnings ratio stocks relies less on the distant future. On the basis of these studies many analysts would argue that low price/earnings ratio stocks would offer both above average returns and below average risk.

While fundamental analysts would not equate high risk with β, expectations of the distant future are more subject to changes in opinion than estimates of near future earnings. Consequently such analysts would not find it surprising that, when β values were calculated, the low price/earnings ratio stocks tended to have, if anything, lower β values than the high price/earnings ratio stocks.

4.3 Basu (1977) investigated the relationship between investment performance and price/earnings ratios by ranking 750 companies listed on the New York Stock Exchange into quintiles based on their price/earnings ratios and reassigning stocks to the appropriate quintile annually. This study covered the period between August 1956 and August 1971. Basu also calculated the β values of the portfolios. His results were as follows:

Table 8

P/E Quintile	Annual return (%)	β
Highest	9.3	1.1121
Highest (excluding negative earnings)	9.6	1.0579
2	9.3	1.0387
3	11.7	0.9678
4	13.6	0.9401
5	16.3	0.9886

In this study, the rate of return proved to be inversely related to price/earnings ratio, as with other studies. It should be noted, however, that the annual returns shown above are 12 times the continuously compounded monthly returns. While this study also shows an inverse relationship between β values and returns, this is an inverse relationship between continuously compounded return and β, not a relationship between the arithmetic average of discrete monthly return and β.

Dreman (1982) also reported that the better performing low price/earnings deciles also tended to have, if anything, lower β values than the lower performing higher price/earnings deciles.

4.4 Fama and French (1992) investigated the relationship between price/book ratio over the period 1960-1990 on the New York Stock Exchange. They concluded that there was a positive relationship between price/book value and return, but that there was no relationship between β values and return. It appears that this study was based on mean discrete returns rather than continuous compounding. Fama and French noted that their results were at odds with the traditional form of the Capital Asset Pricing Model which assumed a linear relationship between β and return.

> *'When the tests allow for variation in β that is unrelated to size, the relation between β and average return for 1941-1990 is weak, perhaps non-existent, even when β is the only explanatory variable. We are forced to conclude that the [Sharpe-Lintner-Black] model does not describe the last 50 years of average stock returns.'*

In view of the foregoing discussion on continuous compounding and arithmetic means, it is difficult to interpret how the Fama and French study would have fared had they used continuous compounding. However, given the bias of arithmetic means towards volatile assets, it seems possible that their findings against any relationship with β might have been stronger had they used continuous compounding.

4.5 Studies such as those of Basu (1977 and also 1983) suggest that the relationship between price/earnings ratios and returns was not due to β. Ball (1978) argued that showing that the superior performance of low price/earnings ratio portfolios did not depend on higher β values was not enough because low price/earnings ratios encapsulate some other form of (as yet unidentified) risk and there therefore remains a relationship between risk and return. It is difficult to understand the logic of Ball's argument as it relies, in effect, on the assumption that β does not measure risk.

4.6 Interestingly, neither Ball (1978) nor Ball and Brown (1980) mentioned the possibility that the use of continuous compounding and arithmetic averages of discrete returns might give different results. There seems to be a widely held view that this does not matter much. To quote Brailsford and Heaney (1998 pp226-227):

> *'There are a number of methods of estimating expected return. The returns could be expressed as arithmetic returns, geometric returns or continuously compounding returns. Although in theory these could affect the results of CAPM tests, the choice of return estimation does not seem to have much affect.'*

The estimates of para 3.5 show that \$1 million invested in 1931 in the portfolio number 2 (with a β value of 1.38) of the Black, Jensen and Scholes (1972) study would have accumulated to approximately \$100 million by the end of 1965. However, performing compound interest calculations with the arithmetic mean results in an estimate of \$2,400 million. It is clear, from this example, that the precise definition of return is not a minor issue.

4.7 A further phenomenon, which may be peculiar to Australia is the poor performance of resource stocks. Following the study of Ball and Brown (1980), the establishment of Australian Stock Exchange accumulation indices in 1979 has enabled an accurate ongoing comparison to be maintained. Over the 20 years 1979 to 1999, the relative performance of the Industrial and Resources indices has shown an apparently inverse relationship between β values and return.

Table 10

Relative Performance of Australian Industrial and Resources Stocks
(ASX All Industrial and All Resources Accumulation Indices)
(continuously compounded Dec 1979 to Dec 1999)

	Industrials	Resources
Index 1979	1,000	1,000
Index 1999	29,387	5,158
β value	0.8	1.2
Mean return (%pa)	16.9	8.2

If there is supposed to be a positive linear relationship between β values and mean then why was it ***large and inverse*** for such an important part of the Australian market over a 20 year period?

It has been argued, as discussed by Ord (1998), that the market in Resources stocks is influenced by international investors and β values should be calculated from an international index. However,

(a) as Ord points out, foreign investors have been just as evident in Industrial stocks; also

(b) if β values should be calculated using international benchmarks, all the evidence in support of a positive linear relationship needs to be recalculated as well – it is unscientific just to recalibrate the data which disagrees with the model without applying the same procedure to data that does agree.

5 Summary and conclusions

5.1 If it is assumed that investors should make investment decisions on the basis of 'maximising their terminal wealth' then this is inconsistent with making assessments on the basis of 'mean return' unless mean return is defined as the mean of continuously compounded returns or its equivalent. As a method of estimating the fortunes of investors, the arithmetic average of discrete rates of return is both inaccurate and biased in favour of volatile assets.

Arithmetic means of discrete rates of return, as distinct from geometric means or means of the force of return, do not correctly measure the fortunes of investment portfolios over successive time intervals. Any model of investment returns therefore needs to establish a relationship between the model's variables and the mean continuously compounded (or force of) return. For this purpose, empirical evidence based on arithmetic averages of discrete rates of return needs to be treated with great caution.

The Capital Asset Pricing Model is sometimes defended on the grounds that it is a 'single period' model. A single period model, especially when the empirical evidence is based on arithmetic averages of discrete rates of return, is unlikely to be suitable for successive multiple periods when the number of periods is large, as often happens in actuarial work.

5.2 When an approximately positive linear relationship between β values and return was 'discovered' the idea of 'risk' related return was then exported from an intra-asset class argument to a plausible explanation of the superior performance of equities over fixed interest securities.

It was argued that a linear relationship between β values and 'return' means that the 'market' portfolio is an 'efficient' portfolio and all other 'efficient' portfolios must consist of linear combinations of the 'market' portfolio and the 'riskless' asset.

This argument adopts a number of assumptions, including the use of volatility as a measure of risk and the explanation usually advanced is that rational investors will only invest in 'riskier' asset classes if their expected return is higher than other classes.

5.3 As experienced observers such as Alan Kohler (2001) have noticed, dominant professional investors, if they are risk averse, can be more concerned with their own business risk than the investment risks to which they are exposing their customers:

> *'Instead of being invested for the long term, the world's (not just Australia's) retirement savings are invested in liquid, short-term assets that are designed to protect the business risks of those doing the investing - not to protect the investment risks of the customers, or even to maximize their long-term returns. ... the principal aim of institutional investment businesses is to reduce their tracking error to the index and so lower their chances of being sacked.'*

5.4 Emerging research into investor behaviour is beginning to suggest that investors may not be 'risk' averse after all. To quote Coleman (2001):-

> *'..over the last 20 years, psychologists and 'behavioural economists' have amassed a large body of evidence showing that most people are not risk averse, they are loss averse which is a very different thing.'*

The proposal that much of the empirical numerical evidence in favour of a 'risk' return relationship might be questionable is supported by the growing evidence that explanations which rely on investor rationality are also questionable.

5.5 The theoretical advantages of continuous compounding are well recognised even though the additional accuracy is not always considered sufficiently important.

Whether for simplicity or for other reasons, most of the empirical academic research supporting a positive linear relationship between β and mean return has been based on arithmetic means of discrete rates of return such as Black, Jensen and Scholes (1972) and Ball, Brown and Officer (1976). On the other hand most of the early research into the predictive power of price/earnings ratios such as McWilliams (1966) and Nicholson (1968) used equivalent compound rates of return which can be directly transformed into continuously compounded rates of return.

The practice adopted by academic research into the relationship between factors such as price/earnings ratios and book values is varied. Basu (1977 and 1983) used continuous compounding while Fama and French (1992) and Lakonishok, Schleifer and Vishny (1994) predominantly used averages of discrete returns.

If the only acceptable basis for considering empirical evidence relating to investment theories and/or models is continuously compounded rates of return, or its equivalent, then a very large part of the existing evidence may need to be reworked or disregarded as misleading.

Repeating the studies of relationship between β and return using continuous compounding is clearly an important area for further research, perhaps best left until there has been some discussion about the validity, or otherwise, of empirical studies and models based on arithmetic means of discrete rates of return.

5.6 Given the different methods of measuring return, it seems likely that the controversy between efficient markets and the predictive power of value indicators such as dividend yield, price/earnings ratios and book values was a failure of communication between two groups using different definitions of mean return.

5.7 In the meantime, the existence and extent of any ***meaningful*** relationship between β and return - as measured by mean continuously compounded rates of return - is far from clear. Form the point of view of a long-term investor, for whom a single-period model is inappropriate, the relationship appears to be considerably less than the 1% per month which seemed to apply to arithmetic means of discrete rates of return as documented in the US by Black, Jensen and Scholes (1972) and in Australia by Ball, Brown and Officer (1976).

Based on the somewhat scant evidence of Basu (1977 and 1983), Dreman (1982) and the Australian resources index since 1979, the relationship between β values and continuously compounded (or time-weighted) rates of return may even be mildly negative.

As there must be some doubt about the relationship between β values and long term return within equities as an asset class, the export of this argument as an explanation for the superior long term past performance of ordinary shares compared to bonds is also open to question. There may well be an equity premium, but volatility and risk averse behaviour by major investors may not be the explanation.

5.8 Much of the discussion about the different definitions of rate of return may be self evident to actuaries. It will be generally known that time-weighted or rates of return can be accurately transformed into mean continuously compounded rates of return and vice-versa. On the other hand, a comparison of the investment performance of two fund managers based on arithmetic averages of discrete rates of return would be highly unusual, if not unprofessional.

However, with the assumptions and calculation methods buried in small print in technical journals, some of us may have adopted investment models which rely on empirical evidence based on such arithmetic averages without being aware of the issues raised in this paper.

5.9 I would like to acknowledge the assistance of Michael Barker, Geoffrey George and Allen Truslove, who commented on drafts of this paper at various stages.

Appendix A

This appendix shows the results of some experiments in the use of a similar formula to that used in section 3.5 to estimate terminal portfolio values when we know the average of the discrete rates of capital growth per month, the standard deviations of such discrete rates of capital growth and the actual outcome.

These experiments are based on the month-end values of the Australian All Ordinaries Index (spliced to its predecessor prior to 1980). The periods shown cover two of the most volatile periods in Australian stock market history - the sharp decline and recovery in the mid-1970s, and the bubble and subsequent crash of October 1987.

If x_i is the discrete rate of capital growth in month i and the mean and standard deviation of the observed values of these discrete rates of monthly capital growth are $\bar{x}$ and s, then the accumulation of an initial investment of \$100,000 ignoring dividends after n months is

$$A_n = 100000 \times \prod_{i=1}^{n} (1 + x_i)$$

Let $a_n = \log\left(10^{-5} A_n\right)$

$$\text{Then } a_n = \sum_{i=1}^{n} \log(1 + x_i) \approx \sum_{i=1}^{n} x_i - \frac{1}{2}\sum_{i=1}^{n} x_i^2 = n \times \bar{x} - \frac{n}{2}\left(s^2 + \bar{x}^2\right)$$

$$\text{Finally } A_n = 10^5 e^{a_n} \approx 10^5 \exp\left\{n \times \bar{x} - \frac{n}{2}\left(s^2 + \bar{x}^2\right)\right\}$$

These experiments were an attempt to test the reliability of the above formula on some live data where the actual results are known and for which the arithmetic means and standard deviations of discrete rates of monthly capital appreciation can be readily calculated.

Table A.1

Experiments with return approximations on ASX SPI capital growth 1961-1995

Period	Jan 1961 Dec 1995	Jan 70 Dec 89	Jan 70 Dec 79	Jan 80 Dec 89
Share price index at end of previous month	185.2	441.8	441.8	500
Share price index at end of period	2203	1649.8	500.0	1649.8
Number of months in period	420	240	120	120
Accumulation of $100,000 at beginning of period without re-investment of dividends ($'000)	1,190	373	113	330
Estimate of capital accumulation using average of discrete monthly rates of growth ($'000)	2,328	637	140	455
Estimate of capital accumulation formula in Appendix 1 ($'000)	1,224	385	113	340
Error of above formula as a rate of return (%pa)	0.08	0.16	0.00	0.30
Error using mean of discrete monthly rates of appreciation as an annual rate of return (%pa)	1.94	2.55	2.17	2.96

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