Loss Aversion and the “Afternoon Effect” in Sequential Auctions

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Abstract

Empirical evidence from sequential auctions shows that prices of identical goods tend to decline between rounds. In this paper, I show how expectations-based reference-dependent preferences and loss aversion can rationalize this phenomenon. I analyze two-round sealed-bid auctions with symmetric bidders having independent private values and unit demand. Equilibrium bids in the second round are history-dependent and subject to a “discouragement effect”: the higher the winning bid in the first auction is, the less aggressive the behavior of the remaining bidders in the second auction. When choosing his strategy in the first round, however, a bidder conditions his bid on being pivotal and hence expects not to be discouraged. Optimal equilibrium behavior, therefore, leads the winner of the first round to underestimate the discouragement effect so that equilibrium prices decline. I also compare sequential and simultaneous auctions and I show that these formats are not bidder-payoff equivalent nor revenue equivalent.

JEL classification: D03; D44; D81; D82.

Keywords: Reference-Dependent Preferences; Loss Aversion; Sequential Auctions; Afternoon Effect.

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1 Introduction

Sequential auctions are a common practice for the sale of multiple lots of the same or similar goods. How should one expect the price to vary from one round to the next? Weber (1983) and Milgrom and Weber (2000) showed that with symmetric, risk-neutral, unit-demand bidders having independent private values, the law of one price should hold and on average prices should be the same across different rounds. Intuitively, if they were not, then demand from the rounds with a higher expected price would shift towards those rounds with a lower expected price, due to arbitrage opportunities. To see why, consider a two-round second-price auction. In the last round, all bidders still participating in the auction will bid their valuations since this is a weakly-dominant strategy. In the first round, it is optimal for bidders to shade their bids to account for the option value of participating in the second round. Bidders with a higher valuation also have a higher option value and, therefore, they shade their bids in the first round by a greater amount than do bidders with a lower valuation. In the second auction, the number of bidders is lower, but the number of objects is lower as well. The first fact has a negative effect on the competition for an object while the second one has a positive effect. Remarkably, in equilibrium these two effects exactly offset each other. As a result, all gains to waiting are arbitraged away and the expected prices in both rounds are the same. The intuition for this result is very general, holds also for more than two rounds and does not depend on the specific type of auction.1

However, this neat theoretical result does not seem to be supported by the data. Ashenfelter (1989), Ashenfelter and Genesove (1992), and McAfee and Vincent (1993) document a puzzling declining price anomaly or afternoon effect (reflecting that later auctions often take place in the afternoon whereas earlier ones are in the morning). Declining price patterns have been also found in Beggs and Graddy (1997), Ginsburgh (1998) and Van den Berg et al. (2001).2 Moreover, while declining prices are more frequent, increasing prices have also been documented; see, for example, Chanel et al. (1996), Deltas and Kosmopoulou (2004), and Raviv (2006).3

Ashenfelter (1989) hypothesized risk aversion as a plausible explanation for the declining-price pattern. However, McAfee and Vincent (1993) argue that risk aversion is not a convincing explanation. They studied two-round first-price and second-price auctions with independent private values, and showed that equilibrium prices decline only if bidders display increasing absolute risk aversion. Under the more plausible assumption of decreasing absolute risk aversion, a monotone symmetric pure-strategy equilibrium fails to exist and prices need not decline.

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1 Technically, with independent private values, the price sequence of any standard auction is a martingale, so that the expected price in round \( k \), conditional on \( p_k \), the price in round \( k \), is equal to \( p_k \).

2 Section 2 summarizes the empirical evidence on the price path in sequential auctions.

3 Milgrom and Weber (2000) showed that if bidders’ signals are affiliated and types are interdependent, then the equilibrium price sequence is a sub-martingale and the expected value of \( p_{k+1} \), conditional on \( p_k \), is higher than \( p_k \). Mezzetti (2011) showed that affiliated types are not necessary to explain increasing-price sequences: interdependent types with informational externalities — that is, when a bidder’s value is increasing in all bidders’ private signals — even with independent signals, push prices to increase between rounds.
In this paper, I study two-round sealed-bid auctions with symmetric bidders having independent private values and unit demand and I show that expectations-based reference-dependent preferences and loss aversion provide an alternative, preference-based, explanation for the afternoon effect. Following the framework developed by Köszegi and Rabin (2006), I assume that in addition to classical consumption utility, a bidder derives gain-loss utility from the comparison of his consumption to a reference point equal to his lagged expectations regarding the same outcomes, with losses being more painful than equal-sized gains are pleasant. Moreover, I develop a dynamic version of the Choice Acclimating Personal Equilibrium (CPE) introduced in Köszegi and Rabin (2007) that I call Sequential Choice Acclimating Personal Equilibrium (SCPE). In a SCPE, a decision maker correctly predicts his (possibly stochastic) strategy at each point in the future, folds-back the game tree using backward induction, and then applies the same (static) CPE as in Köszegi and Rabin (2007) at every stage of the game.4

Expectations-based reference-dependent preferences create an informational externality so that the equilibrium bids are history-dependent, even if bidders have independent private values. Intuitively, learning the type of the winner in the previous auction modifies a bidder’s expectations about how likely he is to win in the current one; and since expectations are the reference point, the optimal bid in the second auction depends also on what a bidder learns from the first one as this modifies his reference point.5 More precisely, I identify what I call the discouragement effect: the higher the winning bid in the first auction is, the less aggressive the bidding strategy of the remaining bidders in the second auction. The intuition is that, from the point of view of a bidder who lost the first auction, the higher the type of the winner is, the less likely he feels to win in the second one; this in turn lowers the bidder’s reference point and therefore reduces his equilibrium bid. When choosing his bid in the first round, however, a bidder conditions his bid on himself having the highest type and hence expects not to be discouraged. The intuition for this result is the following. In the first round it is optimal for a bidder to bid up to a point where he is indifferent between barely winning in the current round (by being tied with his strongest opponent) and winning in the next round. However, ex-post the winner never barely wins. Optimal equilibrium behavior, therefore, leads the winner of the first round to underestimate the discouragement effect so that equilibrium prices tend to decline.

Notice also that the history dependence arising in my model is different from the one stem-

4The original notion of CPE in Köszegi and Rabin (2007) is related to the models of “disappointment aversion” of Bell (1985), Loomes and Sugden (1986), and Gul (1991), where outcomes are also evaluated relative to a reference lottery that is identical to the chosen lottery; likewise the notion of SCPE introduced in this paper is related to the notion of dynamic disappointment aversion proposed in Artstein-Avidan and Dillenberger (2011).

5Jeitschko (1998) studies the role of information transmission and learning in sequential second-price auctions with independent private values by assuming that the distribution of bidders’ values is discrete. Since with a discrete distribution ties among bidders happen with strictly positive probability, a bidder can no longer condition his bid on him having the highest valuation; this in turn triggers the scope for the bidders to update their beliefs about their rivals based on the outcome of the first auction. However, in his model the price of the second auction is still expected to be equal to the price of the first auction, regardless of the outcome of the first auction.
ming from interdependent (common) values. With interdependent values, since in equilibrium he conditions his bid on himself having the highest signal, if a bidder loses the current auction he learns that the winner had a higher signal than his; this in turn makes a losing bidder revise his estimate of the value of the good upward and therefore he will bid more aggressively in subsequent auctions. The discouragement effect instead goes in the opposite direction by pushing bidders to bid less aggressively in later rounds.

With risk-neutral bidders having independent private values, sequential and simultaneous auctions are revenue-equivalent for the seller and payoff-equivalent for the bidders. I show that these equivalences break down if bidders are expectations-based loss-averse. The key difference between sequential and simultaneous auctions is the timing of information. Sequential auctions provide bidders, in between rounds, with the opportunity to update their beliefs about the intensity of competition. Such feedback mechanism, however, is absent in simultaneous auctions. In the classical model this difference is irrelevant since bidding strategies in sequential auctions are history-independent. Loss-averse bidders, instead, update their reference point based on the outcome of the previous round. I show that bidders with high (resp. low) values prefer sequential (simultaneous) auctions since they are more likely to receive good (bad) news between rounds. Furthermore, sequential auctions generate more revenue than simultaneous ones when the number of bidders is large.

For most of the paper, when dealing with sequential sealed-bid auctions, I assume that the winning bid in the first round is publicly announced by the seller prior to the second round. This assumption is inconsequential in the classical reference-free model when bidders have independent private values, but it is not when bidders have reference-dependent preferences. Therefore, in section 7 I analyze sequential auctions without price announcement and I show that the equilibrium strategies are radically different. If the winning bid from the first round is not publicly revealed, a bidder must use his own past bid to update his expectations about how likely he is to win in the second one. As auctions without price announcement provide them with a nosier feedback mechanism, thus exposing them to greater risk, bidders react by bidding less aggressively so that the seller’s expected revenue decreases. Nevertheless, the afternoon effect still arises in equilibrium.

Many different explanations for the afternoon effect have been proposed. I like to divide them into two broad categories: “environment-based” explanations and “preference-based explanations”. Models in the first category attempt to explain the afternoon effect by modifying some of the assumptions of the auction environment of Milgrom and Weber (2000). For example, Black and De Meza (1992) argue that declining prices are no anomaly if the winning bidder in one auction has the option to buy additional units at the same price.\(^6\) Bernhardt and Scoones (1994), Engelbrecht-Wiggans (1994) and Gale and Hausch (1994) consider sequential auctions

\(^6\)However, Ashenfelter (1989) finds declining prices also for the case of bidders with unit demand.
for heterogeneous but “stochastically equivalent” objects — that is, bidders’s valuations are identically distributed across objects, but are not perfectly correlated — and show that in this case equilibrium bidding implies declining prices. Other studies have emphasized demand complementarity (Menezes and Monteiro, 2003), participation costs (Von der Fehr, 1994), supply uncertainty (Jeitschko, 1999), asymmetry among bidders (Gale and Stegeman, 2001) and budget constraints (Pitchick and Schotter, 1988) in accounting for the declining price anomaly.

Models in the second group instead maintain the original set-up of Milgrom and Weber (2000) but modify the bidders’ preferences. Eyster (2002), for example, models the behavior of an agent who has a taste for rationalizing past actions by taking current actions for which those past actions were optimal. He shows that this taste for consistency gives rise to an “unsunk-cost fallacy” that can rationalize declining prices in sequential second-price auctions. Kittsteiner et al. (2004) study sequential sealed-bid auctions with independent private values where bidders have unit demand and their valuation for an object decreases in the rank number of the auction in which it is sold. They show that the sequence of prices constitutes a supermartingale and, after correcting for the decrease in valuations for objects sold in later auctions, conclude that average prices are declining. Their explanation for the afternoon effect, hence, relies on a form of discounting and is equivalent to situations where the objects for sale are identical but bidders are uncertain about the continuation of the auction process. In my model, instead, supply is fixed and certain and bidders do not discount payoffs from future auctions. More recently, Mezzetti (2011) introduced a special case of risk aversion, called aversion to price risk, according to which a bidder prefers to win an object at a certain price rather than at a random price with the same expected value. Under this different notion, in sequential auctions with independent private values a monotone equilibrium in pure strategies always exists and in equilibrium prices decline. Although aversion to price risk and loss aversion are both able to explain the afternoon effect, the intuition behind the result is quite different. In Mezzetti (2011), the afternoon effect is due to the bidders’ dislike of uncertainty over money; in my model, instead, the afternoon effect arises because bidders dislike uncertainty over their consumption value.

While my paper belongs to the second group, I also view it as complementary to the papers in the first one. An explanation based on loss aversion has the advantage of applying very generally, without requiring any additional modification of the auction environment. This is important since, as the evidence suggests, declining prices have been found in many different settings, even with no option to buy additional units and with identical objects.

Similarly to the model of reference-dependent preferences of Köszegi and Rabin (2006), Mezzetti’s notion of aversion to price risk assumes separability of a bidder’s payoff between the utility from winning the object and the disutility from paying the price.

On the other hand, in the case of interdependent values with informational externalities and no aversion to price risk, prices increase along the equilibrium path. When bidders are averse to price risk and values are interdependent, whether equilibrium prices follow an increasing or decreasing path depends on which of the two effects dominates.
The remainder of this paper proceeds as follows. Section 2 discusses the empirical literature on sequential auctions and the afternoon effect. Section 3 describes the model and the bidders’ preferences and introduces the concept of SCPE. Section 4 analyzes two-object sequential first-price auctions. Section 5 studies two-object sequential second-price auctions. Section 6 compares sequential and simultaneous auctions. Section 7 discusses extensions of the model and the robustness of the main results. Section 8 concludes by recapping the results of the model and pointing out some of its limitations as well as possible avenues for future research. All proofs are relegated to Appendix A.

2 Empirical Evidence

The expression “price decline anomaly” first appeared in Ashenfelter (1989), who analyzed sequential English (ascending) auctions for identical bottles of wine sold in same lot sizes at Sotheby’s and Christie’s in London, Christie’s in Chicago and Butterfield’s in San Francisco between August 1985 and December 1987. He found that prices are twice as likely to decrease as to increase. Moreover, for each series of auction he calculated the mean ratio of the price of the second auction to the price of the first one and showed that the ratio is less than one at a statistically significant level. Following Ashenfelter (1989), many other empirical studies have found evidence of declining prices in auctions for wine. For example, the results reported by McAfee and Vincent (1993), Ginsburgh (1998) and Février et al. (2005) are very similar to the ones in Ashenfelter (1989).

Besides wine auctions, the declining price anomaly has been documented for many different goods. Buccola (1982) found it occurring in livestock auctions; Milgrom and Weber (2000) for transponder leases; Engelbrecht-Wiggans and Kahn (1992) and Zulehner (2008) for dairy cattle; Lusht (1994) for commercial real estate; Chanel et al. (1996) for gold jewelry; Pesando and Shum (1996) and Beggs and Graddy (1997) for works of art; Lambson and Thurston (2006) for furs; and Ginsburgh and Van Ours (2007) for Chinese porcelain recovered from shipwrecks.

The studies mentioned so far focus on sequential English (ascending) and second-price auctions; however, the afternoon effect has been observed also for other auction formats. Ashenfelter and Genesove (1992) compared prices paid for similar condominium units in face-to-face bargaining with prices paid in a “pooled” or “right to choose” auction (a first-price auction in which the winner in each round can choose which good to pick) and found that the auction price was higher than the face-to-face price and depended on the order in which the units were auctioned.

9 However, the ratios vary between 0.99 and 0.96, so although statistically significant the effect is quite small in magnitude.

10 However, Ginsburgh (1998) argues that the declining price anomaly is likely to be caused by the fact that most bids are entered by absentees, who use nonoptimal bidding strategies. Février et al. (2005), instead, argue that the reason behind declining prices is a buyer’s option that gives the winner of an auction the right to purchase any number of units at the winning price.
The winning bid declined by about 0.27% with each unit sold, and this drop did not seem to be attributable to quality differences between the units sold. Van den Berg et al. (2001) studied sequential Dutch flower auctions and found that prices decline throughout these auctions. Moreover, at any round, the smaller is the number of remaining unites, the stronger the decline.

A possible explanation for the price decline is the heterogeneity among the objects being auctioned. Beggs and Graddy (1997), for instance, found declining values and an afternoon effect in art auctions. Here winning bids have a tendency to decline relative to estimated market values as the auction progresses. They also showed that in an auction ordered by declining valuations, even with risk-neutral bidders, the price received relative to the estimate for later items in the auction should be less than the price relative to the estimate for earlier items.

There is also experimental evidence on declining prices in sequential auctions. Burns (1985) compared the bidding behavior of professional bidders with multi-unit demands in sequential English auction experiments to that of students and found that prices declined less severely in sequences involving students than in sequences involving professional bidders because the latter used heuristics they follow in real-world markets. Keser and Olson (1996) conducted a series of sequential first-price auction experiments. They report negative prices trends and a significant amount of overbidding, suggesting that the predictions of Milgrom and Weber (2000) are not supported by data. More recently, Neugebauer and Pezanis-Christou (2007) conducted a series of experiments in order to test the effects of an uncertain supply on bidding behavior and prices in sequential first-price auctions with independent private values and unit-demand bidders. They also observed significant overbidding in all but the last round, no matter whether supply is certain or not. Moreover, they find trend-free prices when supply is certain, and significant declining price trends when supply is uncertain. Février et al. (2007) run an experiment on two-unit sequential auctions with and without a buyer’s option (which allows the winner of the first auction to buy the second unit) and considered all the four main auction formats. They find that prices decline when the buyer’s option is available.

Some authors have also found evidence of increasing prices. Among them are Gandal (1997) who looked at sequential auctions for Israeli cable television licenses, and Raviv (2006) who looked at sequential English auction for used cars. Jones, Menezes, and Vella (1996) found that prices could increase or decrease in sequential auctions of wool, as did Chanel et al. (1996) for watches. Deltas and Kosmopoulou (2004) considered a sale of library books and found that expected prices increase over the auction, but that probability of sale decreases. They attribute their findings to “catalogue” effects: how and where an item appears in the pre-sale catalogue.

Summing up, it is quite an interesting result that, in a variety of different types of auctions, price direction throughout an auction can be predicted. Declining prices (on average) have been documented in more types of auctions than have rising prices. Declining prices do not occur in every auction, but they appear to be an empirically robust feature of sequential auctions.
3 Model

3.1 Environment

Consider 2 identical items being sold to \( N \) bidders, \( N > 2 \), via a series of sealed-bid auctions. More specifically, one of the items is sold using a sealed-bid auction and the winning bid is publicly announced. Then, the remaining item is sold again using a sealed-bid auction. Announcing the winning bid from former auctions prior to the current one is in accord with government procurement statutes and with actual practice in some auctions.

I assume bidders want at most one unit and have independent private values. Each bidder’s valuation \( \theta_i, i = 1, \ldots, N \), is drawn independently from the same distribution \( F \) with continuous positive density \( f \) everywhere on the support \([0, \bar{\theta}]\). I will consider two types of games. In the first one, the goods are sold sequentially via a series of first-price auctions. In the second one, the goods are sold sequentially via a series of second-price auctions. Both auctions have a zero reserve price. Throughout the paper, I restrict attention to symmetric equilibria in pure and (strictly) monotone strategies.\(^{11}\) It is convenient to think of the auctions as being held in different periods of the day, the first one in the morning and the second one in the afternoon; however I assume the auctions are held in a short enough time so that bidders do not discount payoffs from the second auction.

3.2 Bidders’ Preferences

Bidders have expectations-based reference-dependent preferences as formulated by Köszegi and Rabin (2006). In this formulation, a bidder’s utility function has two components. If he wins the auction at price \( p \), a type-\( \theta \) bidder experiences consumption utility \( \theta - p \). Consumption utility can be thought of as the classical notion of outcome-based utility. Second, the bidder also derives utility from the comparison of his actual consumption to a reference point given by his recent expectations (probabilistic beliefs).\(^{12}\) I slightly depart from the original model of Köszegi and Rabin (2006) and I assume that bidders have reference-dependent preferences only with respect to their valuation for the item, but not with respect to the price they might pay; in other words, bidders are risk neutral over money.\(^{13}\) Hence, for a riskless consumption outcome

\(^{11}\)In a symmetric equilibrium, the final allocation is efficient: the first good will go to the bidder with the highest value and the second one to the bidder with the second-highest value.


\(^{13}\)Novemsky and Kahneman (2005) and Köszegi and Rabin (2009) argue that reference dependence and loss aversion are indeed weaker in the money than in the consumption dimension.
For each auction in which he participates, after placing a bid, a bidder basically faces a lottery between winning or losing the auction and the probabilities and potential payoffs depend on his own as well as other players’ bids. The final outcome is then evaluated with respect to any possible outcome from this lottery as a reference point. As laid out in Köszegi and Rabin (2007), Choice Acclimating Personal Equilibrium (CPE) is the most appropriate solution concept for such decisions under risk when uncertainty is resolved after the decision is made so
that the decision maker’s strategy determines the distribution of the reference point as well as the distribution of final consumption outcomes.

A strategy for bidder \( i \) is a pair of bidding functions \( \beta_i = (\beta_1, \beta_2) \), one for each auction. Notice that in the second auction the bidding function depends not only on a player’s private type \( \theta \) but also on the publicly announced winning price, \( p_1 \), from the previous auction.

Fixing all other bidders’ strategies, \( \beta_{-i} \), a bidder’s strategy \( \beta_i \), induces a distribution over the set of final consumption outcomes. For \( k = 1, 2 \), let \( \Gamma_k (\beta_i, \beta_{-i}) \) denote the distribution over final consumption outcomes from auction \( k \) point of view. Similarly, let \( EU_k \) denote a bidder expected utility from auction \( k \) point of view if he plans to bid according to \( \beta_i \) and expects his rivals to bid according to \( \beta_{-i} \). To account for the intrinsic dynamic nature of sequential auctions, I introduce a slightly modified version of CPE.

**Definition 1.** A strategy profile \( \beta^* \) constitutes a Sequential Choice Acclimating Personal Equilibrium (SCPE) if for all \( i \), and for \( k = 1, 2 \):

\[
EU_k \left[ \Gamma_k \left( \beta_i^*, \beta_{-i}^* \right) \mid \Gamma_k \left( \beta_i^*, \beta_{-i}^* \right) \right] \geq EU_k \left[ \Gamma_k \left( \overline{\beta}_i, \beta_{-i}^* \right) \mid \Gamma_k \left( \overline{\beta}_i, \beta_{-i}^* \right) \right]
\]

for any \( \overline{\beta}_i \neq \beta_i^* \).

In words, in a SCPE a bidder correctly predicts his (possibly stochastic) strategy at each point in the future, then folds-back the game tree using backward induction and applies the same (static) CPE as in Köszegi and Rabin (2007) at every stage of the game. Notice that, at stage \( k \), a bidder’s reference point is given by his stage-\( k \) expectations, \( \Gamma_k \left( \beta_i^*, \beta_{-i}^* \right) \), about his final consumption at the end of all auctions.\(^{14}\) The following assumption guarantees that all bidders participate in the auction for any realization of their own type, and that the equilibrium bidding functions derived in the next sections are strictly increasing:

**Assumption 1 (No dominance of gain-loss utility)** \( \Lambda \equiv \eta (\lambda - 1) \leq 1 \).

This assumption places, for a given \( \eta (\lambda) \), an upper bound on \( \lambda \) (\( \eta \)) and ensures that an agent equilibrium expected utility is increasing in his type.\(^{15}\) What it requires is that the weight a bidder places on expected gain-loss utility does not (strictly) exceed the weight he puts on consumption utility. Finally, notice that risk neutrality is embedded in this model as a special case; indeed if either \( \eta = 0 \) or \( \lambda = 1 \) then \( \Lambda = 0 \) and the bidders are risk neutral.

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\(^{14}\)The concept of SCPE introduced in this paper coincides with a special case of the dynamic version of Preferred Personal Equilibrium (PPE) introduced by Köszegi and Rabin (2009). In their dynamic model, people have a reference point for every period in which they expects to consume — so that consumption levels in different periods are treated like different dimensions — and are loss-averse over changes in beliefs about present as well future consumption. If consumption takes place only in the last period and the weight on prospective gain-loss utility is equal to 1, my solution concept is equivalent to theirs.

\(^{15}\)Herweg et al. (2010) first introduced Assumption 1 and referred to it as “no dominance of gain-loss utility”. Assumption 1 has been used also by Lange and Ratan (2010), Eisenhuth (2012) and Eisenhuth and Ewers (2012).
4 First-price Auctions

Consider a situation in which two identical items are sold sequentially via first-price auctions. In this case, a symmetric equilibrium consists of two bidding functions \((\beta_1, \beta_2)\), one for each auction. I assume that both functions are strictly increasing and differentiable. The first-period bidding strategy is a function \(\beta_1 : [0, \bar{\theta}] \rightarrow \mathbb{R}_+\) that depends only on the bidder’s value. The bid in the second auction, instead, might depend also on the price paid in the first auction. Since we are focusing on a symmetric equilibrium, it is useful to take the point of view of one of the bidders, say bidder 1 with type \(\theta_1\) and to consider the order statistics associated with the types of the other bidders. Let \(Y_1^{(N-1)} \equiv Y_1\) be the highest of \(N - 1\) values, \(Y_2^{(N-1)} \equiv Y_2\) be the second-highest and so on. Also, let \(F_1\) and \(F_2\) be the distributions of \(Y_1\) and \(Y_2\) respectively, with corresponding densities \(f_1\) and \(f_2\). Since the first-period bidding function \(\beta_1\) is assumed to be invertible, after the first auction is over and its winning price is revealed the valuation of the winning bidder is commonly known to be \(y_1 = \beta_1^{-1}(p_1)\). Thus, the second-period strategy can be described as a function \(\beta_2 : [0, \bar{\theta}] \times [0, \bar{\theta}] \rightarrow \mathbb{R}_+\) so that a bidder with value \(\theta\) bids \(\beta_2(\theta, y_1)\) if \(Y_1 = y_1\). To find an equilibrium that is sequentially rational, I start by looking at the bidder’s problem in the second auction.\(^{16}\)

4.1 Second-period strategy

Consider a bidder with type \(\theta\) who plans to bid as if his type were \(\tilde{\theta} \neq \theta\) when all other \(N - 2\) remaining bidders follow the equilibrium strategy \(\beta_2(\cdot, y_1)\). His expected payoff is

\[
EU_2(\tilde{\theta}, \theta; y_1) = F_2(\tilde{\theta}|y_1) \left[ \theta - \beta_2(\tilde{\theta}, y_1) \right] - \Lambda \theta F_2(\tilde{\theta}|y_1) \left[ 1 - F_2(\tilde{\theta}|y_1) \right]
\]

where \(F_2(\tilde{\theta}|y_1)\) is the probability that \(Y_2\), the second highest valuation among \(N - 1\), is less than \(\tilde{\theta}\) conditional on \(Y_1 = y_1\) being the highest and \(\Lambda \equiv \eta(\lambda - 1)\) captures loss aversion on the item dimension. Differentiating \(EU_2(\tilde{\theta}, \theta; y_1)\) with respect to \(\tilde{\theta}\) yields the first-order condition:

\[
f_2(\tilde{\theta}|y_1) \left[ 1 - 2F_2(\tilde{\theta}|y_1) \right] \theta \Lambda = f_2(\tilde{\theta}|y_1) \left( \theta - \beta_2(\tilde{\theta}, y_1) \right) - \beta_2'(\tilde{\theta}, y_1) F_2(\tilde{\theta}|y_1)
\]

where \(\beta_2'\) is the derivative of \(\beta_2\) with respect to its first argument.

Substituting \(\theta = \tilde{\theta}\) into the first-order condition and re-arranging results in the following differential equation

\[
\frac{\partial}{\partial \tilde{\theta}} \left\{ \beta_2(\theta, y_1) F_2(\theta|y_1) \right\} = f_2(\theta|y_1) \theta \{ 1 - \Lambda \left[ 1 - 2F_2(\theta|y_1) \right] \}
\]

together with the boundary condition that \(\beta_2(0, y_1) = 0\).

\(^{16}\)The analysis in this section and the next one builds on Krishna (2002)’s textbook exposition.
Because the different values are drawn independently, we have that

\[ F_2 (\theta | y_1) = \frac{F (\theta)}{F (y_1)} \]

and substituting into (4) yields

\[ \beta^*_2 (\theta, y_1) = \int_0^\theta x \left( 1 - \Lambda \left[ 1 - \frac{2F(x)}{F(y_1)} \right] \right) dF (x) \frac{N-2}{F (\theta)^{N-2}}. \]  

(5)

The complete bidding strategy is to bid \( \beta^*_2 (\theta, y_1) \) if \( \theta < y_1 \) and to bid \( \beta^*_2 (y_1, y_1) \) if \( \theta \geq y_1 \).\(^{17}\) The latter might occur if a bidder of type \( \theta \geq y_1 \) underbid in the first period causing a lower type to win (of course this is an off-equilibrium event).

The expression in (5) can be re-written as a convex combination of the risk-neutral bid and a term that depends on the bidder’s expectations (reference point):

\[ (1 - \Lambda) \int_0^\theta \frac{x dF (x)^{N-2}}{F (\theta)^{N-2}} + \Lambda \int_0^\theta 2x \frac{F(x)^{N-2}}{F(y_1)^{N-2}} dF (x) \frac{N-2}{F (\theta)^{N-2}}. \]

The first thing worth noticing is that, even with independent private values, the optimal bidding strategy in the second period is history-dependent, as it is a function of \( y_1 \), whereas with risk-neutral preferences (\( \Lambda = 0 \)) this is not the case:

\[ \beta^{RN}_2 (\theta) = \int_0^\theta \frac{x dF (x)^{N-2}}{F (\theta)^{N-2}}. \]

Under risk neutrality, a bidder submits a bid equal to his estimation of the highest valuation of his opponents, conditioning on his valuation being the highest. Because of this conditioning, bids are independent of the prior history of the game. With reference-dependent preferences, instead, the second-round equilibrium bid is decreasing in the first-round price, as shown in the following lemma.

**Lemma 1.** (Discouragement Effect) If \( \Lambda > 0 \), then \( \frac{\partial \beta^*_2 (\theta, y_1)}{\partial y_1} < 0 \forall \theta \).

According to the result in Lemma 1, the higher is the type of the winner in the first round, the less aggressively the remaining bidders will bid in the second round. The rationale for this negative effect, which I call the discouragement effect, is as follows. From the perspective of a bidder who lost the first auction, the higher is the type of the winner, the less likely this bidder is to win in the second auction; with expectations-based reference-dependent preferences a bidder

\(^{17}\) As shown in Eisenhuth and Ewers (2012), \( \Lambda \leq 1 \) is sufficient to ensure that \( \frac{\partial \beta^*_2 (\theta, y_1)}{\partial y_1} > 0 \forall \theta \).
who thinks that most likely he is not going to win does not feel a strong attachment to the item and this pushes him to bid more conservatively. Thus, revealing the first-period winner’s bid (and hence his type) creates like an informational externality. However, notice that the effect of this informational externality on the second-period bids is exactly the opposite of the one that arises with interdependent values. Indeed, with interdependent values the higher is the signal of the first-period winner, the higher is the value of the object to all remaining bidders who in turn bid more aggressively in the second auction. Therefore, by analyzing the distribution of bids in the second auction, one can use the discouragement effect to empirically test the implications of loss aversion against the implications of the classical risk-neutral model with either private values (where there is no history dependence) or with common values (where the higher is the winning price in the first auction, the more aggressively the remaining bidders behave in the second auction).

Furthermore, we have that

\[
\frac{\partial \beta_2^* (\theta, y_1)}{\partial \Lambda} = \int_0^\theta x \left[ 2 \frac{F(x)^N - 1}{F(y_1)^{N-2}} - 1 \right] dF(x)^{N-2}
\]

implying that there exists \(0 < \hat{\theta} < \bar{\theta}\) such that \(\frac{\partial \beta_2^* (\theta, y_1)}{\partial \Lambda} > 0 \Leftrightarrow \theta > \hat{\theta}\). In other words, under loss aversion bidders with relatively high types overbid compared to the risk-neutral benchmark whereas bidders with relatively low types underbid.

### 4.2 First-period strategy

Consider a particular bidder with type \(\theta\) who plans to bid as if his type were \(\hat{\theta} > \theta\) when all other \(N-1\) bidders follow the equilibrium strategy \(\beta_1 (\cdot)\).\(^{18}\) Further, suppose that all bidders expect to follow the equilibrium strategy \(\beta_2^* (\theta, y_1)\) in the second auction, regardless of what happens in the first one (sequential rationality). His expected total utility is

\[
EU_1 (\hat{\theta}, \theta) = F_1 (\hat{\theta}) \left[ \theta - \beta_1 (\hat{\theta}) \right] + \int_0^{\bar{\theta}} F_2 (\theta|y_1) \left[ \theta - \beta_2^* (\theta, y_1) \right] f_1 (y_1) dy_1
\]

\[
- \Lambda \theta \left[ F_1 (\hat{\theta}) + \int_0^{\bar{\theta}} F_2 (\theta|y_1) f_1 (y_1) dy_1 \right] \left[ 1 - F_1 (\hat{\theta}) - \int_0^{\bar{\theta}} F_2 (\theta|y_1) f_1 (y_1) dy_1 \right]
\]

where \(F_1 (\hat{\theta})\) is the probability that \(Y_1\), the highest valuation among \(N-1\), is less than \(\hat{\theta}\), and \(F_2 (\theta|y_1)\) and \(\Lambda\) are defined as before.

The first line in (6) is the sum of expected consumption utilities in period 1 and 2. The second line captures expected gain-loss utility in the product dimension. Indeed, \(F_1 (\hat{\theta}) +\)

\(^{18}\)The analysis is virtually identical for the case \(\bar{\theta} < \theta\).
the sum of the probability with which a bidder of type \( \theta \) expects to win the first auction given that he pretends to be of type \( \tilde{\theta} \) and of his expectation, in the first period, of the probability of winning in the second round given that he pretends to be of type \( \tilde{\theta} \) in the first auction but expects to behave as his real type in the second one. Hence, in accordance with the definition of SCPE in Section 3, a bidder’s reference point in the first period is his overall probability of consumption.

Differentiating \( EU_1(\tilde{\theta}, \theta; y_1) \) with respect to \( \tilde{\theta} \) yields the following first-order condition:

\[
0 = f_1(\tilde{\theta}) [\theta - \beta_1(\tilde{\theta})] - \beta_1(\tilde{\theta}) \ F_1(\tilde{\theta}) - F_2(\theta|\tilde{\theta}) \ [\theta - \beta_2^*(\theta, \tilde{\theta})] \ f_1(\tilde{\theta}) \\
- \Lambda \theta \ \left[ f_1(\tilde{\theta}) - F_2(\theta|\tilde{\theta}) \ f_1(\tilde{\theta}) \right] \ \left[ 1 - F_1(\tilde{\theta}) - \int_{\tilde{\theta}}^{\theta} F_2(\theta|y_1) \ f_1(y_1) \ dy_1 \right] \\
- \Lambda \theta \ \left[ F_1(\tilde{\theta}) + \int_{\tilde{\theta}}^{\theta} F_2(\theta|y_1) \ f_1(y_1) \ dy_1 \right] \ \left[ -f_1(\tilde{\theta}) + F_2(\theta|\tilde{\theta}) \ f_1(\tilde{\theta}) \right].
\]

Substituting \( \theta = \tilde{\theta} \) and re-arranging results in the following differential equation

\[
\frac{d}{d\theta} \ \{ \beta_1(\theta) \ F_1(\theta) \} = f_1(\theta) \ \beta_2^*(\theta, \theta)
\]

together with the boundary condition that \( \beta_1(0) = 0 \). Solving the differential equation yields

\[
\beta_1^*(\theta) = \int_{0}^{\theta} \beta_2^*(s, s) \ f_1(s) \ ds \ \frac{F_1(\theta)}{F_1(\theta)}.
\]

(7)

The first thing worth noticing is that \( \beta_1^*(\theta) \) depends on \( \Lambda \) only indirectly, through \( \beta_2^*(s, s) \). Indeed, just like in the standard case with reference-free preferences, in the first round a bidder chooses his optimal bid conditional on himself having the highest type and being pivotal. This is because a small change in his bid only matters when the bidder wins or is close to winning.\(^20\)

When conditioning on having the highest type, however, a bidder expects that if he were to lose the current auction, he would win the next one for sure and this is why expected gain-loss utility does not directly appear into the first-period bidding function. Furthermore, it is easy to check that for \( \Lambda = 0 \) we get back to the risk-neutral benchmark:

\[
\beta_1^{RN}(\theta) = \int_{0}^{\theta} \beta_2^{RN}(s) \ f_1(s) \ ds \ \frac{F_1(\theta)}{F_1(\theta)}.
\]

\(^{19}\)It is easy to verify that \( \beta_1^*(\theta) \) is increasing in \( \theta \).

\(^{20}\)If a bidder deviates from the symmetric equilibrium strategy by slightly overbidding, there are only two possible consequences. First, if he was already going to win then he still wins, but pays a slightly higher price. Second, his deviation might make him win the current round when he was otherwise going to lose. In the latter case, however, it must be that the highest opposing type is so close that the bidder was almost certain to win in the second round.
where $\beta_2^{RN}(s)$ does not depend of the type of the winner of the first auction.

Let $y_1 = \beta_1^{-1}(p_1)$. Then, the expected equilibrium price in the second auction conditional on the price of the first auction is

$$E[p_2|p_1] = E[p_2|\beta_1(y_1)] = E[\beta_2^*(Y_1^{(N-1)}, y_1)|Y_1^{(N-1)} \leq y_1] = \frac{\int_{0}^{y_1} \beta_2^*(\theta, y_1) f_1(\theta) d\theta}{F_1(y_1)}.$$ 

The following proposition delivers the main result of the paper.

**Proposition 1.** *(Afternoon Effect)* If $\Lambda > 0$, then the price sequence in a two-round sequential first-price auction is a supermartingale and the afternoon effect arises in equilibrium. That is,

$$p_1 = \beta_1^*(y_1) > E[p_2|\beta_1^*(y_1)] = E[p_2|p_1].$$

The intuition behind Proposition 1 is that, just like in the reference-independent case, in equilibrium bidders must be indifferent between winning in the first auction or in the second one. Hence, in the first auction a bidder bids the expectation of the second-round bid of his strongest competitor conditional on himself having the highest type and being the price-setter. However, by conditioning his first-period bid on him having the highest type, a bidder expects not to feel discouraged in the second auction. And because the discouragement effect depresses bids in the second auction, the expectation of his strongest competitor future bid conditional on being the price setter is higher than the unconditional expected bid. In essence, optimal equilibrium behavior leads the current price setter to underestimate the discouragement effect.

It is interesting to compare the logic behind the discouragement effect with the learning effect that arises in common value-auctions with informational externalities. In the symmetric equilibrium of a common-value auction, a bidder conditions his estimate of the value of the item (and hence his bid) on his strongest rival having a (weakly) lower signal than his. Hence, when bidding in the first round a bidder’s expectation of his competitor’s future bid conditional on being the price setter is higher than the unconditional expected bid. In this case, since a bidder revise his estimate of the value of the good upward when losing the first auction, the equilibrium price drifts upward. Conversely, with the informational externalities that arise in a private-value auction with expectations-based loss aversion, when losing the first auction a bidder becomes more pessimistic about how likely he is to win the second one (compared to his first-round expectations); this creates a discouragement effect that pushes bidders to behave less aggressively and, in turn, generates a declining price path in equilibrium.
5 Second-Price Auctions

In this section I assume that two identical items are sold using a series of second-price sealed-bid auctions. I will continue to focus on symmetric equilibrium strategies that are strictly increasing and to assume that the winning bid of the first auction is publicly disclosed by the seller prior to the second round.\(^{21}\) Again, I begin by looking at the bidder’s problem in the second round.

5.1 Second-period strategy

Fixing the bidding strategies of the other players, let \(\Phi(b_2|y_1)\) denote the probability with which a particular bidder expects to win with a bid equal to \(b_2\) conditional on \(y_1\) being the type of the first-round winner. The payment he has to make if he wins the auction is given by the second largest bid and follows the distribution \(\Phi(b|y_1)\). Then, the bidder’s expected utility is

\[
EU_2(b_2, \theta; y_1) = \int_0^{b_2} (\theta - p) d\Phi(p|y_1) - \theta \Lambda \Phi(b_2|y_1) [1 - \Phi(b_2|y_1)]
\]

(8)

Differentiating (8) with respect to \(b_2\) yields the first-order condition:

\[
\theta - b_2 - \theta \Lambda [1 - 2\Phi(b_2|y_1)] = 0.
\]

In a symmetric equilibrium, \(\Phi(b_2|y_1) = F_2(\theta|y_1)\) and hence we obtain:

\[
b_2^*(\theta, y_1) = \theta \left\{ 1 - \Lambda \left[ 1 - 2 \frac{F(\theta)^{N-2}}{F(y_1)^{N-2}} \right] \right\}.
\]

First of all, notice that while it is well known that without loss aversion (\(\Lambda = 0\)) in a symmetric equilibrium a bidder submits a bid equal to his own valuation, the above expression shows immediately that this is not the case with reference-dependent preferences.\(^ {22}\) Furthermore, we have

**Lemma 2.** (Discouragement Effect II) If \(\Lambda > 0\), then \(\frac{\partial b_2^*(\theta, y_1)}{\partial y_1} < 0 \; \forall \theta\).

The intuition behind Lemma 2 is the same as for Lemma 1: the higher the type of the winner in the first auction, the less likely a remaining bidder feels about winning in the second auction and, therefore, he bids less aggressively.

\(^{21}\)Notice that in a second-price auction the winning bid is not the price the winner actually ends up paying. This an important point because if the seller were to reveal the winning price of the first auction, then the bidders would infer the type of the highest remaining bidder and a symmetric equilibrium in monotone strategies would fail to exist.

\(^{22}\)As for the first-price auction, \(\Lambda \leq 1\) is a sufficient condition for \(b_2^*(\theta, y_1)\) to be strictly increasing in \(\theta\); see Lange and Ratan (2010).
5.2 First-period strategy

As shown by Lange and Ratan (2010), if bidders are not averse to losses over money first-price and second-price auctions are revenue-equivalent. Hence, we can use the revenue equivalence theorem to derive the first-round equilibrium bidding function.

In the first auction a type-\( \theta \) bidder wins with probability \( F_1(\theta) \) and, if he wins, the price he pays is \( b_1^*(y_1) \), the bid of the highest rival. Thus, his expected payment in the first round is

\[
F_1(\theta) \int_0^\theta b_1^*(y_1) f_1(y_1|\theta) \, dy_1.
\]

In a first-price auction, instead, the winning bidder pays his own bid and therefore his expected payment in the first round is:

\[
F_1(\theta) \beta_1^*(\theta) = F_1(\theta) \left[ \int_0^\theta \beta_2^*(s,s) f_1(s) \, ds \right].
\]

where the equality follows from (7). From revenue equivalence it follows that

\[
\int_0^\theta b_1^*(y_1) f_1(y_1|\theta) \, dy_1 = \int_0^\theta \beta_2^*(s,s) f_1(s) \, ds
\]

and differentiating both sides of the equality with respect to \( \theta \) yields

\[
b_1^*(\theta) = \beta_2^*(\theta, \theta).
\]

Therefore, the equilibrium bid in the first of two sequential second-price auctions is equal to the second-round bid of a sequential first-price auction where, in the latter, the bidder conditions his bid on him having the highest type. Finally, we have that the afternoon effect arises in equilibrium since, by revenue equivalence, in each round the seller’s expected revenue from a second-price auction is equal to the expected revenue from a first-price auction.

6 Sequential vs. Simultaneous Auctions

In this section I focus on simultaneous auctions; that is, auctions in which all the goods are allocated after only one round of bidding. I first derive the equilibrium bidding strategy in a discriminatory auction for two identical items and compare it with the equilibrium of the sequential first-price auction derived in section 4. Then, I compare the seller’s expected revenue between the two formats. In a discriminatory auction, the bidders submit sealed bids and the
highest bidders each receive one object and each pays his own bid. This procedure generalizes the single-object first-price auction, and is the procedure most commonly used for the sale of U.S. Treasury bills.\textsuperscript{23} As before, I will continue to focus on equilibria in symmetric monotone strategies.

Consider a bidder with type $\theta$ who plans to bid as if his type were $\tilde{\theta} \neq \theta$ when all other $N - 1$ bidders follow the equilibrium strategy $\beta(\cdot)$. His expected utility is

$$EU(\theta, \tilde{\theta}) = F_2(\tilde{\theta}) \left[ \theta - \beta(\tilde{\theta}) \right] - \Lambda \theta F_2(\tilde{\theta}) \left[ 1 - F_2(\tilde{\theta}) \right]$$

where $F_2(\tilde{\theta}) \equiv F_1(\tilde{\theta}) + (N - 1) \left[ 1 - F(\tilde{\theta}) \right] F(\tilde{\theta})^{N-2}$ is the probability that $Y_2$, the second highest valuation among $N - 1$, is less than $\theta$ and $\Lambda$ is defined as before. Notice that it is not necessary for a bidder to outbid all his competitors in order to be awarded an object; it is enough to outbid $N - 2$ of them.

Taking FOC of (9) with respect to $\tilde{\theta}$ yields

$$0 = f_2(\tilde{\theta}) \left[ \theta - \beta(\tilde{\theta}) \right] - \beta'(\tilde{\theta}) F_2(\tilde{\theta}) - \Lambda \theta f_2(\tilde{\theta}) \left[ 1 - 2F_2(\tilde{\theta}) \right].$$

Then, substituting $\theta = \tilde{\theta}$ into the FOC and re-arranging results in the following differential equation

$$\frac{d}{d \tilde{\theta}} \left\{ \beta(\tilde{\theta}) F_2(\tilde{\theta}) \right\} = f_2(\tilde{\theta}) \theta \left[ 1 - \Lambda \left[ 1 - 2F_2(\tilde{\theta}) \right] \right]$$

together with the boundary condition that $\beta(0) = 0$. Solving the differential equation yields

$$\beta^*(\theta) = \int_0^\theta s \left\{ 1 - \Lambda \left[ 1 - 2F_2(s) \right] \right\} f_2(s) ds \frac{F_2(\theta)}{F_2(s)}.$$

Again, the equilibrium bidding function can be re-written as a convex combination of the risk-neutral bid and a term that depends on the bidder’s expectations (reference point):

$$\beta^*(\theta) = (1 - \Lambda) \int_0^\theta \frac{s f_2(s) ds}{F_2(\theta)} + \Lambda \int_0^\theta \frac{2s F_2(s) f_2(s) ds}{F_2(\theta)}.$$

Let $V^{sim}(\theta)$ and $V^{seq}(\theta)$ denote a bidder’s expected utility in equilibrium in a simultaneous and sequential auction, respectively. With independent private values and under risk neutrality ($\Lambda = 0$), it is well known that a bidder’s equilibrium expected utility in simultaneous auction is the same as in a sequential auction. Under loss aversion ($\Lambda > 0$), instead, we have:

\textsuperscript{23}An alternative procedure is the uniform-price auction. In a uniform-price auction, the bidders submit sealed bids and the winning bidders are all charged with the same price, equal to the highest rejected bid. This procedure generalizes the single-object second-price auction. The analysis for the uniform-price auction is virtually identical and hence omitted.
Proposition 2. (*Bidder-payoff Equivalence*) If $\Lambda > 0$, then $V^{\text{seq}}(\theta) \geq V^{\text{sim}}(\theta)$ if and only if

\[
\int_0^\theta F_2(s) s f_2(s) \, ds \geq \int_0^\theta \int_0^x F_2(s|x) s f_2(s|x) \, ds f_1(x) \, dx + \int_0^\theta \int_0^x F_2(s|x) s f_2(s|x) \, ds f_1(x) \, dx.
\]

It is easy to see that the inequality in Proposition 2 cannot bind for every $\theta$ unless $F_2(s) = F_2(s|x)$ implying that, generically, sequential auctions and simultaneous ones are not bidder-payoff equivalent. Notice also that a bidder’s ex-ante probability of obtaining an item is the same under both formats and this implies, trivially, that a bidder’s expected gain-loss utility is also the same under both formats. Hence, the difference between $V^{\text{seq}}(\theta)$ and $V^{\text{sim}}(\theta)$ is simply given by the difference in the expected payments.

Figure 1 shows how $V^{\text{seq}}(\theta) - V^{\text{sim}}(\theta)$ varies with $\theta$ for five different values of $N$ ($3, 4, 5, 10$ and $20$) when the bidders’ types are uniformly distributed on $[0, 1]$ (a darker color corresponds to a higher value for $N$). For a given $N$ there exists a cutoff type $\theta^*$ who is indifferent between the two formats. Furthermore, the value of the cutoff $\theta^*$ is increasing in $N$.

![Figure 1: $V^{\text{seq}}(\theta) - V^{\text{sim}}(\theta)$ for $N = 3, 4, 5, 10$ and $20$ with $\theta$ distributed uniformly on $[0, 1]$.](image)

According to Figure 1, low-type bidders prefer simultaneous auctions whereas high-type ones prefer sequential ones. The intuition for these opposing preferences is reminiscent of the result in Köszegi and Rabin (2009) about a loss-averse decision maker’s dislike of interim partial information because it exposes her to possibly unnecessary bad news due to fluctuations in beliefs. More precisely, Köszegi and Rabin (2009) consider an example with two equiprobable possible consumption levels, $c \in \{0, 1\}$ and an expectations-based loss averse agent who has access to a signal $\zeta \in \{0, 1\}$, where the signal is accurate ($\zeta = c$) with probability $w > \frac{1}{2}$. They show that unless $w = 1$, the agent prefers not to receive the signal. In an auction, however, a bidder’s expected consumption depends on his type and higher types are more likely to win. Hence, during the course of a sequential auction higher types are more likely to receive good...
news (and be less discouraged) whereas lower types are more likely to receive bad news (and be more discouraged). Therefore, higher types prefer sequential auctions while lower types prefer simultaneous ones.

Now I compare the seller’s expected revenue between the two formats. I do the comparison for the first-price and discriminatory auctions, but the same results apply to second-price and uniform-price auctions via revenue equivalence. It is now convenient to take the point of view of the seller and consider the order statistics of the values of all \( N \) bidders. Hence, let \( Z_{1}^{(N)} \equiv Z_{1} \) be the highest of \( N \) values, \( Z_{2}^{(N)} \equiv Z_{2} \) be the second-highest and so on. Also, let \( M_{1} \) and \( M_{2} \) be the distributions of \( Z_{1} \) and \( Z_{2} \) respectively, with corresponding densities \( m_{1} \) and \( m_{2} \). Under risk neutrality (\( \Lambda = 0 \)), these two auction formats are revenue-equivalent, both yielding an expected revenue equal to \( 2E[Z_{3}] \) (Milgrom and Weber, 2000). Under loss aversion (\( \Lambda > 0 \)), instead, we have:

**Proposition 3. (Revenue Equivalence)** If \( \Lambda > 0 \), then \( \mathbb{E}[R_{\text{sim}}] \geq \mathbb{E}[R_{\text{seq}}] \) if and only if

\[
\int_{0}^{\theta} \int_{0}^{\theta} \frac{F_{2}(s) s f_{2}(s)}{F_{2}(\theta)} ds [m_{1}(\theta) + m_{2}(\theta)] d\theta \geq \int_{0}^{\theta} \int_{0}^{\theta} \frac{F_{2}(s|x) s f_{2}(s|x)}{F_{1}(\theta)} ds f_{1}(x) dx m_{1}(\theta) d\theta \\
+ \int_{0}^{\theta} \int_{0}^{\theta} \frac{F_{2}(s|y_{1}) s f_{2}(s|y_{1})}{F_{2}(\theta|y_{1})} ds f_{1}(\theta) d\theta m_{1}(y_{1}) dy_{1}.
\]

As for the previous result about bidder-payoff equivalence, it is easy to see that the condition in Proposition 3 cannot bind for every \( N \) unless \( F_{2}(s) = F_{2}(s|x) \), in which case both sides reduce to \( 2E[Z_{3}] \). Therefore, which format yields a higher revenue depends on the number of bidders. Sequential auctions yield a higher revenue when the number of bidders is relatively high. For example, straightforward calculations show that with two objects if \( \theta \sim [0, 1] \), simultaneous auctions yield a higher expected revenue than sequential ones for \( N = 3 \) whereas for \( N = 4 \) the two formats yield the same expected revenue. For \( N \geq 5 \) sequential auctions yield a higher expected revenue than simultaneous ones.

Gathering together the results from this section and the previous ones, we obtain the following corollary:

**Corollary 1. (Comparison of Different Auction Formats)** If \( \Lambda > 0 \), revenue equivalence holds within formats but not between. That is, sequential first-price auctions are revenue-equivalent to sequential second-price auctions and discriminatory auctions are revenue-equivalent to uniform-price auctions. However, sequential first-price auctions are not revenue-equivalent to discriminatory auctions and sequential-second price auctions are not revenue-equivalent to uniform-price auctions.

Recall that in equilibrium a bidder’s expected probability of consumption and expected gain-loss utility are the same under all four types of auctions considered in this paper. Thus, the
non-equivalence result in Corollary 1 is due to the effect that sequential (partial) information revelation has on the bidding strategy of a loss-averse bidder.

7 Extensions and Robustness

In this section I analyze two extensions of the main model. In the first subsection I consider two-round sequential first-price auctions but I remove the assumption that the seller commits to reveal the first-round winning bid prior to the second round. I show that the equilibrium strategy changes but nonetheless the afternoon effect still arises in equilibrium. Moreover, I also show that in this case the seller’s expected revenue is lower.

In the second subsection I consider sequential first-price auctions with price announcement for more than two objects. In this case the afternoon effect arises only for the last two rounds. Nevertheless, I argue that the model would be able to generate a strictly declining price path by either slightly modifying how the reference point is specified or by removing the assumption that the seller reveals the winning bids after each round.

Even though the analysis in this section is carried out only for first-price auctions, the results apply to second-price auctions as well.

7.1 Sequential Auctions without Price Announcements

In the classical reference-free model with independent private values, the optimal bidding strategy does not depend on the (public) history of the winning prices. As shown in sections 4 and 5, however, this is no longer the case with expectations-based reference-dependent preferences. Hence, some questions naturally arise: Is equilibrium bidding different if the seller commits to not revealing the history of winning bids? Does the rationale for the afternoon effect with expectations-based reference-dependent preferences rely on the history of winning bids being publicly available? And, finally, would the seller be better off by not disclosing the history of winning bids? I answer these questions in the context of sequential first-price auctions.

If at the end of the first auction the seller does not reveal the winning bid, then the remaining \(N - 2\) bidders in the second auction face the following problem:

\[
\max_{\tilde{\theta}} \varphi(\sigma) F(\tilde{\theta})^{N-2} \left[\theta - \beta_2(\tilde{\theta}, \sigma)\right] - \Lambda \theta \varphi(\sigma) F(\tilde{\theta})^{N-2} \left[1 - \varphi(\sigma) F(\tilde{\theta})^{N-2}\right]
\]

(10)

where \(\sigma\) denotes the type that a player pretended to be in the first auction and \(\varphi(\sigma) = \frac{(N-1)(1-F(\sigma))}{1-F(\sigma)}\). Thus, \(\varphi(\sigma) F(\tilde{\theta})^{N-2}\) denotes the probability that the second highest of \(N - 1\) draws is below \(\tilde{\theta}\) given that the highest is above \(\sigma\); or, in other words, the probability that a bidder who pretends to be of type \(\tilde{\theta}\) in the second auction wins this auction given that he

\(^{24}\)This also implies, trivially, that the equilibrium is exactly the same with and without price announcement.
pretended to be of type $\sigma$ in the first auction and lost it. Notice that, crucially, the second-round bid might, in principle, depend also on $\sigma$.

As first conjectured by Milgrom and Weber (2000) and later shown by Mezzetti et al. (2008), with interdependent values it is optimal for a bidder of type $\theta$ to report his type truthfully in the second auction if and only if he reported a type $\sigma \leq \theta$ in the first auction. On the contrary, if a bidder reports a type higher than $\theta$ in the first auction, he might want to over-report in the second auction as well. This happens because, as Milgrom and Weber (2000) pointed out, “a bidder might choose a bid a bit higher in the first round in order to have a better estimate of the winning bid, should he lose”. Recall that, with interdependent values, a better estimate of the winning bid is also a better estimate of the value of the object for sale. In our case, however, values are private and independent; hence, it is optimal for a bidder in the second auction to bid according to his true type, no matter what he did in the first one. Indeed, the first-order condition for the problem in (10) is

$$0 = \theta (N-2) F(\theta)^{N-3} f(\theta) - \theta (N-2) F(\bar{\theta})^{N-3} f(\bar{\theta}) \Lambda \left[ 1 - 2\varphi(\sigma) F(\bar{\theta})^{N-2} \right]$$

$$- \frac{\partial \beta_2(\bar{\theta}, \sigma)}{\partial \theta} F(\bar{\theta})^{N-2} - \beta_2(\bar{\theta}, \sigma) (N-2) F(\bar{\theta})^{N-3} f(\bar{\theta}).$$

Hence, $\varphi(\sigma)$ enters the FOC only through the reference point, but it does not affect the “direct” part of a bidder’s payoff and since $\Lambda \leq 1$ the “direct” part carries a higher weight than the reference-dependent part. Substituting $\theta = \bar{\theta}$ and re-arranging results in the following differential equation

$$\frac{\partial}{\partial \theta} \left\{ \beta_2(\theta, \sigma) F(\theta)^{N-2} \right\} = \theta \left\{ 1 - \Lambda \left[ 1 - 2\gamma(\sigma) F(\theta)^{N-2} \right] \right\} (N-2) F(\theta)^{N-3} f(\theta)$$

together with the boundary condition that $\beta_2(0, \sigma) = 0$. Thus, the equilibrium bidding function is

$$\beta_2(\theta, \sigma) = \int_0^\theta x \left\{ 1 - \Lambda \left[ 1 - \frac{2(N-1)(1-F(\sigma))F(x)^{N-2}}{1-F(\sigma)} \right] F(x)^{N-2} \right\} dF(x)^{N-2}.$$

The equilibrium bidding strategy is a function of what the bidder reported in the previous auction since, if the seller does not publicly reveal the first-round winning bid, a bidder who lost the first auction must use his own bid from the previous round in order to infer where he stands in the ranking of the remaining bidders’ values. Hence, the equilibrium strategy depends on the (private) history of the game and, as the following lemma shows, a slightly different form of discouragement effect arise.

**Lemma 3.** (Discouragement Effect III) If $\Lambda > 0$, then $\frac{\partial \beta_2(\theta, \sigma)}{\partial \sigma} < 0 \forall \theta$.

The intuition for this result slightly differs from the one behind Lemmas 1 and 2. When
the winning bid from the first auction is not publicly revealed, a bidder can only use his own first-round bid to formulate an expectation about how likely he is to win in the second one. The higher the type he pretended to be in the first auction, the less likely he feels to win in the current one since not winning the first auction, given that he pretended to have a high type, is bad news about how fierce competition is. This, in turn, implies that the higher is the type a bidder pretended to be in the first auction, the less aggressive his bidding will be in the second auction. Recall that when the seller announces the winning bid after the first auction, the equilibrium bid in the second auction is:

\[
\beta_2^*(\theta, y_1) = \int_0^\theta \left\{ 1 - \Lambda \left[ 1 - 2 \left( \frac{F(x)}{F(y_1)} \right)^{N-2} \right] \right\} \frac{dF(x)}{F(\theta)^{N-2}}.
\]

Hence, we have the following result.

**Lemma 4.** (Effect of information I) Equilibrium bidding in the second auction is more aggressive when the seller does not reveal the winning bid of the first auction if and only if

\[
\frac{(N - 1) \left[ 1 - F(\sigma) \right]}{1 - F(\sigma)^{N-1}} > \frac{1}{F(y_1)^{N-2}}. \tag{11}
\]

First, notice that condition (11) can hold only if \( y_1 > \sigma \). Furthermore, it is easy to see that the right-hand-side of condition (11) is decreasing in \( y_1 \) implying that, for a fixed \( \sigma \), the higher is the type of the winner in the first auction, the more aggressive second-round bidding behavior is when the winning bid is not revealed. Similarly, the left-hand-side of condition (11) is decreasing in \( \sigma \) implying that, for a fixed \( y_1 \) a bidder who pretended to be a low type in the first auction behaves more aggressively in the second one when the winning bid is not revealed. Of course, as I am about to show next, in equilibrium a bidder will report his type truthfully in both auctions so that \( \sigma = \theta \).

Consider a bidder with type \( \theta \) who plans to bid as if his type were \( \tilde{\theta} > \theta \) when all other \( N - 1 \) bidders follow the equilibrium strategy \( \beta_1^* \).\(^{25}\) Furthermore, suppose that all bidders expect to follow the equilibrium strategy \( \tilde{\beta}_2^* \) in the second auction. Then, the bidder will solve the following problem:

\[
\max_{\tilde{\theta}} F_1(\tilde{\theta}) \left[ \theta - \beta_1^*(\tilde{\theta}) \right] + (N - 1) \left[ 1 - F(\tilde{\theta}) \right] F(\theta)^{N-2} \left[ \theta - \tilde{\beta}_2^*(\theta, \tilde{\theta}) \right] - \Lambda \theta Q(\tilde{\theta}, \theta) \left[ 1 - Q(\tilde{\theta}, \theta) \right]
\]

where \( Q(\tilde{\theta}, \theta) = F_1(\tilde{\theta}) + (N - 1) \left[ 1 - F(\tilde{\theta}) \right] F(\theta)^{N-2} \).

\(^{25}\)The analysis is virtually identical for the case \( \tilde{\theta} < \theta \).
Taking the first-order condition for the problem in (12) yields

\[
0 = f_1(\overline{\theta}) \left[ \theta - \beta_1(\overline{\theta}) \right] - \beta_1(\overline{\theta}) F_1(\overline{\theta}) - (N - 1) f(\overline{\theta}) F(\overline{\theta})^{N-2} \left[ \theta - \beta_2(\theta, \overline{\theta}) \right] - \frac{\partial \beta_2(\theta, \overline{\theta})}{\partial \theta} (N - 1) \left[ 1 - F(\overline{\theta}) \right] F(\overline{\theta})^{N-2}.
\]

Substituting \( \theta = \overline{\theta} \) and re-arranging results in the following differential equation

\[
f_1(\overline{\theta}) \beta_2(\theta, \overline{\theta}) - (N - 1) \left[ 1 - F(\overline{\theta}) \right] F(\overline{\theta})^{N-2} \frac{\partial \beta_2(\theta, \overline{\theta})}{\partial \theta} = \frac{d}{d\theta} \left\{ \beta_1(\theta) F_1(\theta) \right\}
\]

(13)

together with the boundary condition that \( \beta_1(0) = 0 \). Notice that, crucially, \( \frac{\partial \beta_2(\theta, \overline{\theta})}{\partial \theta} \neq 0 \). That is, by misrepresenting his type in the first auction, a bidder is not just affecting the probability of getting to the second auction — like in the classical reference-free model — but he is also affecting his own future bid in the second auction. This occurs because, with no price announcement between auctions, a player’s bid in the current auction affects his reference point in the next one. Solving the differential equation in (13) yields

\[
\hat{\beta}_1(\theta) = \frac{\int_0^\theta \beta_2(s, s) f_1(s) \, ds}{F_1(\theta)} - \frac{\int_0^\theta \left\{ \frac{\partial \beta_2(s, \overline{\theta})}{\partial \theta} \right|_{\theta = s} \frac{1 - F(s)}{f(s)} \right\} f_1(s) \, ds}{F_1(\theta)}.
\]

(14)

Notice that

\[
\frac{\partial \beta_2(\theta, \overline{\theta})}{\partial \theta} = -\frac{2 (N - 1) A f(\overline{\theta}) \left[ 1 - F_2(\overline{\theta}) \right] \int_0^\theta x F(x)^{N-2} dF(x)^{N-2}}{\left[ 1 - F(\overline{\theta}) \right]^{N-1} F(\overline{\theta})^{N-2}} < 0
\]

where \( F_2(\overline{\theta}) = F(\overline{\theta})^{N-1} + (N - 1) \left[ 1 - F(\overline{\theta}) \right] F(\overline{\theta})^{N-2} \).

The following lemma shows that, compared to the case analyzed in section 4, bidders behave less aggressively in the first auction when the seller commits to not revealing the winning bid.

**Lemma 5.** (Effect of information II) Equilibrium bidding in the first round is more aggressive when the seller commits to publicly reveal the winning bid prior to the second round; that is, \( \beta_2^*(\theta) - \hat{\beta}_1(\theta) \geq 0 \ \forall \theta \) and the inequality is strict if \( \theta < \overline{\theta} \).

The intuition behind Lemma 5 is the following. When anticipating that the seller will not reveal the winning bid of the first auction prior to the second one, a bidder knows that his bid in the first auction — in case he does not win — will determine his reference point in the second one. A high bid in the first auction, hence, implies also a high reference point in the second auction. Having a high reference point in the second auction, however, exposes the bidder to
a greater disappointment in case he were to lose the second auction as well. Therefore, if the seller commits to not revealing the first-round winning price, bidders bid less aggressively in the first auction.

Furthermore, the seller’s total expected revenue is higher when she commits to revealing the first round’s winning bid.

**Proposition 4. (Revenue)** The seller’s expected revenue is higher when she commits to disclose the winning bid from the first auction prior to the second one.

From an ex-ante perspective, bidders going into the second round without knowing the type of the winner in the first round are exposed to much more uncertainty about competition compared to bidders who know the type of the first-round winner. Indeed, in the latter case every bidder going into the second auction knows that all of his competitors’ types are below a certain cutoff type while in the former a bidder only knows that the winner had a higher type than his. An expectations-based loss-averse bidder dislikes uncertainty in his consumption outcomes because he dislikes the possibility of a resulting loss more than he likes the possibility of a resulting gain (so he is “first-order” risk averse; see Kőszegi and Rabin, 2007). As auctions without price announcements expose bidders to a greater background risk, they react by bidding less aggressively. Therefore, compared to the analysis in section 4, if the seller does not reveal the winning price of the first auction prior to the second one, her expected revenue decreases.\(^{26}\) Nevertheless, the afternoon effect still arises in equilibrium.

**Proposition 5. (Afternoon Effect II)** If \( \Lambda > 0 \), then the price sequence in a two-round first-price auction without price announcement is a supermartingale and the afternoon effect arises in equilibrium. That is,

\[
p_1 = \tilde{\beta}_1(y_1) > \mathbb{E} \left[ p_2 | \tilde{\beta}_1(y_1) \right] = \mathbb{E} [ p_2 | p_1 ] .
\]

Like in Section 4 in equilibrium prices decline because of the discouragement effect, but the intuition is slightly different. When the seller commits to not disclosing the winning bid, in the first round bidders are willing to pay a positive premium — equal to the second term on the right-hand-side of (14) — in order to avoid having to go to the second round and being discouraged.

To conclude, with expectations-based reference-dependent preferences, the equilibrium bidding strategy changes depending on whether the seller commits to publicly reveal the winning bids from the previous rounds. With price announcement first-round bids are always higher whereas bids in the second round can be higher or lower than without price announcement.

\(^{26}\)This result is akin to the famous “Linkage Principle”: auctioneers have an incentive to pre-commit to revealing all available information. (Milgrom and Weber, 1982).
Furthermore, the seller’s expected revenue is higher when she commits to disclose the previous round’s winning bid. In either case, however, equilibrium prices follow a declining path.

7.2 More than Two Units

The equilibrium derived in sections 4 and 5 easily generalizes to more than two units (rounds). However, in this case the afternoon effect reduces to a “last-round” effect. To see why, suppose that $K > 2$ units are sold in a sequence of $K$ first-price auctions. The winning prices $p_1, p_2, \ldots, p_{K-1}$ are commonly known to the remaining bidders in auction $k > 1$. Let $Y_k \equiv Y^{(N-1)}_k$ denote the $k$th-highest order statistic among $N - 1$ draws, with distribution $F_k$ and corresponding density $f_k$. As before, I will derive the symmetric bidding strategies by working backwards from the last auction.

In the $K^{th}$ auction, the last one, following the same reasoning as in section 4, the bidding strategy is

$$
\beta_K^* (\theta, y_{K-1}) = \int_0^\theta x \{1 - \Lambda [1 - 2F_K (x|y_{K-1})]\} f_K (x|y_{K-1}) \, dx \over F_K (\theta|y_{K-1}).
$$

Once again, there is a discouragement effect as it is easy to see that $\beta_K^* (\theta, y_{K-1})$ is decreasing in $y_{K-1}$ — the type of the previous round’s winner. Next, consider a bidder’s expected utility in round $K - 1$:

$$
EU_{K-1} (\tilde{\theta}, \theta; y_{K-2}) = F_{K-1} (\tilde{\theta}|y_{K-2}) [\theta - \beta_{K-1} (\tilde{\theta}, y_{K-2})]
+ \int_\theta^{y_{K-2}} F_K (\theta|y_{K-1}) [\theta - \beta_K (\theta, y_{K-1})] f_{K-1} (y_{K-1}|y_{K-2}) \, dy_{K-1}

- \Lambda \theta \left[ F_{K-1} (\tilde{\theta}|y_{K-2}) + \int_\theta^{y_{K-2}} F_K (\theta|y_{K-1}) f_{K-1} (y_{K-1}|y_{K-2}) \, dy_{K-1} \right] \times

\left[ 1 - F_{K-1} (\tilde{\theta}|y_{K-2}) - \int_\theta^{y_{K-1}} F_K (\theta|y_{K-1}) f_{K-1} (y_{K-1}|y_{K-2}) \, dy_{K-1} \right].
$$

Using familiar steps by now, it is easy to show that the equilibrium bid in round $K - 1$ is

$$
\beta_{K-1}^* (\theta) = \int_0^\theta \beta_K (x, x) f_{K-1} (x|y_{K-2}) \, dx \over F_{K-1} (\theta|y_{K-2}).
$$

Notice, crucially, that $\beta_{K-1}^*$ is independent of $y_{K-2}$. This might look surprising at first since an essential feature of the analysis in section 4 was that expectations-based loss aversion induces history dependence even with independent private values. The reason for this is that a bidder’s reference point, in each auction, equals his overall probability of consumption. In other words, a bidder only cares about whether or not he will get an item, but not about in which round he actually gets it. And since in every auction, except the last one, a bidder conditions his bid on having the highest remaining type, in equilibrium a bidder believes that if he loses the current
auction he will win the next one for sure. Hence, the discouragement effect is only present in the last stage $K$, when the bidder will find out for sure whether he gets an item. This, in turn, implies that

$$\mathbb{E}[p_{j+1}|p_j] = p_j$$

for $j = 1, \ldots, K - 2$. For the last two rounds, instead, we have

$$\mathbb{E}[p_K|p_{K-1}] > p_{K-1}.$$ 

Does this mean that for sequential auctions of more than two rounds expectations-based loss aversion cannot generate a strictly declining price path in equilibrium? Not necessarily. A reference point equal to the overall probability of consumption, like the one assumed in this paper, is most appropriate for auctions that are held in relatively short time, like in the same day. For longer sequential auctions which take place over several days or weeks, however, a model in which a bidder’s reference point in a given auction equals his probability of getting the item in that same auction might be more appropriate. In this case, as argued in Köszegi and Rabin (2009), a decision-maker would experience expected gain-loss utility with respect to the outcome of each single auction. Moreover, he would experience expected gain-loss utility also with respect to the change in beliefs about how likely he is to win any of the future auctions based on the outcome of the current one. Finally, it is easy to see that if the winning bids from previous rounds are not publicly disclosed, equilibrium bids are (private) history-dependent in every round, in which case a strictly declining price path would arise in equilibrium.

8 Conclusions

In this paper I have proposed a novel, preference-based explanation, for the afternoon effect observed in sequential auctions by positing that bidders are loss-averse. Expectations-based reference-dependent preferences create an informational externality, the discouragement effect, that makes the equilibrium strategy history-dependent: the higher is the type of the winner in the first auction, the less aggressively the remaining bidders will bid in the second one. The effect of this informational externality on the second-round bids is the opposite of the one that arises in models with common values where the higher the signal of the first-round winner, the higher is the estimated value of the object for all remaining bidders who in turn bid more aggressively. Therefore, by looking at the distribution of bids in the second auction, one can use the discouragement effect to empirically test the implications of loss aversion against the implications of the classical model with either private values (no history dependence) or common values (the higher the winning price in the first auction, the more aggressively bidders behave in the second auction).
In equilibrium a bidder must be indifferent between winning in the first auction or in the second one. Hence, in the first auction he chooses the optimal bid conditional on having the highest type and being the price-setter. By conditioning his bid in the first auction on having the highest type, however, a bidder expects not to feel discouraged in the second auction. Thus, in equilibrium bidders underestimate the discouragement effect and bid more aggressively in the first auction so that prices follow a declining path.

In addition to rationalizing the afternoon effect, loss aversion delivers new testable implications that are interesting in their own rights. For example, when bidders are expectations-based loss-averse simultaneous and sequential auctions are not revenue-equivalent nor bidder-payoff equivalent. Furthermore, in a sequential auction the seller always achieves a higher expected revenue by committing to reveal the winning bid after each round.

However, despite being able to explain the afternoon effect and to generate new testable predictions, the model suffers from some limitations. First, I have departed from the original model of expectations-based reference-dependent preferences proposed by Köszegi and Rabin (2006) by assuming that bidders are loss-averse only over consumption, but not over money. Admittedly not very realistic, this assumption however considerably simplifies the analysis. For example, with loss aversion over money, first-price and second-price auctions are not revenue equivalent anymore and the analysis of the second-price auction becomes much more intricate. Furthermore, in some of the auctions discussed in the Introduction and Section 2 the goods up for sale are not sought after by the bidders for their consumption value, but rather for commercial purposes (i.e., a production or a resale motive). If this is the case, then what bidders care about is the monetary value of the good and a model of reference-dependent preferences where gains and losses are evaluated with respect to the overall gains from trade \((\theta - p)\) might be more appropriate.

There are several interesting directions for future research. One would be to study sequential dynamic (open) auctions, like English and Dutch auctions. If bidders are risk-neutral and have independent private values, it is well-known that the English (resp. Dutch) auction is strategically equivalent to a second-price (resp. first-price) sealed-bid auction. This equivalency is unlikely to hold when bidders are expectations-based loss-averse.\(^{27}\)

Another interesting extension would be to analyze a model where loss-averse bidders have interdependent values. Mezzetti (2011) showed that the informational externality arising from the interdependency between the bidders’ values generates an increasing price sequence. The current paper argues that expectations-based loss aversion also creates an informational externality which, however, pushes prices to decline. Which effect will dominate would likely depend on the strength of loss aversion as well as on how sensitive a bidder’s value is to his rivals’ signals.

\(^{27}\)See Ehrhart and Ott (2014) for a first analysis of single-object dynamic auctions with expectations-based loss-averse bidders.
A Proofs

Proof of Lemma 1: We have
\[
\frac{\partial \beta^*_2(\theta, y_1)}{\partial y_1} = -\frac{2\Lambda (N-2)^2 F(y_1)^{N-3} f(y_1) \int_0^\theta x F(x)^{2N-5} f(x) \, dx}{[F(y_1) F(\theta)]^{2(N-2)}} < 0. \tag*{■}
\]

Proof of Proposition 1: We have
\[
\beta^*_1(y_1) = \int_0^{y_1} \frac{\beta^*_2(\theta, \theta) f_1(\theta) d\theta}{F_1(y_1)} > \int_0^{y_1} \frac{\beta^*_2(\theta, y_1) f_1(\theta) d\theta}{F_1(y_1)} = \mathbb{E}[p_2|p_1]\]
where the inequality follows from Lemma 1. ■

Proof of Lemma 2: We have
\[
\frac{\partial \beta^*_2(\theta, y_1)}{\partial y_1} = -\frac{2\Lambda (N-2) \theta F(\theta)^{N-2}}{F(y_1)^{N-1}} < 0. \tag*{■}
\]

Proof of Proposition 2: We have that
\[
V^{sim}(\theta) = F_2(\theta) [\theta - \beta^*_1(\theta)] - \Lambda \theta F_2(\theta) [1 - F_2(\theta)]
\]
and
\[
V^{seq}(\theta) = F_1(\theta) [\theta - \beta^*_1(\theta)] + \int_\theta^{\bar{\theta}} F_2(\theta|y_1) [\theta - \beta^*_2(\theta, y_1)] f_1(y_1) \, dy_1
\]
\[
-\Lambda \theta \left[ F_1(\theta) + \int_{\theta}^{\bar{\theta}} F_2(\theta|y_1) f_1(y_1) \, dy_1 \right] \left[ 1 - F_1(\theta) - \int_{\theta}^{\bar{\theta}} F_2(\theta|y_1) f_1(y_1) \, dy_1 \right].
\]
Hence,
\[
V^{seq}(\theta) - V^{sim}(\theta) = F_2(\theta) \beta^*_1(\theta) - F_1(\theta) \beta^*_1(\theta) - \int_{\theta}^{\bar{\theta}} F_2(\theta|y_1) \beta^*_2(\theta, y_1) f_1(y_1) \, dy_1
\]
\[
- (1 - \Lambda) \int_0^{\theta} s f_2(s) \, ds + \Lambda \int_0^{\theta} 2F_2(s) s f_2(s) \, ds
\]
\[
- (1 - \Lambda) \int_0^{\theta} \int_0^{x} s f_2(s|x) \, ds f_1(x) \, dx - \Lambda \int_0^{\theta} \int_0^{x} 2F_2(s|x) s f_2(s|x) \, ds f_1(x) \, dx
\]
\[
- (1 - \Lambda) \int_{\theta}^{\bar{\theta}} \int_0^{x} s f_2(s|x) \, ds f_1(x) \, dx - \Lambda \int_{\theta}^{\bar{\theta}} \int_0^{x} 2F_2(s|x) s f_2(s|x) \, ds f_1(x) \, dx.
\]
Notice that

\[ \int_0^\theta s f_2(s) \, ds = \int_0^\theta \int_0^x s f_2(s|x) \, ds f_1(x) \, dx + \int_0^\theta \int_0^x s f_2(s|x) \, ds f_1(x) \, dx \]

implying that

\[ V^{\text{seq}}(\theta) - V^{\text{sim}}(\theta) \geq 0 \iff \]

\[ \int_0^\theta F_2(s) s f_2(s) \, ds \geq \int_0^\theta \int_0^x F_2(s|x) s f_2(s|x) \, ds f_1(x) \, dx + \int_0^\theta \int_0^x F_2(s|x) s f_2(s|x) \, ds f_1(x) \, dx \]

and this concludes the proof. ■

**Proof of Proposition 3:** We have that

\[ \mathbb{E}[R^{\text{seq}}] = \int_0^\theta \beta_1^*(\theta) m_1(\theta) \, d\theta + \int_0^\theta \int_0^{y_1} \frac{\beta_2^*(\theta, y_1) f_1(\theta)}{F_1(y_1)} \, d\theta m_1(y_1) \, dy_1 \]

and

\[ \mathbb{E}[R^{\text{sim}}] = \int_0^\theta \beta_2^*(\theta) [m_1(\theta) + m_2(\theta)] \, d\theta. \]

Hence,

\[ \mathbb{E}[R^{\text{sim}}] - \mathbb{E}[R^{\text{seq}}] = \]

\[ \int_0^\theta \left[ (1 - \Lambda) \int_0^\theta s f_2(s) \, ds + \Lambda \int_0^\theta 2 F_2(s) s f_2(s) \, ds \right] \left[ m_1(\theta) + m_2(\theta) \right] \, d\theta \]

\[ - \int_0^\theta \left[ (1 - \Lambda) \int_0^\theta x f_2(x|s) \, dx f_1(s) \, ds + \Lambda \int_0^\theta 2 F_2(x|s) x f_2(x|s) \, ds f_1(s) \, ds \right] \frac{m_1(\theta) \, d\theta}{F_1(\theta)} \]

\[ - \int_0^\theta \left[ (1 - \Lambda) \int_0^{y_1} \frac{x f_2(x|y_1)}{F_2(\theta|y_1)} \, dx f_1(\theta) \, d\theta + \Lambda \int_0^{y_1} \int_0^\theta 2 F_2(x|y_1) \frac{x f_2(x|y_1)}{F_2(\theta|y_1)} \, dx f_1(\theta) \, d\theta \right] \frac{m_1(y_1) \, dy_1}{F_1(y_1)}. \]

Notice that

\[ \int_0^\theta \int_0^\theta s f_2(s) \, ds [m_1(\theta) + m_2(\theta)] \, d\theta = \int_0^\theta \int_0^\theta s f_2(s|x) \, ds f_1(x) \, dx m_1(\theta) \, d\theta 
+ \int_0^\theta \int_0^{y_1} \int_0^\theta s f_2(s|y_1) \, ds f_1(\theta) \, d\theta m_1(y_1) \, dy_1 \]

implying that

\[ \mathbb{E}[R^{\text{sim}}] - \mathbb{E}[R^{\text{seq}}] \geq 0 \iff \]

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\[
\int_0^\theta \int_0^\theta \frac{F_2(s) s f_2(s)}{F_2(\theta)} \, ds \, [m_1(\theta) + m_2(\theta)] \, d\theta \geq \int_0^\theta \int_0^\theta \frac{F_2(s|\theta) s f_2(s|\theta)}{F_1(\theta)} \, ds \, f_1(\theta) \, dxm_1(\theta) \, d\theta
\]

\[
+ \int_0^\theta \int_0^\theta \int_0^\theta \frac{F_2(s|\theta_1) s f_2(s|\theta_1)}{F_2(\theta_1)} \, ds \, f_1(\theta_1) \, d\theta m_1(\theta_1) \, d\theta_1
\]

and this concludes the proof. \[\blacksquare\]

**Proof of Lemma 3:** We have

\[
\frac{\partial \hat{\beta}_2(\theta, \sigma)}{\partial \sigma} = -\frac{2(N-1) \Lambda f(\sigma) [1 - F_2(\sigma)] \int_0^\theta xF(x)^{N-2} \, dF(x)^{N-2}}{[1 - F(\sigma)^{N-1}]^2} < 0
\]

where \(F_2(\sigma) = F(\sigma)^{N-1} + (N-1) [1 - F(\sigma)] F(\sigma)^{N-2}\). \[\blacksquare\]

**Proof of Lemma 4:** Immediate by inspection. \[\blacksquare\]

**Proof of Lemma 5:** We have

\[
\beta^*_1(\theta) - \hat{\beta}_1(\theta) = \int_0^\theta \left[ \beta^*_2(s, s) - \hat{\beta}_2(s, s) + \frac{\partial \hat{\beta}_2(s, \theta)}{\partial \theta} \bigg|_{\theta = s} \frac{1 - F(s)}{f(s)} \right] f_1(s) \, ds.
\]

A sufficient condition for the above expression to be non-negative is

\[
\beta^*_2(s, s) - \hat{\beta}_2(s, s) \geq -\frac{\partial \hat{\beta}_2(s, \theta)}{\partial \theta} \bigg|_{\theta = s} \frac{1 - F(s)}{f(s)}
\]

\[\Leftrightarrow \quad 2\Lambda \left\{ 1 - \frac{N-1 [1 - F(s)]}{1 - F(s)^{N-1}} \right\} \int_0^\theta xF(x)^{N-2} \, dF(x)^{N-2} \geq 2\Lambda (N-1) [1 - F(s)] [1 - F_2(s)] \int_0^\theta xF(x)^{N-2} \, dF(x)^{N-2}
\]

\[\Leftrightarrow \quad \frac{1}{F(s)^{N-2}} - \frac{(N-1)[1 - F(s)]}{1 - F(s)^{N-1}} \geq \frac{(N-1)[1 - F(s)][1 - F_2(s)]}{1 - F(s)^{N-1}}
\]

\[\Leftrightarrow \quad \frac{1 - F_2(s)}{F(s)^{N-1}} \geq \frac{1 - F_2(s)}{1 - F(s)^{N-1}}
\]

\[\Leftrightarrow \quad 1 \geq F_2(s)
\]

and this concludes the proof. \[\blacksquare\]

**Proof of Proposition 4:** We know from Lemma 5 that first-round bidding is more aggressive with price announcement and this implies, trivially, that the first-round expected revenue is higher with price announcement. Hence, it suffices to show that the second-round expected revenue is also higher with price announcement; that is:
Substituting and re-arranging yields:

\[
(1 - \Lambda) \int_0^{\bar{\theta}} x \frac{X (\theta) f_1 (\theta) \, d\theta}{F (1)} \, m_1 (y_1) \, dy_1 + \Lambda \int_0^{\bar{\theta}} \frac{2X (\theta) f_1 (\theta) \, d\theta}{F (1) (N - 2)} \, m_1 (y_1) \, dy_1 \geq \int_0^{\bar{\theta}} \beta_2 (\theta) \, m_2 (\theta) \, d\theta
\]

where \(X (\theta) = \int_0^\theta \frac{xdF (x)}{F (x)}\) and \(\bar{X} (\theta) = \int_0^\theta \frac{xF (x) \, dF (x)}{F (x)}\).

Notice that

\[
\int_0^{\bar{\theta}} y \frac{X (\theta) f_1 (\theta) \, d\theta}{F (1)} \, m_1 (y_1) \, dy_1 = \int_0^{\bar{\theta}} X (\theta) \, m_2 (\theta) \, d\theta
\]

which further implies that

\[
\int_0^{\bar{\theta}} \frac{\bar{X} (\theta) f_1 (\theta) \, d\theta}{F (1)} \, m_1 (y_1) \, dy_1 = \int_0^{\bar{\theta}} \bar{X} (\theta) \, m_2 (\theta) \, d\theta.
\]

The result then follows since

\[
\int_0^{\bar{\theta}} \frac{2\Omega (y_1)}{F (1) (N - 2)} \, m_1 (y_1) \, dy_1 \geq \int_0^{\bar{\theta}} \frac{2(N - 1) [1 - F (\theta)]}{1 - F (\theta) (N - 1)} \, \bar{X} (\theta) \, m_2 (\theta) \, d\theta
\]

as

\[
\frac{1}{F (s)} \geq \frac{(N - 1) [1 - F (s)]}{1 - F (s) (N - 1)} \quad \Leftrightarrow \quad 1 \geq F_2 (s)
\]

which holds \(\forall s \in [0, \bar{\theta}]\).

**Proof of Proposition 5:** We have

\[
\tilde{\beta}_1 (y_1) = \int_0^{\bar{\theta}} \beta_2 (\theta) \, f_1 (\theta) \, d\theta \quad \frac{F (1)}{y_1} - \int_0^{\bar{\theta}} \left\{ \frac{\partial \beta_2 (\theta \tilde{\theta})}{\partial \tilde{\theta}} \bigg|_{\tilde{\theta} = \bar{\theta}} \frac{1 - F (\theta)}{F (1)} \right\} f_1 (\theta) \, d\theta
\]

\[
> \int_0^{\bar{\theta}} \beta_2 (\theta \tilde{\theta}) \, f_1 (\theta) \, d\theta
\]

\[
= \mathbb{E} [p_2 | p_1]
\]

where the inequality follows from Lemma 3.
References


