THE UNIVERSITY OF MELBOURNE

Department of Economics

Working Paper Series

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July 2010

Research Paper Number 1099

ISSN: 0819-2642
ISBN: 978 0 7340 4452 5

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Modelling Aggregate Personal Income Tax Revenue in Multi-Schedular and Multi-Regional Structures

By

John Creedy and José Félix Sanz-Sanz[1]

Abstract

This paper derives analytical expressions for aggregate personal income tax revenue obtained from a multi-schedular and multi-regional personal income tax system, with revenue divided among central and regional governments. Aggregate income tax revenue is expressed as a function of characteristics of the distribution of taxable income, making it possible to identify the sources of revenue differences among regions. The approach is applied to the tax structure in Spain, and the effects of income distribution differences among the Spanish regions is examined.

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1 Introduction

A number of countries have separate tax structures, with different tax rates and income thresholds, for different sources of income, in addition to different rules regarding eligible deductions. Thus they are said to have multiple ‘schedules’. In addition, tax revenue in some countries is divided between central government and autonomous regions or states. Some transfer payments, in the form of tax credits, may also be administered at both the central and regional level. One feature of fiscal policy in a multi-regional economy involves transfers among regions having quite different population sizes and income distributions, relating not only to the forms of the relevant distributions but the extent to which incomes are derived from different sources. These elements introduce a number of complexities in modelling aggregate tax revenue.

For tax planning purposes it is important to be able to model aggregate revenue allowing for these complications and in particular to be able to estimate the implications for total income tax revenue of a range of exogenous changes. Those changes include simple changes in tax rates and thresholds and, importantly, changes in the income distributions of different income sources within regions.

The aim of this paper is therefore to provide a method of examining the implications for total tax revenue of a number of changes, including changes in the distribution of income and in the tax structure itself, in a multi-schedular and multi-regional economy. It is shown how an expression for aggregate tax revenue depends on proportions of people, within each region, and proportions of total income between the income thresholds of the income tax functions.

The methods used are widely applicable. However, after deriving the general results, the approach is applied to the Spanish personal income tax structure. This provides an interesting case study as it has undergone significant reforms, in additional to the

type of base-broadening and rate-reducing changes which have been common in many other countries.[2] Furthermore, income taxation since 2002 is shared between central and regional governments, consisting of 15 autonomous regions within the Common Territory. In addition, different tax rates and thresholds apply to a range of income sources in a multi-schedular tax structure. Various tax credits exist at central and regional levels, some of which depend on non-income as well as income characteristics of tax units.

Section 2 formulates analytical expressions for the tax liability of each tax unit. Aggregate tax revenue is then examined in more detail in Section 3. The method is applied to Spain in Section 4. Brief conclusions are given in Section 5.

2 A Tax Structure with Several Schedules and Central and Regional Rates

This section describes a personal income tax structure containing several schedules where, in addition, tax revenue is divided between each region and the central government.

2.1 Income Taxation of a Tax Unit

Let x_{hi} denote the taxable income of tax unit *h* from source $i = 1,...,I$. In transforming from gross to taxable income, there are typically tax-deductible expenditures and non-income allowances. The operation of these allowances is not considered here, and taxable income is taken as the starting point, rather than gross income.

Suppose that the income tax structure has marginal tax rates t_{ki} and thresholds a_{ki} for $k = 1,...,K$, where t_{ki} applies between a_{ki} and $a_{k+1,i}$ (with $a_{K+1,i} = \infty$). Where separate tax rates are imposed at the central and regional government levels, the

[2] On the recent reforms, see OECD (2006).

income thresholds are typically common. This is the case examined here. Thus, letting superscripts *C* and *R* refer to central and regional rates respectively:

$$t_{ki} = t_{ki}^{C} + t_{ki}^{R} \tag{1}$$

For a multi-step tax structure with *K* steps, $T(x) = 0$ for $a_0 = 0 < x < a_1$, $T(x) = t_1(x - a_1)$ for $a_1 < x < a_2$, and $T(x) = t_1(a_2 - a_1) + t_2(x - a_2)$ for $a_2 < x < a_3$, and so on. Then in general, if $a_k < x < a_{k+1}$, Creedy and Gemmell (2006, p. 25) show that:

$$T(x) = t_k(x - a'_k) \tag{2}$$

where:

$$a'_k = \frac{1}{t_k}\sum_{j=1}^{k} a_j(t_j - t_{j-1}) \tag{3}$$

Hence in the present context, if $a_{ki} < x_{hi} < a_{k+1,i}$, unit *h* is in the *k*th tax bracket for source *i* and the following expressions describe income taxation at central and regional levels.

$$T_i^{C}\left(x_{hi} \middle| a_{ki} < x_{hi} < a_{k+1,i}\right) = t_{kih}^{C}\left(x_{hi} - a'^{C}_{kih}\right) \tag{4}$$

$$T_i^{R}\left(x_{hi} \middle| a_{ki} < x_{hi} < a_{k+1,i}\right) = t_{kih}^{R}\left(x_{hi} - a'^{R}_{kih}\right) \tag{5}$$

The terms a'^{C}_{ki} and a'^{R}_{ki} are the corresponding thresholds such that tax liability in a multi-threshold tax structure can be expressed in terms of an equivalent single-rate structure. In writing the expressions (4) and (5) the marginal tax rate terms, *t*, along with the effective thresholds, a', need the *h* subscripts, in order to clarify the point that the tax rates and thresholds indicated are those that apply to the tax unit in question, depending on the tax bracket into which the unit falls.

Suppose, in addition, there are central and regional government non-refundable tax credits of C_C and C_R. Total tax paid by unit *h* is expressed as:

$$T(\sum_i x_{hi}) = \max\left\{0, \sum_{i=1}^{I} T_i^{C}(x_{hi}) - C_C\right\} + \max\left\{0, \sum_{i=1}^{I} T_i^{R}(x_{hi}) - C_R\right\} \tag{6}$$

The existence of non-refundable tax credits means that several cases must be distinguished. The most common situation where tax unit *h* is such that

$\sum_{i=1}^{I} T_i^C(x_{hi}) > C_C$ and $\sum_{i=1}^{I} T_i^R(x_{hi}) > C_R$. The expression given in (6) above for tax liability is thus simplified to:

$$T(\sum_i x_{hi}) = \sum_{i=1}^{I} \left\{ T_i^C(x_{hi}) + T_i^R(x_{hi}) \right\} - \left(C_R + C_C \right) \tag{7}$$

and:

$$T(\sum_i x_{hi}) = \sum_{i=1}^{I} \left\{ t_{kih} x_{hi} - (t_{kih}^C a'^C_{kih} + t_{kih}^R a'^R_{kih}) \right\} - \left(C_R + C_C \right) \tag{8}$$

A further simplification is available where, as here, central and regional income thresholds are the same. Using the above expression for a'_k, it can be shown that:

$$t_{kih}^C a'^C_{kih} + t_{kih}^R a'^R_{kih} = \sum_{j=1}^{k_{ih}} a_{ji} \left(t_{ji} - t_{j-1,i} \right) \tag{9}$$

The other cases follow directly from the above results.

3 Total Tax Revenue

This section derives aggregate revenue for the tax system described above. It also demonstrates how the effects on revenue of changes in income distribution, along with changes in the tax rates and income thresholds can easily be examined.

3.1 Aggregation over Individuals

Suppose there are n_{C+R} taxpayers whose central and regional tax exceeds the relevant credits, equations (8) and (9) can be used to write their tax as:

$$T(\sum_{i=1} x_{hi}) = \sum_{i=1}^{I} \left\{ t_{kih}(x_{hi}) - \sum_{j=1}^{k_{ih}} a_{ji}(t_{ji} - t_{j-1,i}) \right\} - (C_R + C_C) \tag{10}$$

which becomes:

$$T\left(X\right) = \sum_{i=1}^{I} \left\{ t_{kih}\left(x_{hi} - a'_{jih}\right) \right\} - (C_R + C_C) \tag{11}$$

where $a'_{jih} = \frac{1}{t_{kih}} \sum_{j=1}^{k_{ih}} a_{ji}(t_{ji} - t_{j-1,i})$. Similarly, suppose there are n_R and n_C tax units whose tax exceeds regional and central government tax credits respectively. Then there are *N* taxpayers, that is whose tax liability is positive, with $N = n_{C+R} + n_R + n_C$).

The total tax revenue can therefore be expressed as:

$$T = \sum_{h=1}^{N} \sum_{i=1}^{I} \left\{ t_{kih} (x_{hi} - a'_{jih}) \right\} - N(\bar{C}_R + \bar{C}_C) \tag{12}$$

Where, as above, x_{hi} is the taxable income for income source i for tax unit h. The terms $\bar{C}_C$ and $\bar{C}_R$ denote the appropriate average value defined over taxpayers, remembering the tax schedule asymmetry whereby tax must be positive. The first term in (12) can be rewritten as:

$$\sum_{i=1}^{I} \sum_{h=1}^{N} \left\{ t_{kih} (x_{hi} - a'_{jih}) \right\} \tag{13}$$

In the case of a single source of income, with a multi-step function, the tax per person can be expressed in terms of summary information about the distribution of taxable income, which determines the proportion of tax units falling into the various marginal tax rate groups.[3] For example, suppose that $F(x)$ denotes the distribution function of taxable income, *x>0*. Tax per unit is thus:

$$\sum_{k=1}^{K} \left[t_k \int_{a_k}^{a_{k+1}} (x - a'_k) \, dF(x) \right] \tag{14}$$

Define $F_1(x)$ as the first-moment distribution function, that is the proportion of total income of units below *x*, and introduce the general term $G(a_k)$, defined as:

$$G(a_k) = \left\{ F_1(a_{k+1}) - F_1(a_k) \right\} - \frac{a'_k}{\bar{x}} \left\{ F(a_{k+1}) - F(a_k) \right\} \tag{15}$$

The first term in curly brackets gives the proportion of total income between adjacent thresholds, and the second term in curly brackets is the number of tax units between those thresholds. The expression in (16) can also be written as:

[3] For further discussion, see Creedy and Gemmell (2006).

$$G(a_k) = \{F(a_{k+1}) - F(a_k)\}\left[\frac{F_1(a_{k+1}) - F_1(a_k)}{F(a_{k+1}) - F(a_k)} - \frac{a'_k}{\bar{x}}\right] \tag{16}$$

The first term inside the square brackets of (16) is the slope of the Lorenz curve of the relevant distribution of income, between the two points associated with adjacent income tax thresholds. The Lorenz curve has a slope of 45 degrees at the arithmetic mean; that is, $\frac{dF_1(\bar{x})}{dF(\bar{x})} = 1$. The second term in the square brackets is simply the ratio of the 'effective' threshold to arithmetic mean income. And of course the term in curly brackets in (16) is the proportion of people within the tax bracket.

Total revenue per person is thus:

$$\bar{x}\sum_{k=1}^{K} t_k G(a_k) \tag{17}$$

Hence, for the case of several income sources, each with its own tax schedule, total tax revenue over all individuals and sources becomes:

$$T = N\sum_{i=1}^{I} \bar{x}_i \left[\sum_{k_i=1}^{K_i} \left\{t_{k_i} G_i\left(a_{k_i}\right)\right\}\right] - N\left[(\bar{C}_R + \bar{C}_C)\right] \tag{18}$$

The first term in equation (18) can usefully be written in vector notation. Define the column vectors:

$$G_i = \begin{bmatrix} G_i(a_{1i}) \\ G_i(a_{2i}) \\ . \\ G_i(a_{Ki}) \end{bmatrix} \tag{19}$$

and:

$$t_i = \begin{bmatrix} t_{1i} \\ t_{2i} \\ . \\ t_{Ki} \end{bmatrix} \tag{20}$$

Then, if a prime indicates that the vector is written as a row vector:

$$\sum_{k_i=1}^{K_i} \left\{t_{k_i} G_{k_i}\left(a_{k_i}\right)\right\} = t'_i G_i \tag{21}$$

These values may be placed in a column vector, denoted $\left[t'G\right]$. Then if $\overline{x}$ represents a column vector whose *i*th element consists of the arithmetic mean, $\overline{x}_i$, then:

$$N\sum_{i=1}^{I}\overline{x}_i\left[\sum_{k_i=1}^{K_i}\left\{t_{k_i}G_{k_i}\left(a_{k_i}\right)\right\}\right]=N\overline{x}'\left[t'G\right] \qquad (22)$$

where, as before, a prime indicates transposition. This allows the effects of tax and income distribution changes to be easily examined. Several illustrations of such potential effects are given in the following section, in the context of the Spanish personal income tax.

4 An Application: The Spanish Tax Structure

The Spanish tax structure displays the characteristics considered in the previous section, having recently undergone significant tax reforms. In particular, income taxation since 2002 is shared between central and regional governments, consisting of 15 autonomous regions within the Common Territory. In addition, different tax rates and thresholds, and other rules influencing the difference between gross and taxable income, apply to a range of income sources: this involves the use of a multi-schedular tax structure.

The empirical results presented here were obtained using the Personal Income Tax information reported for a sample of 896,390 Spanish tax units. The original dataset comes from a cross-sectional dataset collected by the Spanish Tax Agency for year 2002. The data were then adjusted to tax year 2007 and the personal income tax structure examined is the one that came into force in January 2007. Following the 2007 Spanish code, attention is concentrated on just two sources of income. The first income source includes: labour income; alimony; self employment income; income from property and income applications to shareholders coming from Corporations under the *fiscal transparency regime*.[4] The second income source includes: capital gains and any form of income derived from financial savings such as interest rates from bank accounts and deposits, share dividends, bond interest or any other type of

[4] This is similar to S-Corporations in the US.

yield earned from debt saving instruments. Incomes include both monetary compensations and fringe benefits.

The marginal rates and thresholds for the first income source are shown in Table 1. For the second source, tax is paid at fixed central and regional (marginal and average) rates of 0.111 and 0.069.

Table 1 Tax Structure for Income Source 1

Income Threshold (€s)	Central Govt MTR	Regional Govt MTR	Total MTR
0	0.1566	0.0834	0.24
17,360	0.1827	0.0973	0.28
32,360	0.2414	0.1286	0.37
52,360	0.2713	0.1587	0.43

Table 2 gives, for each region and for all regions combined, the number of individuals who pay positive amounts of tax, along with the arithmetic means of the two income sources. The final two columns of Table 2 show the arithmetic means of the central and regional tax credits, which together give the last term in equation (18). There are clearly substantial differences in arithmetic mean incomes among regions.

The expression for aggregate revenue in each region requires the various proportions of people and proportions of income below each of the tax thresholds. This is simple for the second source of income, since the tax function is linear. For the first income source, Tables 3 and 4 report values of the first moment distribution and the distribution function respectively, for the required income thresholds. These two tables thus together give three points along the Lorenz curve of the first source of income in each region. For example, for all regions combined there are approximately five per cent of tax units (those paying positive tax) above the top threshold for income source 1, and they are responsible for about 20 per cent of total income from that source. However, for Madrid, eight per cent of positive taxpayers are above the top threshold, and they are responsible for about 30 per cent of total income from source 1.

Table 2 **Number of Taxpayers (with positive tax) and Arithmetic Means of Taxable Incomes and Tax Credits (€s)**

	N	$\overline{x}_1$	$\overline{x}_2$	$\overline{C}_C$	$\overline{C}_R$
National	12,229,939	20,816.74	2,220.59	1,228.84	646.27
Andalucía	1,772,425	19,148.92	1,686.22	1,254.13	660.46
Aragón	480,016	19,239.42	2,338.24	1,190.27	626.56
Asturias	362,701	19,894.59	1,648.66	1,203.68	634.44
Baleares	311,943	20,006.38	2,313.47	1,176.61	619.04
Canarias	472,742	20,330.13	1,439.27	1,259.68	661.04
Cantabria	181,796	19,546.91	2,287.14	1,220.81	642.94
Castilla-León	790,965	18,619.11	1,889.49	1,208.71	636.20
Castilla-La Mancha	469,160	17,565.85	1,563.90	1,216.59	640.83
Cataluña	2,473,158	22,266.55	2,506.11	1,225.06	643.65
Valencia	1,391,005	18,720.08	2,278.14	1,197.66	630.46
Extremadura	244,384	17,149.91	1,378.86	1,190.61	628.12
Galicia	730,355	18,520.83	1,763.29	1,161.04	612.92
Madrid	2,131,743	25,885.79	3,096.36	1,279.47	670.80
Murcia	309,242	18,780.95	1,729.87	1,271.32	669.52
Rioja	108,306	18,529.28	2,540.61	1,212.06	637.13

Table 3 **Proportions of Total Taxable Income Below Thresholds (First Income Source)**

	$F_1(a_2)$	$F_1(a_3)$	$F_1(a_4)$
National	0.29019	0.63216	0.80771
Andalucía	0.33006	0.69497	0.85926
Aragón	0.32503	0.69809	0.86190
Asturias	0.31424	0.70250	0.86856
Baleares	0.32095	0.62923	0.80426
Canarias	0.29832	0.63865	0.82860
Cantabria	0.32786	0.68411	0.85049
Castilla-León	0.34005	0.71888	0.88787
Castilla-La Mancha	0.38301	0.73908	0.89074
Cataluña	0.25939	0.59872	0.77946
Valencia	0.34565	0.68570	0.84315
Extremadura	0.39003	0.74727	0.89608
Galicia	0.34662	0.69615	0.86208
Madrid	0.19713	0.50417	0.70211
Murcia	0.34329	0.69763	0.86529
Rioja	0.36322	0.70585	0.87476

All that is required to obtain the values of *G* are the values of the effective tax thresholds, a'_k. For central and regional rates combined, the relevant values are 2,480; 9,748.11 and 15,693.95. The resulting values of *G* are given in Table 5. From the analytical results derived above, for any tax bracket, multiplying *G* by the relevant tax rate gives the ratio of tax raised per capita by the bracket to the total income per capita. As the tax rates are common across regions, comparisons of the extent of revenue within each region arising from income source 1 can be made by moving down the columns. The table shows the relative importance of the top tax bracket in Madrid, and the unimportance of the bottom tax bracket, compared with other regions.[5] Finally total tax revenue is reported in Table 6.

Table 4 **Proportions of Taxpayers below Thresholds (First Income Source)**

	$F(a_2)$	$F(a_3)$	$F(a_4)$
National	0.56074	0.86344	0.95491
Andalucía	0.59035	0.88808	0.96763
Aragón	0.58030	0.88792	0.96704
Asturias	0.55800	0.88570	0.96864
Baleares	0.60398	0.86906	0.95701
Canarias	0.57119	0.86188	0.95896
Cantabria	0.58835	0.88265	0.96492
Castilla-León	0.59488	0.89430	0.97353
Castilla-La Mancha	0.64104	0.90839	0.97528
Cataluña	0.52079	0.84388	0.94432
Valencia	0.61925	0.89186	0.96510
Extremadura	0.65168	0.91198	0.97600
Galicia	0.61687	0.89160	0.96861
Madrid	0.45962	0.79380	0.92136
Murcia	0.60652	0.89010	0.96956
Rioja	0.62178	0.89268	0.97164

[5] Separate values of *G* for Central and Regional tax schedules are shown in Appendix A.

Table 5 Values of G for each Tax Threshold (First Income Source According to Total Tax Schedule)

	$G(a_1)$	$G(a_2)$	$G(a_3)$	$G(a_4)$
National	0.290193	0.305909	0.132714	0.158296
Andalucía	0.330057	0.326354	0.123800	0.114204
Aragón	0.325034	0.333407	0.123714	0.111220
Asturias	0.314239	0.347408	0.125427	0.106700
Baleares	0.320947	0.275424	0.132178	0.162015
Canarias	0.298323	0.304869	0.143401	0.139719
Cantabria	0.327856	0.318916	0.125351	0.121344
Castilla-León	0.340052	0.338948	0.127502	0.089822
Castilla-La Mancha	0.383015	0.318322	0.114534	0.087180
Cataluña	0.259386	0.303344	0.136769	0.181298
Valencia	0.345647	0.303937	0.119315	0.127588
Extremadura	0.390029	0.319601	0.112424	0.081951
Galicia	0.346616	0.312747	0.125395	0.111321
Madrid	0.197128	0.273870	0.148977	0.249379
Murcia	0.343290	0.316890	0.126414	0.109277
Rioja	0.363215	0.306374	0.127377	0.101215

Table 6 Estimated Total Tax Revenue (€s)

	Total Revenue	Central Government.	Regional Government.
National	51,148,217,111	33,061,697,229	18,086,519,880
Andalucía	6,155,788,630	3,972,274,107	2,183,514,522
Aragón	1,776,894,526	1,145,725,113	631,169,413
Asturias	1,352,992,058	874,123,198	478,868,860
Baleares	1,271,754,868	818,791,512	452,963,356
Canarias	1,810,355,523	1,167,961,396	642,394,127
Cantabria	683,181,263	440,400,756	242,780,507
Castilla-León	2,672,907,005	1,725,380,941	947,526,063
Castilla-La Mancha	1,410,927,079	910,996,715	499,930,364
Cataluña	11,679,310,225	7,520,720,015	4,158,590,210
Valencia	4,981,828,923	3,209,163,568	1,772,665,355
Extremadura	705,599,802	456,083,010	249,516,793
Galicia	2,521,071,133	1,627,923,016	893,148,116
Madrid	12,712,845,321	8,281,203,401	4,431,641,920
Murcia	1,034,493,203	667,339,810	367,153,393
Rioja	378,267,552	243,610,671	134,656,881

For example, for all regions combined, the total income tax revenue, gross of the tax credits, is given from (18) by the total number of positive taxpayers multiplied by the term (where values for *G* are rounded to two decimal places for convenience):

$$\begin{bmatrix} 20,816 & 2,220 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.24 & 0.28 & 0.37 & 0.43 \end{bmatrix} \begin{bmatrix} 0.29 \\ 0.31 \\ 0.13 \\ 0.16 \end{bmatrix} \\ 0.18 \end{bmatrix} \qquad (19)$$

When the total amount of tax credits per (positive) taxpayer, of 1,228+646, is deducted from this result, the net tax per capita is obtained. Multiplying this value by the total number of taxpayers gives the value in the first row of Table 6.[6] Results are obtained for the regions simply by changing the vector of arithmetic mean incomes and the vector of *G* values in (19), and then using the appropriate values of *N* and the average tax credits.

The effects on gross tax revenue of changes in the average income from the second source, or changes in the relative dispersion of income from the first source (which changes the Lorenz curve and thus the *G* values), or changes in the marginal tax rate structure, are thus easily examined using modifications to expressions of the form shown in (19). For example, elimination of the top marginal income tax bracket simply means that the row vector of tax rates has only three elements and the column vector of *G* values is reduced to three elements with the third element replaced by 0.29.

Changes in the tax thresholds have the effect of changing the *G* values. Hence a 'ready reckoner' could be produced by replacing Tables 4 and 5 by larger tables giving values of the distribution function and first moment distribution function for a

[6] As a useful check on the programming of the calculations, aggregate revenue was obtained both using the formulae and by simply adding all the individual tax unit amounts, giving exactly the same results.

range of income levels. The introduction of additional tax brackets for the second income source could be accommodated by producing similar tables for that source.

The effects of change in the distribution of income within a region can be examined using the same kind of summary information. For example, if mean income increases, whereby incomes in a region are assumed to increase by the same proportion, this is equivalent to a reduction in the tax thresholds, so that information about the Lorenz curve (the F and F_1 values) can be used to obtain the appropriate G value. A change in inequality can be accommodated by specifying the way in which the Lorenz curve for the region changes.

The difficulty of dealing with the central and regional tax credits and thus aggregate net income tax revenue remains, as an analytical expression for aggregate credits has not been obtained.

4.1 Further Comparisons

As suggested above, the effects of changing only the tax rates are easily examined in this framework, as only the vector of marginal rates needs to be altered in expressions corresponding to (19). For example, the previous discussion has not allowed for the small change in the tax rate structure in Madrid in 2007, making it unique among the Spanish regions. The income thresholds for the first income source are the same as in Table 1 above, and the central government rates are the same, but the marginal tax rates for Madrid became 0.0794, 0.0943, 0.1266 and 0.1577 for the four income brackets. This involves a slight reduction in all the rates, with the largest reductions being for the first and second tax brackets. Given the nature of the distribution of income in Madrid for the first source, it is anticipated that this would have relatively little effect on total revenue. But in view of the differences among regions in their income distributions, the same could not be said of the other regions.

The effects on total tax revenue, and revenue within each region, if all regions were to adopt the Madrid structure, can easily be obtained using the information given above. The percentage changes in total (central plus regional) tax revenue and in the

regional tax revenue alone are shown in Table 7. In producing these values, it was assumed that the average tax credits within each region remain unchanged. Clearly the poorer regions, where a much larger proportion of total income is obtained by those who fall into the first two tax brackets, would experience substantially larger reductions in tax revenue.

Table 7 Percentage Reduction in Tax Revenue from Adoption of Madrid's 2007 Tax Rate Structure

	Total Revenue	Regional Revenue
National	1.7268	5.0096
Andalucía	1.9868	5.8113
Aragon	1.8754	5.4658
Asturias	1.9240	5.6338
Baleares	1.7081	4.9486
Canarias	1.8697	5.4546
Cantabria	1.8631	5.4260
Castilla-Leon	2.0137	5.8968
Castilla-LaMancha	2.1540	6.3276
Cataluña	1.6141	4.6696
Valencia	1.8701	5.4398
Extremadura	2.2008	6.4844
Galicia	1.9373	5.6687
Madrid	1.3815	3.9630
Murcia	2.0302	5.9393
Rioja	1.9282	5.6124

4.2 Regional Comparisons

The previous subsection considered the effects on all regions of adopting a different regional structure of marginal income tax rates. As explained above, it is possible to use the same basic approach to consider the effects of a range of changes in the taxable income distributions of each region. This is particularly useful in the present context where it is clear that different regions have different fiscal capacities. Such disparities in regional revenue-raising abilities are especially evident when, as here, progressive taxes are assigned partially to regional governments.

Central governments normally carry out some form of regional fiscal equalization. Under these circumstances, sound design of these inter-regional transfers requires a clear understanding of the precise sources of divergence of regional fiscal capacities. The present approach can thus contribute to the debate on regional transfers by clarifying precisely how regions differ with respect to the tax structure and the distribution of taxpayers. This is because the above analytical results make it evident that differences in revenue hinge on basically four factors: the number and distribution of taxpayers, the distribution of taxable incomes and the specific tax parameters that define the structure: these are the marginal tax rates, tax bracket thresholds and average tax credits.

The present approach makes it possible to construct a matrix in which each region's tax revenue can be computed under the assumption that it shares one or more of the characteristics of other regions. Thus a '15 by 15' matrix is obtained such that each entry shows the revenue obtained by a row region, under the assumption that it has a particular characteristic of the column region. The leading diagonal of such a square matrix obviously shows the actual revenue obtained by the region. This matrix is augmented by an additional row and column for the country as a whole. Similarly, the information can be displayed in relative terms, showing the percentage differences in revenue which could be raised by each region, given different assumed characteristics (so that each corresponding leading diagonal element is zero).

To illustrate the kind of information which can be produced along these lines, Tables 8 to 10 report three such hypothetical '16 by 16' matrices for Spanish regions in the 2007 fiscal year. Each matrix shows the relative impact on the revenue collection of the row region if it were to replicate the specific characteristic of the column region. Specifically, Table 8 presents the revenue impact of differences in arithmetic mean taxable incomes. Table 9 depicts the effects of differences in the form of the relative taxable income distributions; that is, the arithmetic means are unchanged but the proportions of people in each tax bracket, and the corresponding proportions of total taxable income within each bracket, are assumed to the those of the region in the

columns. Finally, Table 10 shows the revenue consequences of simultaneous changes in both the arithmetic mean taxable income and the relative distributions of income.[7]

For example, Table 8 shows that if Andalucía were to have the same arithmetic mean taxable income as Aragon (a given percentage change in all incomes), it would have 4.17 per cent higher income tax revenue. However, from Table 9 if Andalucia were to have its actual arithmetic mean, but the same relative form of income distribution as Aragon, it would have slightly less revenue: there would be a reduction of 0.40 per cent. Table 10 indicates that if the distribution of taxable income in Andalucia were the precisely the same as in Aragon (in both absolute and relative terms), its revenue would be 3.77 per cent higher. In fact these effects are additive, so that the elements of Table 10 effectively equal the sum of the corresponding elements in Tables 8 and 9.

[7] In producing these results it has been assumed that average tax credits remain unchanged

Table 8 Effects of Varying Arithmetic Mean Taxable Income (Row Region has Mean Income of Column Region)

	Nat.	And.	Arag.	Ast.	Bal.	Can.	Cant.	Cast.-L.	Cast.-LM.	Cat.	Val.	Extr.	Gal.	Mad.	Murc.	Rioja
National	0	-14.73	-11.26	-9.33	-5.65	-6.98	-9.19	-17.81	-27.07	12.04	-15.39	-30.96	-19.09	41.57	-17.29	-15.69
Andalucia	17.31	0	4.17	6.30	10.72	9.01	6.58	-3.56	-14.43	31.42	-0.67	-19.02	-5.07	66.02	-2.98	-0.97
Aragon	12.32	-3.91	0	2.00	6.15	4.54	2.26	-7.25	-17.44	25.55	-4.54	-21.74	-8.67	57.99	-6.70	-4.82
Asturias	10.23	-5.86	-1.98	0	4.11	2.52	0.27	-9.17	-19.27	23.35	-6.47	-23.53	-10.57	55.50	-8.63	-6.75
Baleares	5.80	-9.34	-5.77	-3.79	0	-1.38	-3.64	-12.50	-22.00	18.16	-10.01	-26.01	-13.81	48.49	-11.96	-10.31
Canarias	7.62	-8.43	-4.63	-2.55	1.48	0	-2.37	-11.77	-21.85	20.73	-9.13	-26.10	-13.16	52.89	-11.21	-9.44
Cantabria	9.96	-6.10	-2.24	-0.24	3.85	2.28	0	-9.42	-19.51	23.07	-6.74	-23.76	-10.82	55.20	-8.87	-7.03
Castilla-Leon	21.17	3.60	7.87	9.98	14.51	12.71	10.31	0	-11.04	35.50	2.96	-15.69	-1.54	70.60	0.58	2.67
Castilla-LaMancha	35.79	16.25	21.04	23.33	28.40	26.34	23.74	12.27	0	51.71	15.59	-5.18	10.55	90.71	12.90	15.29
Cataluña	-10.88	-24.18	-21.09	-19.29	-16.00	-17.15	-19.20	-26.99	-35.34	0	-24.82	-38.86	-28.13	26.70	-26.50	-25.11
Valencia	17.50	0.66	4.71	6.80	11.09	9.44	7.06	-2.81	-13.38	31.24	0	-17.84	-4.28	64.91	-2.24	-0.30
Extremadura	42.51	22.23	27.21	29.57	34.85	32.69	30.02	18.11	5.38	59.02	21.56	0	16.33	99.47	18.76	21.26
Galicia	22.46	5.09	9.28	11.41	15.86	14.13	11.70	1.52	-9.39	36.62	4.43	-13.99	0	71.34	2.10	4.13
Madrid	-30.66	-41.48	-39.04	-37.47	-34.87	-35.68	-37.49	-43.81	-50.61	-21.79	-42.09	-53.46	-44.73	0	-43.39	-42.37
Murcia	20.99	3.08	7.41	9.60	14.19	12.40	9.90	-0.60	-11.85	35.60	2.40	-16.59	-2.17	71.40	0	2.09
Rioja	17.96	0.91	5.05	7.11	11.49	9.76	7.42	-2.59	-13.29	31.87	0.28	-17.81	-4.08	65.94	-2.02	0

Table 9 Effects of Varying Relative Incomes: Row Region has F and F_1 of Column Region

	Nat.	And.	Arag.	Ast.	Bal.	Can.	Cant.	Cast.-L.	Cast.-LM.	Cat.	Val.	Extr.	Gal.	Mad.	Murc.	Rioja
National	0	-1.98	-2.32	-2.90	0.89	-0.70	-1.80	-3.15	-3.11	0.13	-0.84	-3.30	-1.69	1.69	-1.99	-2.44
Andalucia	1.89	0	-0.40	-1.09	3.00	1.15	0.14	-1.21	-0.99	1.77	1.32	-1.15	0.40	2.89	0.04	-0.41
Aragon	2.18	0.37	0	-0.65	3.21	1.47	0.51	-0.77	-0.57	2.07	1.61	-0.73	0.74	3.17	0.41	-0.02
Asturias	2.99	1.02	0.65	0	4.01	2.25	1.18	-0.19	-0.06	3.00	2.27	-0.23	1.37	4.36	1.04	0.58
Baleares	-0.93	-2.76	-3.10	-3.70	0	-1.61	-2.61	-3.88	-3.77	-0.91	-1.61	-3.93	-2.44	0.38	-2.74	-3.17
Canarias	0.74	-1.29	-1.66	-2.29	1.73	0	-1.12	-2.52	-2.43	0.82	-0.06	-2.62	-0.96	2.32	-1.29	-1.76
Cantabria	1.72	-0.14	-0.51	-1.15	2.73	1.00	0	-1.30	-1.14	1.66	1.08	-1.30	0.21	2.87	-0.11	-0.55
Castilla-Leon	2.96	1.17	0.77	0.07	4.11	2.22	1.29	0	0.28	2.74	2.51	0.14	1.60	3.65	1.24	0.81
Castilla-LaMancha	2.36	0.72	0.27	-0.50	3.68	1.61	0.80	-0.45	0	1.91	2.18	-0.11	1.23	2.38	0.83	0.42
Cataluña	-0.30	-2.38	-2.68	-3.21	0.46	-0.99	-2.17	-3.55	-3.63	0	-1.33	-3.84	-2.15	1.86	-2.42	-2.89
Valencia	0.45	-1.26	-1.64	-2.31	1.53	-0.25	-1.15	-2.38	-2.12	0.26	0	-2.26	-0.87	1.16	-1.20	-1.62
Extremadura	2.30	0.74	0.27	-0.53	3.68	1.55	0.79	-0.42	0.10	1.74	2.24	0	1.28	2.01	0.87	0.47
Galicia	1.30	-0.42	-0.81	-1.50	2.43	0.59	-0.31	-1.55	-1.27	1.07	0.89	-1.40	0	1.91	-0.35	-0.77
Madrid	-2.98	-5.22	-5.47	-5.91	-2.43	-3.65	-4.96	-6.39	-6.68	-2.39	-4.33	-6.92	-5.10	0	-5.32	-5.81
Murcia	1.79	-0.06	-0.47	-1.18	2.95	1.04	0.07	-1.26	-1.00	1.60	1.29	-1.15	0.36	2.59	0	-0.45
Rioja	2.05	0.35	-0.04	-0.72	3.16	1.34	0.46	-0.78	-0.49	1.82	1.63	-0.63	0.76	2.66	0.41	0

Table 10 Effects of Varying Mean Taxable Income and its Distribution: Row Region has Distribution of Column Region

	Nat.	And.	Arag.	Ast.	Bal.	Can.	Cant.	Cast.-L.	Cast.-LM.	Cat.	Val.	Extr.	Gal.	Mad.	Murc.	Rioj.
National	0	-16.30	-13.18	-11.99	-4.75	-7.66	-10.72	-20.19	-28.76	12.38	-15.78	-32.54	-20.16	45.86	-18.72	-17.39
Andalucia	19.69	0	3.77	5.20	13.96	10.44	6.74	-4.71	-15.06	34.65	0.63	-19.63	-4.66	75.11	-2.92	-1.32
Aragon	14.94	-3.54	0	1.35	9.56	6.26	2.79	-7.95	-17.66	28.97	-2.94	-21.95	-7.91	66.93	-6.28	-4.78
Asturias	13.49	-4.85	-1.34	0	8.15	4.87	1.43	-9.23	-18.86	27.42	-4.26	-23.12	-9.19	65.08	-7.57	-6.08
Baleares	4.89	-11.89	-8.68	-7.46	0	-3.00	-6.15	-15.90	-24.72	17.63	-11.36	-28.61	-15.86	52.09	-14.38	-13.02
Canarias	8.39	-9.47	-6.05	-4.75	3.19	0	-3.36	-13.74	-23.12	21.96	-8.89	-27.27	-13.69	58.65	-12.12	-10.67
Cantabria	11.97	-6.23	-2.74	-1.42	6.67	3.42	0	-10.58	-20.14	25.80	-5.64	-24.37	-10.54	63.19	-8.93	-7.45
Castilla-Leon	25.08	4.84	8.71	10.19	19.18	15.56	11.76	0	-10.64	40.45	5.49	-15.34	0.05	82.03	1.83	3.48
Castilla-LaMancha	40.13	17.39	21.74	23.40	33.51	29.44	25.17	11.95	0	57.41	18.12	-5.28	12.01	104.13	14.01	15.86
Cataluña	-11.00	-25.48	-22.71	-21.66	-15.22	-17.81	-20.53	-28.95	-36.56	0	-25.02	-39.92	-28.91	29.75	-27.63	-26.46
Valencia	18.49	-0.61	3.04	4.44	12.93	9.51	5.92	-5.18	-15.21	32.99	0	-19.65	-5.13	72.22	-3.45	-1.89
Extremadura	47.30	23.61	28.14	29.87	40.40	36.17	31.72	17.95	5.50	65.29	24.37	0	18.00	113.96	20.09	22.02
Galicia	24.51	4.69	8.48	9.93	18.74	15.19	11.47	-0.05	-10.46	39.55	5.32	-15.06	0	80.26	1.75	3.36
Madrid	-31.83	-43.15	-40.98	-40.16	-35.13	-37.15	-39.27	-45.85	-51.79	-23.24	-42.78	-54.42	-45.82	0	-44.83	-43.90
Murcia	23.48	3.04	6.95	8.44	17.53	13.87	10.03	-1.85	-12.60	39.01	3.69	-17.34	-1.80	81.01	0	1.66
Rioja	20.90	1.31	5.06	6.49	15.20	11.69	8.02	-3.37	-13.66	35.77	1.94	-18.20	-3.32	76.00	-1.59	0

5 Conclusions

The aim of this paper was to derive analytical expressions for aggregate revenue and the revenue elasticity of complex personal income tax systems, as applied to tax units and in aggregate. The complexity arises from the schedular nature of the system, the role of central and regional governments, and the existence of a range of tax credits and eligible expenditures and deductions.

Formal expressions for aggregate tax revenue were derived, in terms of the distribution of taxable income within each region. It was possible to separate total revenue into components relating to the income tax structure and summary measures of the distribution of taxable income, in particular the proportions of taxpayers, and of total taxable income, in each tax bracket. This made it possible to examine the sources of differences among regions.

The general approach was then applied to the personal income tax structure in Spain which, since reforms dating from 2002, has similar characteristics. The role of substantial differences in the income distributions of different Spanish regions was examined.

Appendix A. Separate values of *G* for Central and Regional Governments

Central Government Tax Schedule				
	$G(a_1)$	$G(a_2)$	$G(a_3)$	$G(a_4)$
National	0.290193	0.305909	0.132724	0.161007
Andalucía	0.330057	0.326354	0.123810	0.116320
Aragón	0.325034	0.333407	0.123724	0.113364
Asturias	0.314239	0.347408	0.125436	0.108673
Baleares	0.320947	0.275424	0.132188	0.164704
Canarias	0.298323	0.304869	0.143412	0.142245
Cantabria	0.327856	0.318916	0.125361	0.123590
Castilla-León	0.340052	0.338948	0.127512	0.091601
Castilla-La Mancha	0.383015	0.318322	0.114543	0.088942
Cataluña	0.259386	0.303344	0.136780	0.184428
Valencia	0.345647	0.303937	0.119325	0.129922
Extremadura	0.390029	0.319601	0.112433	0.083703
Galicia	0.346616	0.312747	0.125405	0.113442
Madrid	0.197128	0.275025	0.149916	0.254015
Murcia	0.343290	0.316890	0.126424	0.111306
Rioja	0.363215	0.306374	0.127387	0.103131

Regional Government Tax Schedule				
	$G(a_1)$	$G(a_2)$	$G(a_3)$	$G(a_4)$
National	0.290193	0.305909	0.132695	0.153661
Andalucía	0.330057	0.326354	0.123782	0.110587
Aragón	0.325034	0.333407	0.123696	0.107555
Asturias	0.314239	0.347408	0.125408	0.103328
Baleares	0.320947	0.275424	0.132159	0.157417
Canarias	0.298323	0.304869	0.143380	0.135400
Cantabria	0.327856	0.318916	0.125333	0.117504
Castilla-León	0.340052	0.338948	0.127483	0.086780
Castilla-La Mancha	0.383015	0.318322	0.114517	0.084169
Cataluña	0.259386	0.303344	0.136749	0.175948
Valencia	0.345647	0.303937	0.119298	0.123599
Extremadura	0.390029	0.319601	0.112407	0.078957
Galicia	0.346616	0.312747	0.125377	0.107694
Madrid	0.197128	0.271630	0.147188	0.241403
Murcia	0.343290	0.316890	0.126396	0.105809
Rioja	0.363215	0.306374	0.127358	0.097940

References

Agencia Tributaria (2008). *Manual Práctico Renta y Patrimonio 2007*. Ministerio de Economía y Hacienda.

Caminada, K. and Goudswaard, K. (1996) Progression and revenue effects of income tax reform. *International Tax and Public Finance*, 3, pp. 57-66.

Creedy, J. (1996) Fiscal Policy and Social Welfare: An Analysis of Alternative Tax and Transfer Systems. Aldershot: Edward Elgar.

Creedy, J. and Gemmell, N. (2006) *Modelling Tax Revenue Growth*. Cheltenham: Edward Elgar.

Dye, R.F. and McGuire, T.J. (1991) Growth and variability of state individual income and general sales taxes. *National Tax Journal*, 44, pp. 55-66.

OECD (2006) *Fundamental Reform of Personal Income Tax*. OECD Tax Policy Studies No.13.

Romero, D. and Sanz-Sanz, J.F. (2007). Personal income taxation: the cornerstone of the tax system in a democracy. In *Fiscal Reform in Spain. Accomplishments and Challenges* (ed. by J. Martínez-Vazquez and J.F. Sanz-Sanz), pp. 151-207. Studies in Fiscal Federalism and State-Local Finance. Cheltenham: Edward-Elgar.