The Economic Value of Exploiting Time-Varying Return Moments

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Abstract

This paper evaluates out-of-sample dynamic portfolio performance to examine the economic value of exploiting time variation in the risk premium and in the volatility of stock returns to a multiperiod investor. We find that ignoring time variation in these return moments leads to significant utility costs. The time-varying risk premium plays a more important role than time-varying volatility in forming portfolio weights and in improving portfolio performance because the conditional expected return is a more persistent process than conditional volatility. Strategies using the dividend yield as a state variable perform poorly out of sample, although the good in-sample performance of these strategies has been documented in the literature. Unlike prior studies, this paper finds that the utility cost of myopic behavior is small.

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1 Introduction

Merton (1971) theorizes that if investment opportunities are time-varying, the portfolio choice of a multiperiod investor can differ from that of a single-period investor because of hedging motives. Recent empirical evidence of return predictability has stimulated interest in dynamic portfolio choice. Appropriate conclusions about the importance of return predictability on dynamic investment decisions depend on portfolio performance. Based on an out-of-sample analysis in a static setting, DeMiguel, Garlappi, and Uppal (2007) find that a simple equally-weighted portfolio strategy \( \frac{1}{N} \) surprisingly outperforms many optimal mean-variance rules including those designed to reduce parameter uncertainty. To the best of our knowledge, little research has been done on out-of-sample portfolio performance in a dynamic setting.\(^1\)

This paper evaluates out-of-sample portfolio performance to examine the economic value of exploiting time variation in the risk premium and in the variance of stock returns to a multiperiod investor. The multiperiod investor in this paper mimics an individual investor in the real world who selects portfolio strategies by using information available at the time of decision making. Because the investor does not know the parameters of return distributions, the errors caused by the need to estimate the parameters give rise to additional ‘estimation risk’ that should be considered in portfolio analysis. Over a 5-year investment horizon, she incorporates new information to update the estimates of the parameters of a return generating process (RGP) and correspondingly to update portfolio rules. The investor is assumed to maximize expected power utility over her terminal wealth. She encounters a tradeoff between accounting for return predictability and estimation risk. Properly incorporating the time variation in the first and second return moments in dynamic portfolio choice can improve portfolio performance. On the other hand, additional estimation risk that is caused by an increase in the number of estimated parameters in modeling time-varying return distributions leads to an adverse effect on investment performance. This paper investigates the following questions. Should the real-world investor account for predictability of the risk premium and the volatility of stock returns in the presence of estimation risk? If return predictability should be considered, what is the utility loss if she ignores the time variation in the first and/or second

\(^1\)Cooper, Gutierrez, and Marcum (2005) and Handa and Tiwari (2006) also explore out-of-sample portfolio performance in a myopic setting.
conditional return moments? What is the cost if she behaves myopically?

This paper extends the literature in two aspects. First, it appropriately accounts for the time-varying risk premium and time-varying volatility of a risky return in a two-factor model, which is more consistent with empirical evidence than other models that have been used in evaluating the performance of dynamic portfolio strategies.² Most of prior studies in the dynamic portfolio choice literature investigate the impact of predictable variation in expected returns on multiperiod portfolio decisions.³ Few studies examine the effect of time-varying volatility of stock returns on portfolio choice,⁴ although volatility predictability has been well documented in the literature.⁵ Inspired by Merton’s intertemporal capital asset pricing model (ICAPM), we account for the time variation in the both first and second moments of stock returns in the two-factor model. By letting some parameters of the two-factor model be zero, we can obtain models with simplified return generating processes that have either a constant risk premium or constant volatility. Comparing the investment performance of the portfolio rules optimal to these alternative models can shed light on the economic value of ignoring time variation in the risk premium and/or in volatility.

Second, this paper conducts an out-of-sample performance comparison of dynamic portfolio strategies. Most of prior studies ignore parameter uncertainty in examining dynamic portfolio choice, while some recent papers implement the Bayesian approach in dynamic portfolio decisions to account for estimation risk (Barberis, 2000; Xia, 2001; Brandt, Goyal, Santa-Clara, and Stroud, 2005; Skoulakis, 2007). In evaluating portfolio performance of different investment rules, these recent papers apply portfolio weights to next period hypothetical stock returns generated by an assumed return model with uncertain parameters that are drawn from a given distribution. In fact, a real-world investor can only apply portfolio weights to next period actual stock returns whose underlying generating processes are unknown. Therefore, prior studies tend to overestimate the performance of portfolio strategies derived from assumed

²Consistent with findings by Whitelaw (1994) and Brandt and Kang (2004), these two conditional return moments are exposed to different shocks.
³See Brennan, Schwartz, and Lagnado (1997), Balduzzi and Lynch (1999), Campbell and Viceira (1999), Campbell, Chan, and Viceira (2003), and others.
⁵Bollerslev, Chou, and Kroner (1992) review the research on volatility predictability.
return models. In an out-of-sample performance analysis of this paper, portfolio weights are applied to next-period actual returns. The investor has two ways to incorporate new information in forming portfolio weights: (1) She can compute portfolio weights based on the current values of state variables combined with newly updated model parameters, which are estimated annually using rolling regressions. (2) She can implement the Bayesian approach in forming portfolio weights by learning some parameters of the two-factor model over time to account for parameter uncertainty.

We find that ignoring time variation in conditional return moments leads to statistically and economically significant utility costs. If the investor does not use the dynamic portfolio rule obtained based on the two-factor model but instead uses the rule for an i.i.d. return model, she would endure a utility cost of at least 1.7% in terms of an annualized certainty equivalent return. We also show that the theoretical optimal policy for the two-factor model, which is obtained by assuming that estimated model parameters are the true parameters, performs worse than other policies that help to reduce estimation risk. The adverse effect of parameter uncertainty on the theoretical optimal policy comes mainly from considerable estimation error in the coefficient on conditional variance in the two-factor model.

Unlike the findings by DeMiguel, Garlappi, and Uppal (2007) that the rule $1/N$ performs better than other mean-variance rules, the out-of-sample performance of simple portfolio rules, which invest fixed fractions of 50%, 60%, or 100% of wealth in the risky asset at each rebalancing period, is significantly inferior to the performance of dynamic strategies incorporating time-varying return moments. The reason for the difference in findings is that there is only one risky asset with a small number of parameters to be estimated in this paper. The gain from optimal allocation by accounting for time-varying return moments exceeds the loss caused by estimation risk.

This paper shows that compared with time-varying volatility, the time-varying risk premium plays an important role in forming portfolio weights and in improving portfolio performance for the real-world investor who can rebalance her portfolio quarterly. The portfolio rule in a model specifying only a time-varying risk premium performs far better than the rule in a model specifying only time-varying volatility. One possible reason is that the conditional expected return is a more persistent process than conditional volatility. A small increase in
future expected return can have a long-lasting effect on portfolio choice. While high conditional volatility converges to the average level soon, its effect on portfolio weights for a 5-year investment horizon is relatively weak.

We find that portfolio rules relying on the dividend yield perform quite poorly out of sample. Using these rules leads to a utility loss of roughly 2% in terms of an annualized certainty equivalent return compared with using the strategy for the i.i.d. return model. This result stands in stark contrast to significant in-sample utility gains of applying these rules, which has been documented in the literature. Conducting an out-of-sample evaluation is, therefore, necessary to assess the value of a dynamic portfolio strategy because in-sample analysis may represent an ideal that is difficult to materialize in reality and can be misleading.

This study shows that myopic behavior leads to a small utility loss for a real-world investor. Balduzzi and Lynch (1999) and Campbell and Viceira (1999), however, find that the utility costs of myopic policies are large based on in-sample analysis. These papers use the dividend yield as a state variable, which is a very persistent process, and the persistence of state variables leads to large utility losses in sample. Because portfolio strategies based on the dividend yield perform even worse out of sample than portfolio strategies without considering return predictability, a real-world investor is worse off if adopting the strategies based on the dividend yield. The state variables used in this paper are relatively less persistent and strategies based on these state variables perform well out of sample. Under these circumstances, this study shows that utility gains from hedging demands is modest. In addition, dynamic policies are more susceptible to parameter uncertainty than myopic ones. This effect further reduces the utility costs of myopic behavior.

To handle estimation error, two major methods have been used in the literature. Brandt (1999), Ait-Sahalia and Brandt (2001), Brandt and Santa-Clara (2006), and Brandt, Santa-Clara, and Valkanov (2009) directly draw inference about portfolio weights from the data without modeling return distributions. Brennan (1998), Barberis (2000), Xia (2001), Brandt, Goyal, Santa-Clara, and Stroud (2005), and Skoulakis (2007) implement the Bayesian approach to learn some model parameters over time in the process of investment decisions. Because of the complexity of introducing Bayesian updating, these works simplify return dis-

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6See, for example, Balduzzi and Lynch (1999), Brandt (1999), Campbell and Viceira (1999), and Xia (2001).
7Brandt (2004) summarizes recent research in this area.
tributions that are the special cases employed by the investor in this paper. Unlike this paper, none of the above studies conducts an out-of-sample analysis of the performance of different dynamic portfolio strategies and compare the economic value of time variation in the risk premium and in the volatility of stock returns.

The remainder of this paper is organized as follows: Section 2 lays out a dynamic portfolio choice problem. Section 3 estimates different models for the risky return and illustrates various portfolio strategies. In-sample and out-of-sample utility comparisons of these strategies are conducted in Sections 4 and 5, respectively. The conclusions are drawn in the final section.

2 A Dynamic Portfolio Choice Problem

This paper considers an investor with power utility of terminal wealth who can trade one risky asset and one risk-free asset quarterly. For simplicity, the risk-free rate is constant. The investor first specifies the return process of the risky asset and then chooses portfolio weights to maximize her expected power utility. Portfolio weights are computed based on a minor modification to the simulation-based method proposed by Brandt, Goyal, Santa-Clara, and Stroud (2005).

2.1 Modeling the Risky Asset Return

Let $r_t^e$ be the excess return on the risky asset at time $t$, defined as the risky return in excess of the risk-free rate. Let $\mu_t$ and $\sigma_t$ denote the conditional mean and volatility of the excess return, respectively. The excess return then can be expressed as

$$r_{t+1}^e = \mu_t + \sigma_t \epsilon_{t+1},$$

where $\mathbb{E}_t(\epsilon_{t+1}) = 0$ and $\text{var}_t(\epsilon_{t+1}) = 1$. Modeling the excess return becomes specifying the conditional risk premium $\mu_t$ and the conditional variance $\sigma_t^2$. The investor can simply assume that $\mu_t$ or $\sigma_t^2$ is a constant or that $\mu_t$ or $\sigma_t^2$ changes over time as a function of state variables. The combination of different assumptions about $\mu_t$ and $\sigma_t^2$ leads to a variety of return models. Comparing portfolio performances delivered by these models can shed light on the costs of simplifying RGP.
The simplest model sets both $\mu_t$ and $\sigma_t$ as constants. According to this specification, the excess return is independently and identically distributed. This model has been widely taken as a benchmark model in the dynamic portfolio choice research.

(1) IID model (constant first and second moments):

$$\mu_t = d_1$$

$$\sigma_t^2 = \exp(b_1).$$

Many prior studies investigating the effect of return predictability on dynamic portfolio choice simply assume that the excess return has a time-varying mean and constant volatility. Hence the second return model studied in this paper considers time variation only in the risk premium by using $cay$ to forecast the excess return. $cay$ is an empirical proxy for the log consumption-wealth ratio and captures deviations from the common trend in consumption, asset wealth, and labor income. It was first constructed by Lettau and Ludvigson (2001) to predict future stock market returns. There are two reasons for choosing $cay$ as a predictor for future returns. First, the desire to smooth out consumption paths over time drives consumption adjustment out of current asset wealth and labor income corresponding to a change in future investment opportunities. Thus deviations from the common trend, $cay$, summarize expectations of future returns on the market portfolio (Lettau and Ludvigson, 2001). Second, $cay$ has a pretty good out-of-sample predictive power (Lettau and Ludvigson, 2007). $cay$ is therefore a good choice for a real-world investor to forecast future returns based on information available only at the time of decision making.

(2) CAY model (constant volatility):

$$cay_{t+1} = c_1 + c_2 cay_t + e_{cay,t+1}$$

$$\mu_t = d_1 + d_2 cay_t$$

$$\sigma_t^2 = \exp(b_1).$$

This model specifies the conditional risk premium as a linear function of $cay$. $cay$ follows a first-order AR process, and $c_2$ reflects the persistence of this process.

Chacko and Viceira (2005) and Gomes (2007) examine the impact of volatility predictability on dynamic portfolio choice by simply assuming a constant expected return on the risky
asset. Hence the third model considers time variation only in the conditional variance. We use daily returns to construct the quarterly realized variance of the risky asset because variance can be estimated accurately using data of a high frequency (Merton, 1980). Let \( \hat{\sigma}_t^2 \) be the quarterly realized variance. We estimate \( \hat{\sigma}_t^2 \) as the sum of squared daily returns and the multiplication of adjacent returns:

\[
\hat{\sigma}_t^2 = \sum_{i=1}^{N_t} (r_{it})^2 + 2 \sum_{i=1}^{N_t-1} r_{it} r_{i+1,t},
\]

where \( N_t \) is the number of daily returns within the quarter \( t \). The second term on the right-hand side of equation (7) is used as an adjustment to overcome the non-synchronous trading issue involved in a stock index return (French, Schwert, and Stambaugh, 1987), which is a proxy for the risky asset return.\(^8\) This paper transforms the realized variance to the log realized variance because the logarithm transformation removes the skewness of the variance and makes the log realized variance an appropriate regressand. Imposing a constant expected return, we get

(3) **VOL model (constant expected return):**

\[
\log(\hat{\sigma}_{t+1}^2) = b_1 + b_2 \log(\hat{\sigma}_t^2) + \sigma_{\sigma,t+1}
\]

\[
\mu_t = d_1
\]

\[
\hat{\sigma}_t^2 = \exp[b_1 + b_2 \log(\hat{\sigma}_t^2) + 0.5\text{var}(\sigma_t)].
\]

Notice that conditional variance \( \sigma_t^2 \) is formed based on information available at time \( t \).

The capital asset pricing model (CAPM) has been used to account for time-varying expected returns and volatility in the portfolio allocation research. For example, Chacko and Viceira (2005) and Liu, Longstaff, and Pan (2003) use the CAPM to study dynamic asset allocation in a continuous-time setting.

\(^8\)Following French, Schwert, and Stambaugh (1987), the sample mean is not subtracted to compute the quarterly realized variance because this adjustment is very minor.
(4) CAPM:

\[
\log(\hat{\sigma}_{t+1}^2) = b_1 + b_2 \log(\hat{\sigma}_t^2) + b_3 \text{cay}_t + e_{\sigma,t+1} \tag{11}
\]

\[
\text{cay}_{t+1} = c_1 + c_2 \text{cay}_t + c_3 \log(\hat{\sigma}_t^2) + e_{\text{cay},t+1} \tag{12}
\]

\[
\mu_t = d_1 + d_3 \sigma_t^2 \tag{13}
\]

\[
\sigma_t^2 = \exp[b_1 + b_2 \log(\hat{\sigma}_t^2) + b_3 \text{cay}_t + 0.5\text{var}(e_\sigma)]. \tag{14}
\]

In this specification, \(\text{cay}\) helps to forecast the conditional volatility of the risky return process, consistent with findings by Guo and Whitelaw (2006) and Lettau and Ludvigson (2007). Notice that \(\text{cay}\) and \(\log(\hat{\sigma}^2)\) evolve jointly in a VAR model with one lag. The CAPM compensates high risk exposure with a high risk premium through the term \(d_3\sigma_t^2\) if \(d_3 > 0\).

Whitelaw (1994) and Brandt and Kang (2004) find that a conditional expected return and conditional variance are exposed to difference shocks. Clearly, the CAPM is inconsistent with these empirical findings because it simplifies the conditional expected return as a linear function of the conditional variance. Motivated by these empirical findings, we include both \(\text{cay}\) and conditional variance to forecast the next-period return.

(5) CAY-VOL model:

\[
\log(\hat{\sigma}_{t+1}^2) = b_1 + b_2 \log(\hat{\sigma}_t^2) + b_3 \text{cay}_t + e_{\sigma,t+1} \tag{15}
\]

\[
\text{cay}_{t+1} = c_1 + c_2 \text{cay}_t + c_3 \log(\hat{\sigma}_t^2) + e_{\text{cay},t+1} \tag{16}
\]

\[
\mu_t = d_1 + d_2 \text{cay}_t + d_3 \sigma_t^2 \tag{17}
\]

\[
\sigma_t^2 = \exp[b_1 + b_2 \log(\hat{\sigma}_t^2) + b_3 \text{cay}_t + 0.5\text{var}(e_\sigma)]. \tag{18}
\]

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We also consider the CAPM in the following specification:

\[
\log(\hat{\sigma}_{t+1}^2) = b_1 + b_2 \log(\hat{\sigma}_t^2) + e_{\sigma,t+1}
\]

\[
\mu_t = d_1 + d_3 \sigma_t^2
\]

\[
\sigma_t^2 = \exp[b_1 + b_2 \log(\hat{\sigma}_t^2) + 0.5\text{var}(e_\sigma)].
\]

Both specifications of the CAPM lead to the same results analyzed later in this paper, so we do not report the results related to this form. We select the other specification because \(\text{cay}\), in addition to the lagged log variance, has forecasting power for future volatility.

We choose this model for parsimony because a VAR model including the second lag, in addition to the first one, slightly improves the explanatory power of the regressions.
This CAY-VOL model incorporates time variation in both the risk premium and the volatility. The expected excess return $\mu_t$ changes over time depending on the levels of $\sigma_t^2$ and $cay_t$. Similar to the CAPM, the CAY-VOL model compensates high risk with a high return through the term $d_3\sigma_t^2$ if $d_3 > 0$. The term $d_2cay_t$ can be understood as the premium to hedge against shifts in future investment opportunities, as indicated in the intertemporal capital asset pricing model (ICAPM) proposed by Merton (1973). Notice that the CAPM is obtained from the CAY-VOL model by imposing the restriction that $d_2 = 0$. Similarly, the IID, CAY, and VOL models can be considered as restricted versions of the CAY-VOL model.

This paper also studies the following two models with the dividend yield as a predictive variable, which has been used extensively in the dynamic portfolio choice literature. Let $dy_t$ be the dividend yield at period $t$.

(6) DY model\(^{14}\) (constant volatility):

\[
\begin{align*}
    dy_{t+1} &= k_1 + k_2 dy_t + e_{dy,t+1} \\
    \mu_t &= k_3 + k_4 dy_t \\
    \sigma_t^2 &= k_5^2.
\end{align*}
\]

This model specifies the conditional risk premium as a linear function of the dividend yield. It is easy to see that if the dividend yield is replaced with $cay_t$, the DY model becomes the CAY model. Both the CAY and DY models ignore time variation in the conditional variance. However, examining the CAY and DY models is motivated differently. The purpose of studying the DY model is to examine whether in-sample outperformance of the DY model over the IID model shown in prior studies still can hold out of sample, whereas the purpose of studying the CAY model is to investigate the economic value of neglecting time-varying risk premia.

\(^{11}\)The equation (18) is exact if $e_\sigma$ is normally distributed. Neither a t-test nor a Lillifors test can reject the null that $e_\sigma$ is normally distributed at the conventional 5% significance level.

\(^{12}\)Guo and Whitelaw (2006) adopt a similar specification to test the ICAPM, but they use the level of variance rather than the log of variance as the dependent variable in equation (15).

\(^{13}\)Campbell and Viceira (2001) summarize the literature on strategic asset allocation, in which the dividend yield is often used as a proxy for time variation in future investment opportunity sets and as a state variable to form optimal asset-allocation decisions in a long-horizon setting.

\(^{14}\)This model has been studied by Balduzzi and Lynch (1999), Campbell and Viceira (1999), Barberis (2000), and Xia (2001), among others.
For the latter purpose, the DY model cannot substitute for the CAY model. The reason is that some recent research fails to find out-of-sample forecasting power of the dividend yield and puts the predictability of the dividend yield under debate (Goyal and Welch, 2003; Inoue and Kilian, 2004). In contrast, cay shows an ability to forecast future returns out of sample (Lettau and Ludvigson, 2007).

(7) **DYH model**\(^{15}\) (heteroskedasticity):

\[
\begin{align*}
d_{y,t+1} &= k_1 + k_2 d_{y,t} + e_{dy,t+1} \\
\mu_t &= k_3 + k_4 d_{y,t} \quad (22) \\
\sigma_t^2 &= (k_5 + k_6 d_{y,t})^2 \quad (23)
\end{align*}
\]

This model is different from the DY model by accounting for heteroskedasticity.

### 2.2 The Investor’s Problem

Consider the portfolio choice at time \(t\) for the investor who maximizes the expected power utility of the terminal wealth \(W_T\) by trading the risky asset and the risk-free asset at times \(t, t+1, \ldots, T-1\). Let \(W_t\) be the investor’s wealth at time \(t\), and let \(R^f\) denote the constant gross risk-free rate. The investor with constant relative risk aversion \(\gamma\) encounters the following maximization problem:

\[
V_t(W_t, Z_t) = \max_{\{x_s\}^{T-1}_{s=t}} \mathbb{E}_t[u(W_T)] = \max_{\{x_s\}^{T-1}_{s=t}} \mathbb{E}_t \left( \frac{W_T^{1-\gamma}}{1 - \gamma} \right), \quad (25)
\]

subject to a sequence of budget constraints

\[
W_{s+1} = W_s(x_s r_{s+1}^e + R^f), \quad \forall s \geq t \quad (26)
\]

where \(x_t\) is the time \(t\) portfolio weight, the fraction of wealth allocated to the risky asset, and \(Z_t\) represents the information set at time \(t\) that is used to compute the portfolio weight.

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\(^{15}\)This model has been studied by Lynch and Balduzzi (2000). Following Whitelaw (1994), we use \(\sqrt{\pi/2}|e_{t+1}|\) to represent an unbiased estimate of the standard deviation of \(e_{t+1}\) with an assumption of normality.
Et denotes the expectation at time $t$ conditional on $Z_t$. The function $u(\cdot)$ refers to the functional form of power utility. This paper imposes short sale and borrowing constraints, so portfolio weights range from 0 to 1. The maximized expected utility is denoted by $V_t$, the value function. It measures the maximal expected power utility, conditional on the information set $Z_t$ available at time $t$, of the terminal wealth $W_T$ generated by the current wealth $W_t$ and the subsequent optimal portfolio weights. According to the principle of optimality, when the investor selects the optimal weight $x_t$ at time $t$ given the current wealth $W_t$ and the state information $Z_t$, the investor takes into account the fact that the subsequent portfolio weights $x_s$, $s > t$, are optimal based on the then-available wealth $W_s$ and information $Z_s$.

The investor forms portfolio policies after selecting a return model. She may behave myopically or dynamically by optimizing a one-period or a multiperiod problem. For the one-period problem, $t = T - 1$ in equation (25). Dynamic portfolio weights in excess of myopic weights are hedging demands to hedge against changes in future investment opportunities. If the return follows the IID model, there are no hedging demands because the dynamic portfolio weight for the IID model is the same as the myopic weight (Merton, 1969).

### 2.3 Solving the Dynamic Problem

The principle of optimality facilitates decomposing the intertemporal portfolio choice problem into a one-period problem of the form

$$V_t(W_t, Z_t) = \max_{\{x_s\}_{s=t}^{T-1}} \max_{0 \leq x_s \leq 1} E_t[u(W_T)] = \max_{\{x_s\}_{s=t}^{T-1}} \max_{0 \leq x_s \leq 1} E_t\left(\frac{W_T^{1-\gamma}}{1 - \gamma}\right) = \max_{0 \leq x_t \leq 1} E_t\left[\max_{\{x_s\}_{s=t+1}^{T-1}} E_{t+1}\left(\frac{W_T^{1-\gamma}}{1 - \gamma}\right)\right] = \max_{0 \leq x_t \leq 1} E_t\{V_{t+1}(W_t(x_t r_{t+1}^e + R^f), Z_{t+1})\}. \tag{27}$$

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16An information set $Z_t$ is a $\sigma$-algebra generated by state variables at time $t$. Conditional expectation based on the information set $Z_t$ is equal to conditional expectation based on the state variables that generate $Z_t$. Thus, without confusion, $Z_t$ in this paper denotes both the information set and the state variables that generate the $\sigma$-algebra.
Now the multiperiod portfolio choice problem turns into choosing the portfolio weight $x_t$ on the risky asset conditional on the wealth $W_t$ and information set $Z_t$ available at time $t$ with an assumption that the next-period optimization problem is solved with the value function of $V_{t+1}(W_{t+1}, Z_{t+1})$.

Applying a fourth-order Taylor expansion, we can rewrite the value function in the preceding one-period problem around the current wealth growing at the risk-free rate $W_t R_f$:

$$V_t(W_t, Z_t) = \max_{0 \leq x_t \leq 1} E_t\{V_{t+1}[W_t(x_t r_{t+1}^e + R^f), Z_{t+1}]\}$$

$$= \max_{0 \leq x_t \leq 1} E_t \left[ V_{t+1}(W_t R_f, Z_{t+1}) + \partial_1 V_{t+1}(W_t R_f, Z_{t+1}) W_t x_t r_{t+1}^e + \frac{1}{2} \partial_1^2 V_{t+1}(W_t R_f, Z_{t+1}) W_t^2 x_t^2 (r_{t+1}^e)^2 + \frac{1}{6} \partial_1^3 V_{t+1}(W_t R_f, Z_{t+1}) W_t^3 x_t^3 (r_{t+1}^e)^3 \right]$$

$$+ \frac{1}{24} \partial_1^4 V_{t+1}(W_t R_f, Z_{t+1}) W_t^4 x_t^4 (r_{t+1}^e)^4$$

$$= \max_{0 \leq x_t \leq 1} \left[ E_t(A_{t+1}) + E_t(B_{t+1}) W_t x_t + E_t(C_{t+1}) W_t^2 x_t^2 + E_t(D_{t+1}) W_t^3 x_t^3 + E_t(E_{t+1}) W_t^4 x_t^4 \right]$$

(28)

where $\partial_1^i V_{t+1}(\cdot, \cdot)$ is the $i$th-order partial derivative with respect to the first argument of the value function. The formula for $A_{t+1}, B_{t+1}, C_{t+1}, D_{t+1},$ and $E_{t+1}$ can be found in the Appendix. Based on the simulation-based approach proposed by Brandt, Goyal, Santa-Clara, and Stroud (2005), we first simulate a sizable number of hypothetical sample paths of returns and state variables according to their joint distributions. Then we project $A_{t+1}, B_{t+1}, C_{t+1}, D_{t+1},$ and $E_{t+1}$ across paths onto information set $Z_t$ available at time $t$, given the optimal portfolio choices for all future periods, to get the conditional expectations of $E_t(A_{t+1}), E_t(B_{t+1}), E_t(C_{t+1}), E_t(D_{t+1})$, and $E_t(E_{t+1})$. Specifically, these conditional expectations are obtained by running only five regressions across paths with $A, B, C, D,$ and $E$ as regressands. The regressors are a vector of a second-order polynomial expansion in the state variables.\(^{18}\) The future optimal

\(^{17}\)Brandt, Goyal, Santa-Clara, and Stroud (2005) show that a fourth-order Taylor expansion around $W_t R_f$ can approximate the value function very well because $W_t x_t r^e$ is a very small amount relative to $W_t R_f$ due to $r^e \ll R_f$ and $0 \leq x_t \leq 1$.

\(^{18}\)We also compute portfolio weights using a third-order polynomial expansion in the state variables as regressors. These weights are similar to those reported in this paper.
portfolio choices are reflected in the construction of $\hat{W}_T$ along each simulated path because $\hat{W}_T = W_t R_f \prod_{s=t+1}^{T-1}(x_s^* r_{s+1}^e + R_f^e)$ uses the optimal weight $x_s^*$ for all $s \geq t + 1$.\(^{19}\)

Because the investor selects portfolio weights $x_t$ to maximize the expected terminal utility by anticipating that the optimal weights are chosen for all future periods, this dynamic maximization problem is solved recursively backward. Starting from one period before the end of the investment horizon, $x_{T-1}$ is first chosen along each simulated path to maximize the fourth-order Taylor expansion of the expected terminal utility in equation (28). Then this process is repeated until $x_{t+1}$ is selected. At time $t$, the problem becomes finding $x_t$ to maximize equation (28) that is a fourth-order polynomial in $x_t$ with $W_t$, $E_t(A_{t+1})$, $E_t(B_{t+1})$, $E_t(C_{t+1})$, $E_t(D_{t+1})$, and $E_t(E_{t+1})$ available. Because these five conditional expectations depend on $W_t$ and $Z_t$, $x_t$ is a function of $W_t$ and $Z_t$ as well. Brandt, Goyal, Santa-Clara, and Stroud (2005) propose an iteration method to search for a solution to equation (28). This method is subject to the following limitations: (a) the convergent solution may not be the point at which the global maximum of equation (28) is obtained, and (b) it has difficulty imposing the short-sale and borrowing constraints during the search process. To overcome these limitations, we instead first solve equation (28) by finding all points that make the first-order condition zero. Since the first-order condition is a cubic polynomial in $x_t$, which is a scalar, solutions can be found analytically. Then we substitute into equation (28) the solution points that satisfy the portfolio weight constraints and the boundary points implied from those weight constraints. By comparing function values at those solution points and boundary points, the point that gives the maximal function value is selected. This point satisfies portfolio weight constraints and is the point at which the global maximum is obtained. Clearly, this modified method can accommodate portfolio choice problems with or without weight constraints.\(^{20}\) In addition, this modified method can inherit the virtues mentioned in Brandt, Goyal, Santa-Clara, and Stroud (2005), such as a short time needed to solve for portfolio weights and the ability to handle a dynamic portfolio choice problem with a large number of state variables. Without loss of generality, the initial wealth can be normalized as $W_t = 1$ because the power utility

\(^{19}\)\(\hat{W}_T\) is used in the expressions of $A_{t+1}$, $B_{t+1}$, $C_{t+1}$, $D_{t+1}$, and $E_{t+1}$ in the Appendix.

\(^{20}\)Although the simulation-based approach proposed by Brandt, Goyal, Santa-Clara, and Stroud (2005) combined with the grid-search technique can avoid the second limitation, it would be time-consuming to search for portfolio weights with a fine grid or a wide range of grids.
function is homothetic in wealth.

2.4 A Utility Metric to Measure Portfolio Performance

A certainty equivalent return (CER) is adopted as a metric to measure the portfolio performance of a rebalancing rule. A CER associated with a rule is interpreted as the annual rate of return on wealth that, if earned with certainty, would provide the investor with the same utility as the given portfolio rule.\textsuperscript{21} The CER is obtained by solving

\[ u(W_t(1 + \text{CER})^\frac{T-t}{4}) = E_t\left(\frac{W_{T}^{1-\gamma}}{1 - \gamma}\right). \]

(29)

Notice that CER is a function of state variables \(Z_t\), as is the expected utility on the right-hand side of equation (29). An unconditional CER associated with a portfolio rule is computed the same way as shown in equation (29) by instead using unconditional expected utility, which is obtained by integrating the conditional utility levels over the distribution of states of the economy.\textsuperscript{22}

3 Empirical Estimation and Portfolio Strategies

This section calibrates models discussed in Section 2.1 to the U.S. historical data. All parameters in the models are estimated using the generalized method of moments (GMM). This section then examines properties associated with different portfolio strategies for these models. Portfolio performance of these strategies will be compared in the latter sections.

\textsuperscript{21}This metric is an extension of the CER used by Kandel and Stambaugh (1996) in a single-period setting.

\textsuperscript{22}We also use the certainty equivalent wealth level as a metric to compare performances of different portfolio policies. We do not report the results based on this measure because the results are the same as reported in this paper when CER is used as the measure. The certainty equivalent wealth level for a portfolio rule measures the certain amount of wealth that makes the investor indifferent between receiving it for certain at the horizon and investing her current wealth using the portfolio rule up to the horizon. The certainty equivalent wealth level \(CE\) can be computed from the following relation:

\[ \left(\frac{CE^{1-\gamma}}{1 - \gamma}\right) = E_t\left(\frac{W_{T}^{1-\gamma}}{1 - \gamma}\right). \]

(30)
3.1 Data

All return data in this study come from CRSP and cover the period from January 1952 to December 2006. All returns used in estimating models are deflated by the consumer price index inflation rate. The quarterly deflated risk-free rate is the average of the 3-month T-bill rate in excess of the corresponding inflation rate. The daily value-weighted return on the NYSE, NASDAQ, and AMEX markets is used as a proxy for the daily return on the risky asset. The daily data are used to construct the realized volatility of the risky return. Because the daily frequency of the risk-free rate is not directly available, it is computed using the continuously compounded formula as the average of the monthly risk-free rate. The quarterly excess stock market return is the sum of the daily stock market return in excess of the daily risk-free rate within each quarter. Without further specification, all the returns hereafter refer to the deflated returns.

Since \( c_{ay} \) is available only quarterly, all models in this paper are estimated at the quarterly frequency. In the in-sample analysis, \( c_{ay} \) is generated based on the full sample of data—quarterly aggregate consumption, labor income, and asset holdings, which can be downloaded from the web sites of Martin Lettau and Sydney Ludvigson. In the out-of-sample evaluation, we consider the second case in which \( c_{ay} \) is recursively constructed using these data available only at the time of decision making.

To calculate the dividend yield, we first construct the dividend payout series using the value-weighted return including dividends and the price index series associated with the value-weighted return excluding dividends. Following the standard convention in the literature, we take the dividend series to be the sum of dividend payments over the past year to remove seasonality. The dividend yield then is measured as the past-year dividend payments divided by the price index in a given quarter (Fama and French, 1989; Ferson and Harvey, 1991).

3.2 Empirical Estimation of Return Models

Table 1 reports the estimated coefficients and their t-statistics of the five models discussed in Section 2.1 based on the quarterly data from 1952 to 2006. The IID model indicates that the unconditional mean and unconditional standard deviation of the quarterly excess return on
the risky asset are 2.1% and 8.1%, respectively.\textsuperscript{23} In the CAY model, \textit{cay} is a very persistent process. Its own lag has a significant predictive power, with an $R^2$ of roughly 73%. Consistent with the results in Lettau and Ludvigson (2001), \textit{cay} affects the excess stock market return positively and significantly with an $R^2$ of 5.9%.

Following the stock market crash in October 1987, the realized stock market variance in the fourth quarter of 1987 rose to an extremely high level. Then, in the following quarter, it dropped to normal. The finite-sample predictive power of the log realized variance degrades substantially when the observation during the crash period is included because the regression is very sensitive to outliers. Following Campbell, Lettau, Malkiel, and Xu (2001) and Guo and Whitelaw (2006), we set the realized stock market variance in the fourth quarter of 1987 to the second highest value in the sample.\textsuperscript{24}

In the log realized variance equation of the VOL model, its own lag explains roughly 39% of variation in next-period log realized variance. The log realized variance is much less persistent than \textit{cay}, and it is less accurately predicted than \textit{cay} based on $R^2$ levels. The constant term of the return equation tells us that the average quarterly excess return is 1.9%, which is statistically significant. In the log realized variance equation of the CAPM, the log realized variance has little impact on forecasting \textit{cay}. In contrast, \textit{cay} affects the log realized variance statistically and negatively, consistent with the results shown by Guo and Whitelaw (2006) and Lettau and Ludvigson (2007). Nevertheless, the marginal explanatory power from \textit{cay} is small because the $R^2$ increases by only 1%. In the return regression, we can see that the coefficient on the conditional variance is 4.4, positive but insignificant. The very low explanatory power in the return equation of the CAPM suggests that conditional volatility, compared with \textit{cay}, captures a small portion of the variation in the risk premium.

The CAY-VOL model captures time variation in both the first and second conditional moments, and these first two conditional moments are exposed to different shocks. In the return equation, the coefficient on \textit{cay} and the coefficient on the conditional stock market variance are statistically significant at the 1% and 10% levels, respectively. Nevertheless, the conditional stock market variance and \textit{cay} jointly explain only about 7.4% of the variation

\begin{footnotesize}
\begin{enumerate}
\item\textsuperscript{23}$\exp(-5.039/2) = 8.1\%.$
\item\textsuperscript{24}The estimation results for the log realized variance are very similar no matter whether the observation in the fourth quarter of 1987 is removed or reset by the second highest value in the sample.
\end{enumerate}
\end{footnotesize}
in the stock market return, consistent with the literature stating that most portion of fluctuation in stock returns stems from unexpected returns. The overidentified specification of the model has a J-statistic of 1.94 with a p-value of 0.58, so the model cannot be rejected at a conventional significance level. The coefficient on the conditional variance is 4.2, similar to the corresponding coefficient in the CAPM specification. Both these models indicate that high risk exposure is compensated with a high return. In the CAY-VOL model, the expected return also varies with forward-looking $cay$, which can be understood as the premium to hedge against shifts in future investment opportunities.

The coefficient on $cay$ in the excess return equation of the CAY-VOL model is higher by 0.23 than that in the CAY model. This is reasonable. In the CAY-VOL model, a high value of $cay$ leads to a low variance in the next period, which forecasts a low excess return. On the other hand, because $cay$ affects the excess return positively, a high level of $cay$ predicts a large excess return. The second effect dominates the first one, resulting in a positive but lower coefficient on $cay$ in the CAY model, which does not model conditional volatility explicitly. Comparing the $R^2$ of the excess return equation in the CAY model and that in the CAY-VOL model, we can see that most of the variation in the stock market excess return can be explained by $cay$ alone. The explanatory power is reduced from 7.4% to 5.9% by removing time-varying volatility from the CAY-VOL model.

Table 2 exhibits the coefficient estimation for the DY and DYH models, which use the dividend yield as a single forecasting variable. Consistent with the literature, the dividend yield is a highly persistent process, with the coefficient on its own lag as high as 0.962. The dividend yield affects next-period return positively and significantly at a 10% significance level. Comparing the $R^2$ of 1.7% in these two models with the $R^2$ of 7.4% in the CAY-VOL specification indicates that $cay$ combined with the conditional variance is much better at capturing the variation in the excess risky return than the dividend yield. Poor predictability of the dividend yield on the risk premium has been documented by Lettau and Ludvigson (2005), Ang and Bekaert (2007), and Goyal and Welch (2003). Boudoukh, Michaely, Richardson, and Roberts (2007) and Campbell and Thompson (2005) find that total payout to price ratio and earnings-based valuation ratios, respectively, perform better than the dividend-price ratio.

25Because the stock market return increases with $cay$ and the stock market variance falls with it, there is a negative correlation of -0.32 between the two components of the expected stock return in the CAY-VOL model.
These findings may be caused by changes in payout policy as firms have shifted from paying dividends to repurchasing shares. Lettau and Ludvigson (2005) find that fluctuations in expected returns are positively correlated with fluctuations in expected dividend growth and that these fluctuations have offsetting effects on the log dividend-price ratio. As a result, the log dividend-price ratio has difficulty revealing the variation in market risk premium over business cycle frequencies. The DYH model takes heteroskedasticity into account. The dividend yield explains less than 1% of variation in the standard deviation of the excess return; in contrast, the lagged log variance and \( cay \) combined account for 40% of the fluctuations in the log stock market variance.

3.3 Portfolio Weights

According to the algorithm discussed in Section 2.3, portfolio weights for different models are computed under the assumption that the investor has power utility with constant relative risk aversion of 5 and can rebalance her portfolio quarterly in a 5-year investment horizon. Merton (1969) shows that dynamic portfolio weights are the same as the myopic ones for the IID model because of no hedging demands. Using the IID model, the investor invests 72% of her wealth in the risky asset over the investment horizon. The first row of Figure 1 exhibits the portfolio weights associated with the CAY, VOL, and CAPM models that use single state variables to specify RGPs. This figure illustrates how these single state variables determine myopic, hedging, and total portfolio weights at the beginning of a 5-year investment horizon. Focusing on the CAY model first, notice that the total weights increase with \( cay \), as do the myopic and hedging weights, except that short-sale and borrowing constraints affect this monotonic relationship. A high level of \( cay \) predicts a high excess return; it therefore implies a high conditional Sharpe ratio. Thus \( cay \) affects the myopic weights positively. Because \( cay \) is negatively correlated with realized risky return shocks and affects the expected return positively, holding more stocks can hedge against an adverse shift in future investment opportunities, thus leading to a positive hedging demand.

When time-varying volatility is considered in the model specifications, such as the VOL

\( ^{26} \)Applying the algorithm described in Section 2.3, we compute portfolio weights by simulating 1,000,000 sample paths for each model.
and CAPM models, we find that the hedging demand is negative and small, consistent with the findings of Chacko and Viceira (2005). Figure 1 also shows that the higher is the conditional volatility, the lower are both myopic and total portfolio weights. The rationale is that holding less of the risky asset can reduce risk exposure when volatility is expected to be higher. Even though the expected return increases with the conditional volatility in the CAPM, the potential reward is not large enough to drive the investor to hold more of the risky asset when the risky return is expected to be more volatile. However, given a level of conditional volatility, the reward for exposing high risk in the CAPM does make the investor allocate a larger fraction of her wealth to the risky asset than she would in the VOL model, which has a constant expected return.

The second row of Figure 1 depicts how the investor chooses portfolio weights relying on the dividend yield for the DY and DYH models. Similar to the CAY model, both myopic and total weights rise linearly with the dividend yield, consistent with the findings by Campbell and Viceira (1999). Generally speaking, investment strategies for these two models are very similar.

Figure 2 exhibits how myopic, hedging, and optimal portfolio weights are determined by the two state variables at the beginning of a 5-year investment horizon in the CAY-VOL specification. Left to right, the first row plots these weights with respect to \( cay \) given that the conditional variance is one standard deviation below, at, or one standard deviation above the mean. Notice that both myopic and optimal portfolio weights increase with \( cay \). These tendencies are similar to those in the CAY model, but these weights respond more sensitively to \( cay \) when volatility is expected to be lower. The reason is that the low conditional volatility tells the investor that realized returns are close to expected ones with small deviations. The investor can rely on the conditioning information to a large extent because of small risk.

Left to right, the bottom row of Figure 2 shows how myopic, hedging, and optimal portfolio weights are related to conditional volatility given that \( cay \) is one standard deviation below, at, or one standard deviation above the mean. In contrast with the relationship with \( cay \), myopic and optimal portfolio weights increase with the conditional volatility when \( cay \) is low but decrease when \( cay \) is high. This relationship stems from the fact that the conditional
Sharpe ratio increases (decreases) with the conditional volatility when \( cay \) is low (high).\(^{27}\) We also notice that the relationship between the portfolio weights and the conditional variance in the CAY-VOL model is similar to those in the VOL and CAPM models only when \( cay \) is relatively large.

Clearly, the optimal weights for the CAY-VOL model have a rich and complex relation with \( cay \) and conditional variance. If the model is misspecified by ignoring the time-varying expected return or volatility, or even if the CAPM is employed, portfolio weights can be so different based on these alternative models from the weights based on the CAY-VOL model given the levels of state variables. The structure in the CAY-VOL model of allowing the conditional expected return and conditional volatility to be driven by two different shocks is essential to making these differences in portfolio weights.

### 3.4 Horizon Effects of Hedging Demands

The preceding subsection shows that the optimal portfolio weights for the CAY model include high and positive hedging demands, while the optimal portfolio rule for the VOL model has small and negative hedging demands. The question naturally arises: is it still true that in the CAY-VOL model, \( cay \) induces positive and large hedging demands and that conditional volatility contributes small and negative hedging demands?

The answer to this question can shed light on the relative importance of the first and second conditional return moments in determining dynamic portfolio weights and costs of myopic behavior when predictability of both the first and second return moments is specified.

To find the answer, we convert the discrete-time problem into a continuous-time one so that we can appeal to the Bellman equation to distinguish the hedging demands contributed by \( cay \) and by conditional variance. Figure 3 illustrates the horizon effect of the hedging demands induced by these two state variables. Because of short-sale and borrowing constraints, hedging

\[^{27}\text{Let us look at the conditional Sharpe ratio, the expected excess return per unit of risk:}\]

\[
SR_t = \frac{\mu_t}{\sigma_t^2} = \frac{b_7 + b_8cay_t + b_9\sigma_t^2}{\sigma_t^2} = \frac{b_7 + b_8cay_t}{\sigma_t^2} + b_9
\]

where \( b_7 < 0 \) and \( b_8 > 0 \). The conditional Sharpe ratio has the following properties:

1. When \( cay_t \) is negative, \( b_7 + b_8cay_t < 0 \). \( SR_t \) increases as \( \sigma_t^2 \) goes up, and
2. when \( cay_t \) is positive and large, \( b_7 + b_8cay_t > 0 \). \( SR_t \) declines as \( \sigma_t^2 \) increases.
demands for some initial values of the state variables cannot be seen in this figure, so Figure 4 exhibits the horizon effect without imposing short-sale and borrowing constraints. Figures 3 and 4 describe the relationship between investment horizons and the hedging demands induced by $cay$ in the upper panel and by conditional volatility in the lower panel for nine different initial states of the economy. Left to right, the plots illustrate the cases in which the variance at the beginning of the investment horizon is one standard deviation below, at, and one standard deviation above the sample mean. The solid, dashed, and dash-dot lines represent cases in which $cay$ at the beginning of the investment horizon is one standard deviation below, at, and one standard deviation above the sample mean. Obviously, time-varying volatility contributes little to the hedging demands compared with $cay$. Without weight constraints imposed, we see that the higher the initial value of $cay$ and the lower the initial volatility, the higher is the contribution of $cay$ to the hedging demands. The hedging demands induced by $cay$ are positive and have a pronounced horizon effect, but not those induced by the conditional volatility. Except that the first plot in the second row shows small negative hedging demands with a clear horizon effect when volatility is expected to be low at the time of investment, the other two plots in the second row show much noise. One possible reason is that the hedging demands contributed by variance are small, so relatively substantial noise is introduced in computing the derivative of the conditional expectation of the value function with respect to the variance.

Why does $cay$ contribute to hedging demands significantly compared to conditional volatility? The reason is that $cay$ is more persistent than the variance. The half-life of the variance is 1.32 quarters, and the half-life of $cay$ is 4.24 quarters. In other words, the variance is expected to be half its original value after four months, whereas even after a year, half the original level of $cay$ is expected to stay. Hence $cay$ is an important conditioning variable that makes a significant contribution to hedging demands, but its hedging ability is weaker when

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28 Barberis (2000) shows the horizon effect of multiperiod portfolio rules conditional on the dividend yield. In Barberis’s paper, the DY model is used with constant volatility.

29 Conditional expectation of the value function is computed as the average across 10,000,000 simulation paths with the same initial values of state variables $cay$ and volatility based on the simulation-based approach proposed in Brandt, Goyal, Santa-Clara, and Stroud (2005).

30 The half-life is defined as the number of periods required for the impulse response to a unit shock to a time series to dissipate by half. It is used widely as a measure of persistence.
the value of \( cay \) at the beginning of investment horizons is lower and the volatility at that time is expected to be higher.

4 In-Sample Portfolio Performance Analysis

Table 3 presents the in-sample annualized CERs associated with the dynamic and myopic strategies for the IID, CAY, VOL, CAPM, and CAY-VOL models. The utility measures are computed under the assumption that the CAY-VOL specification is the correct RGP with known parameters reported in the last column of Table 1.

The dynamic policy and the myopic policy for the IID model are the same, with a CER of 5.9% per year. If adopting the dynamic policies conditional on \( cay \) in the CAY model, the investor can earn an annualized CER of 7.86%. If the investor also takes into account time variation in conditional variance using the CAY-VOL model, she can obtain an annualized CER of 8.06%. These results show that time-varying conditional return moments have a large effect on portfolio performance. Ignoring time variation in these return moments, the investor can suffer a loss of roughly 2% in terms of CER per year. This result is stronger than the findings by Balduzzi and Lynch (1999) that, without transaction costs, utility costs of ignoring return predictability are roughly 23% for a 20-year investment horizon, roughly 1% per year.

The annualized CERs of the dynamic policies conditional on time-varying variance in the VOL and CAPM models are 5.75% and 6.33%, respectively. The dynamic rule for the CAPM outperforms the corresponding rule for the VOL model because the CAPM compensates high risk with a high return. Notice that once dynamic portfolio rules are computed conditional on \( cay \), utility measures rise considerably. These results imply that incorporating time variation in the expected return can improve portfolio performance substantially compared with considering time-varying variance. The rationale is as follows: First, \( cay \) is a more powerful forecasting variable than conditional variance to predict future stock excess returns because \( cay \), compared to conditional variance, can explain a large portion of the variation in the excess return. Second, high conditional variance drives the investor to reduce risky investments because of risk-averse preferences. At the same time, it leads the investor to increase risky investments because of the risk-reward relationship implied by the CAY-VOL model. These two effects counteract, resulting in a weak power of the conditional volatility in determining
portfolio weights. Third, \textit{cay} is a more persistent process than conditional variance. \textit{cay} therefore has a substantial hedging ability, whereas conditional volatility does not, as shown in section 3.4.

The investor following the myopic strategies for the CAY and CAY-VOL models can earn annualized CERs of 7.65\% and 7.95\%, respectively, with 30 bp in difference. This result means that variance timing can improve investment performance when the investor behaves myopically, consistent with the findings based on mean-variance portfolio rules by Fleming, Kirby, and Ostdiek (2001, 2003). In contrast, the investor's utility increases by only 11 bp when switching from the dynamic policy for the CAY model to that for the CAY-VOL model. These comparisons tell us that considering time-varying variance in a myopic setting is more valuable in improving performance than that in a dynamic setting. The explanation is that variance can be predicted well in a short run because variance has a half-life of about four months. In a myopic problem, a good forecast of next-period variance helps to improve investment performance. But dynamic portfolio weights are computed by forecasting not only next-period variance but also expected variance at subsequent periods until the end of the 5-year investment horizon. Because high next-period conditional variance converges to the average level soon, utility gains, which equal total utility gains divided by the number of investment horizons, using conditional volatility as a state variable is small in a dynamic setting. Put differently, in a dynamic portfolio choice problem, the relatively large improvement in portfolio performance by using well-predicted next-period variance is offset by little improvement on predicted variance near the end of the investment horizon, leading to a smaller utility gain per year of relying on conditional volatility.

The annualized CER decreases by 11 bp if the investor switches to the myopic policy from the dynamic policy for the CAY-VOL model. Although there exists a noticeable horizon effect of portfolio weights, as shown in Section 3.4, short-sale and borrowing constraints rule out or reduce the hedging demands for many initial states of the economy, as exhibited in Figure 3, resulting in modest unconditional utility costs over the distribution of initial states of the economy if the investor behaves myopically. This means that market timing has a significant effect on investment decisions when short-sale and borrowing constraints are imposed. Balduzzi and Lynch (1999) and Campbell and Viceira (1999) conduct in-sample analysis and show
that the utility costs of ignoring hedging demands are large. These papers use the dividend yield as a state variable, which is much more persistent than the state variables we use, and the persistency of state variables drives the large utility costs in sample.

5 Out-of-Sample Portfolio Performance Analysis

The in-sample evaluation of portfolio performance assumes that the RGP used to compute portfolio weights is the same one that is used to generate future returns over investment horizons. In fact, a real-world investor uses only currently available data to compute portfolio weights and applies the weights to future actual returns whose RGPs are unknown. Therefore, in-sample analysis on portfolio performance overestimates the value of portfolio policies optimal to assumed models. This section attempts to investigate an out-of-sample measure of portfolio performance for different portfolio rules.

5.1 An Out-of-Sample Experiment Design

The data used in this paper start in the first quarter of 1952 and end in the fourth quarter of 2006. The following experiment is designed to test the out-of-sample performance of each portfolio rule. Assume that there are a group of investors who are identical except that they start their investments at different periods of time. These investors encounter a dynamic portfolio allocation problem specified by equations (25) and (26) and have relative risk aversion of 5. In every quarter from the first quarter of 1982 to the first quarter of 2002, there is one investor who starts his or her investment with a 5-year horizon. These investors can rebalance their portfolios quarterly and can access the historical data only over the periods from the first quarter of 1952 to the fourth quarter of the previous year. They use these available data to estimate parameters in the models they employ and to form portfolio weights based on the newly estimated parameters and current values of state variables. For each portfolio rule, there are 81 investments, which start at every quarter beginning in the first quarter of 1982 and last for five years. We then average over time the utilities of the terminal wealth obtained by using each portfolio rule to get the unconditional expected terminal utility and compute the certainty equivalent utility measure following the method discussed in Section 2.4.
"cay" is a constructed variable and determines portfolio weights as a state variable for some portfolio rules, so we consider two ways to construct "cay." In the first way, the cointegration parameters used to construct "cay" are fixed and estimated based on the full sample.\footnote{\textsuperscript{31}The "cay" we constructed using the full sample is the same as the "cay" provided by Lettau and Ludvigson on their web sites.} In the second way, the cointegration parameters are estimated recursively by using data only available at the time of decision making, and "cay" is constructed using these newly estimated parameters. The first way is interesting because the cointegration coefficients can be estimated accurately if the full sample is used (Inoue and Kilian, 2004). Recursively constructed "cay" can have significant errors (Lettau and Ludvigson, 2001, 2005), hurting portfolio performance associated with the strategies based on these recursively constructed "cay" values. But it is the second way, not the first one, that is realistic. Of course, the second way is more likely to be stringent because the cointegration coefficients can be estimated more accurately now than in the past as more data are available.

5.2 Statistical Significance of Outperformance

To examine the statistical significance of the out-of-sample utility comparison of a pair of portfolio rules, we construct a statistic to test the null hypothesis that the performance of rule $i$ is not greater than the performance of rule $j$. Let $Z_t = U^i_t - U^j_t$, where $U^i_t$ is the power utility of the terminal wealth for a 5-year investment starting at period $t$ and using rule $i$.\footnote{If rule $i$ drives a higher expected terminal utility than rule $j$, then it also has a higher CER. Therefore, we only need to assess the statistical significance of relative performance of two rules based on their expected terminal utilities.} Under the null hypothesis, the Studentized statistic

\[ S = \frac{\bar{Z}}{\sigma(\bar{Z})} \]  

has a standard normal distribution asymptotically, where $\bar{Z} = \sum_{s \geq t} Z_s/(T - t)$ and $\sigma(\bar{Z})$ is the standard error of $\bar{Z}$ and is calculated according to the Newey-West formula. Because the sample size is small in the out-of-sample evaluation, the inference can be of a large bias when p-values are drawn according to the asymptotic distribution of the statistic in (31). Hence we
also calculate empirical p-values based on bootstrap samples.\footnote{We bootstrap 1000 sample paths from the historical data using the stationary bootstrap proposed by Politis and Romano (1994) and choose the average block size according to Politis and White (2004). Along each sample path, there are 220 periods of data to mimic the historical sample. An empirical p-value is obtained as one minus the quantile of $S$ in $(\tilde{Z} - \bar{Z})/\sigma(\tilde{Z})$, where $\tilde{Z}$ is the bootstrap analogue of $\bar{Z}$ and $\sigma(\tilde{Z})$ is the bootstrap standard error of $\tilde{Z}$. These statistics are asymptotically pivotal and constructed according to the guidelines proposed by Hall and Wilson (1991). To get empirical p-values when $cay$ is recursively constructed, we use the methods described in Chang, Park, and Song (2006) and Parker, Paparoditis, and Politis (2006) to bootstrap 1000 sample paths of consumption, asset wealth, and labor income, which are available on the web sites of Martin Lettau and Sydney Ludvigson. Along each sample path, $cay$ would be recursively constructed using consumption, asset wealth, and labor income available at the time of making investment decisions.

\footnote{We also consider the DY and DYH models by correcting the small sample bias proposed by Stambaugh (1999). The out-of-sample utilities of portfolio rules for these two models after correcting the small sample bias are slightly lower than the utilities before the correction. For brevity, we do not report the utilities for the case with the bias correction.}}

### 5.3 Out-of-Sample Performance Comparison

Table 4 presents the out-of-sample utility comparison of the dynamic and myopic investment rules for various models discussed in Section 2.1. First, notice that the dynamic policies for the DY and DYH models do not beat the policy for the IID model, not to mention the policies for the other models.\footnote{The investor using the IID model would get an annualized CER roughly 2% higher than the investor using the DY and DYH models. The poor out-of-sample performance of the DY and DYH models is consistent with the recent debate that the dividend yield fails to demonstrate out-of-sample predictability by Bossaerts and Hillion (1999), Goyal and Welch (2003), and Ang and Bekaert (2007). These results suggest that in-sample utility gains based on the dividend yield, which have been anticipated by prior studies on dynamic portfolio choice, may represent an ideal that is difficult to materialize in reality. These results also imply that conducting an out-of-sample evaluation is necessary to assess the value of a dynamic portfolio rule because in-sample analysis can be misleading.}

Similar to the in-sample results, the dynamic rule for the CAY model outperforms the rules for the IID and VOL models and the CAPM. However, the out-of-sample performance of the dynamic strategy for the CAY-VOL model stands in contrast to the in-sample performance reported in Table 3. This strategy performs poorly out of sample, with its annualized CER
almost 1% lower than the CER associated with the dynamic rule for the CAY model, no matter whether \( cay \) is constructed recursively or based on the full sample.

Parameter uncertainty can contribute substantially to the poor out-of-sample performance of the dynamic policy for the CAY-VOL model. In particular, the coefficient on the conditional variance in the CAY-VOL model is estimated with a relatively large amount of noise, and the estimate fluctuates substantially over time in the rolling regressions using data from the first quarter of 1952 to the fourth quarter of every year from 1981 to 2006. In fact, dynamic portfolio weights depend not only on the values of state variables but also on parameters in a given model. For example, if the true coefficient on conditional variance in the CAY-VOL model is very negative, a high level of conditional volatility forecasts a low expected return. Thus the preference of avoiding high risk and a low return leads investors to invest a small amount, if not none, of their wealth in the risky asset. Otherwise, if the true coefficient is positive and high, a high level of conditional volatility may induce investors to hold more of the risky asset because a high expected return can compensate risk and makes the risky asset attractive. Hence large fluctuation in the estimate of this coefficient can result in a high possibility that portfolio weights are “mistakenly” determined by conditional volatility if the estimated coefficient is far from the true value. The improvement in portfolio performance achieved by considering time-varying volatility cannot counterbalance a decline in the performance caused by estimation risk. As a result, the dynamic policy for the CAY-VOL model underperforms the dynamic policy for the CAY model out of sample.

This paper proposes two methods to deal with the adverse effect of parameter uncertainty on out-of-sample portfolio performance. The first method is to form a portfolio policy conditional on \( cay \) alone in the CAY-VOL model, which is denoted by \( cay\text{-CAY-VOL} \). Let us rewrite equation (25), in which investors choose portfolio weights to maximize their expected terminal utilities based on \( cay \) and conditional volatility, as follows:

\[
V_t(W_t, cay_t, \sigma_t^2) = \max_{\{x_s\}_{s=0}^{T-1}} \mathbb{E} \left[ u(W_T) \mid cay_t, \sigma_t^2 \right] = \max_{\{x_s\}_{s=0}^{T-1}} \mathbb{E} \left( \frac{W_T^{1-\gamma}}{1-\gamma} \right) \mid cay_t, \sigma_t^2).
\]

Then the dynamic policy conditional on \( cay \) alone in the CAY-VOL model is to maximize investors’ terminal utilities conditional solely on \( cay \) by integrating the above conditional
terminal utility over the distribution of time-varying volatility:

\[ V_t(W_t, cay_t) = \max_{\{x_s\}_{s=t}^{T-1}} \mathbb{E}\left[ \mathbb{E}\left( W_T^{1-\gamma} | cay_t, \sigma_t^2 \right) | cay_t \right] = \max_{\{x_s\}_{s=t}^{T-1}} \mathbb{E}\left( W_T^{1-\gamma} | cay_t \right). \]

For the comparison purpose, we also allow the investors to form a dynamic policy conditional on time-varying variance alone in the CAY-VOL model, denoted by \( \sigma_t^2 \)-CAY-VOL, which can be obtained by exchanging \( cay_t \) with \( \sigma_t^2 \) in the preceding maximization problem.

The second method naturally takes the Bayesian approach that uncertainty about parameters is summarized by the posterior beliefs reflecting the information contained in the historical data and investors’ prior beliefs about the parameters. In the CAY-VOL model, the most uncertain estimate is the estimated coefficient on conditional variance in the mean equation. Due to computational complexity for out-of-sample performance analysis, investors only account for uncertainty of the parameters in the mean equation. As more data are available over an investment horizon, investors learn the parameters in the mean equation and simply reestimate the other parameters as given.

Let \( x_t = [1, cay_t, \sigma_t^2] \), \( X_{0,t-1} = [x_0, ..., x_{t-1}]' \), and \( Y_{1,t} = [r_{1}^{t}, ..., r_{t}^{t}]' \). Then the time series of the excess return regression in the CAY-VOL model can be written as

\[ Y_{1,t} = X_{0,t-1}d + C_{0,t-1}\epsilon \]

where \( C_{0,t-1}C_{0,t-1} = \Sigma_{0,t-1}, \Sigma_{0,t-1} = \text{diag}[\sigma_0^2, ..., \sigma_{t-1}^2] \), and \( d = [d_1, d_2, d_3]' \). Investors choose a diffuse prior about \( d \) and assume that \( \epsilon \) has a standard normal distribution. Then the posterior beliefs about \( d \) are characterized by

\[ p(d|Y_{1,t}, X_{0,t-1}) \sim N\left((X'_{0,t-1}\Sigma_{0,t-1}^{-1}X_{0,t-1})^{-1}(X'_{0,t-1}\Sigma_{0,t-1}^{-1}Y_{1,t}), (X'_{0,t-1}\Sigma_{0,t-1}^{-1}X_{0,t-1})^{-1} \right). \]

Instead of using the future return distribution based on point estimates of \( d \), investors form an investment rule, denoted by learning-CAY-VOL, using the predictive distribution for future returns, which can be obtained by integrating the posterior distribution of returns over uncertainty in the parameters captured by the posterior beliefs of \( d \). With learning, the predictive

\(^{35}\text{We also allow investors to learn only the coefficient of the conditional variance in the mean return equation; the results are similar. Brennan (1998) and Xia (2001) also study learning about an individual element of a parameter vector.}\)
distribution of returns change over time as new information is incorporated into investors’ beliefs. The posterior beliefs of \(d\) therefore become part of the state space describing the conditional distribution of returns. Because the sufficient statistics for the posterior beliefs of \(d\) are its mean and variance, sufficient state variables for the dynamic portfolio choice problem are the unique elements of the matrices \(X_{0, t-1}' \Sigma_{0, t-1}^{-1} X_{0, t-1} \) and \(X_{0, t-1}' \Sigma_{0, t-1}^{-1} Y_{1, t}\) along with \(cay\) and conditional variance.\(^{36}\)

Table 5 presents the out-of-sample portfolio performance of \(cay\)-CAY-VOL, \(\sigma^2\)-CAY-VOL, and learning-CAY-VOL. For the comparison purpose, it also includes results reported in Table 4 about the performance of the alternative models nested by the CAY-VOL model. Panel A shows annualized CERs for the case in which \(cay\) is constructed based on the full sample, and panel B shows annualized CERs for the case using recursively constructed \(cay\). In the panel A, notice that the third and fourth columns exhibit the utilities associated with portfolio rules relying solely on \(cay\) and conditional variance, respectively. Clearly, among the policies conditional on the same state variables, the annualized CERs for the CAY-VOL model are higher than those for the alternative models with simplified RGPs. Table 6 shows the difference in CERs of the strategies for the CAY-VOL model relative to those for the other models when \(cay\) is constructed based on the full sample.\(^{37}\) We can see that the CER of the rule \(cay\)-CAY-VOL is higher than the CERs of the dynamic rules for the other models, and the differences are statistically significant. Furthermore, all policies for the CAY-VOL model perform much better than the unconditional policy for the IID model. For example, investors would suffer a significant utility cost of 3.09% in the annualized CER by using the policy for the IID model rather than switching to \(cay\)-CAY-VOL. These results highlight that the CAY-VOL model explores superior information about the future return distribution and that ignoring time variation in the risk premium and in volatility leads to economically and statistically significant utility costs. These results also indirectly indicate out-of-sample return predictability from

\(^{36}\)When applying the simulation-based approach to computing portfolio weights, we update the posterior beliefs of \(d\) along each simulated path given the previously simulated data and then generate a realization of \(d\) from the posterior beliefs, along with the other parameters, to simulate next-period data.

\(^{37}\)Both asymptotic and empirical p-values for the one-sided test are reported except that only asymptotic p-values are presented for the performance comparison involving the policy learning – CAY – VOL because more than one month is needed to compute the empirical p-values even using Linux clusters for the performance comparison involving the policy learning – CAY – VOL.
the perspective of a multiperiod economic agent.

Panel B of Table 5 presents annualized CERs associated with the CAY and CAY-VOL models and the CAPM, in which \( cay \) is a predictive variable and constructed recursively. It is not surprising that portfolio performances of strategies conditional on \( cay \) degrade significantly when the cointegration parameters in \( cay \) are estimated recursively. Significant errors in recursively constructed \( cay \) during the early estimation recursions are introduced when only a part of the sample is used to estimate the cointegration parameters. Nevertheless, all conclusions stay the same as drawn for the case in which \( cay \) is constructed based on the full sample. For instance, as shown in Table 7, adopting the rule \( cay \)-CAY-VOL can raise the annualized CER by at least 1.7% and 3.5% compared with employing the IID model and the DY/DYH models, respectively.

Comparing the out-of-sample evaluation in panel A with that in panel B of Table 5 indicates that the quality of a state variable can be an important factor affecting out-of-sample performance of portfolio rules relying on that state variable. If an investor using the CAY-VOL model could and would construct \( cay \) using parameters estimated based on the full sample, annualized CER would increase by roughly 1.5% when the rule \( cay \)-CAY-VOL or the Bayesian rule learning-CAY-VOL is adopted.

Notice that the Bayesian rule learning-CAY-VOL performs better than the theoretical optimal rule without considering parameter uncertainty for the CAY-VOL model because the Bayesian approach reduces the adverse portfolio performance due to estimation risk. However, this Bayesian rule performs worse than the rule \( cay \)-CAY-VOL. These results are not surprising. DeMiguel, Garlappi, and Uppal (2007) show that although mean-variance policies combined with the Bayesian approach to account for parameter uncertainty perform better than policies without considering estimation risk, they cannot outperform the simplest equally-weighted policy \((1/N)\) when the size of historical data is small. Intuitively, investors need a large number of observations to learn well about uncertain parameters. With a small amount of historical data, the uncertainty about parameters, even incorporating learning, is still large, leading to relatively poor out-of-sample performance.

Kan and Zhou (2007) prove that in a static setting, some portfolios that reduce estimation risk perform better than theoretical optimal policies without considering parameter uncertain-
ty and Bayesian policies under a diffuse prior. By removing the dependence on conditional volatility, the policy cay-CAY-VOL is more immune to the adverse effect of estimation risk than the Bayesian learning policy with poor learning due to the lack of historical data. Although cay-CAY-VOL may not be the optimal policy in the presence of estimation risk, which is beyond the scope of this paper, it is not an ad-hoc policy because simulation results show that cay-CAY-VOL delivers quite a good out-of-sample performance in the presence of parameter uncertainty when the CAY-VOL model is the true RGP. The outperformance of cay-CAY-VOL shows that the CAY-VOL model can explore superior information about future returns compared with other models with simplified RGPs. Of course, the rules for the CAY-VOL model have to take good account of parameter uncertainty. Otherwise, estimation risk can hurt the performance of these rules.

We also examine the out-of-sample performance of the simple strategies that invest fixed fractions of 50%, 60%, or 100% of wealth in the risky asset at each rebalancing period. Unlike the findings by DeMiguel, Garlappi, and Uppal (2007), the annualized CERs of these rules are much inferior to the CERs of other rules incorporating time-varying return moments. The reason for the difference in findings is that there is only one risky asset with a small number of parameters to be estimated in this paper. The gain from optimal allocation by accounting for time-varying return moments exceeds the loss caused by estimation risk.

Table 8 shows that the policy conditional on cay is more profitable than the policy relying on the conditional variance in the CAY-VOL model. Similar to the in-sample analysis, the dynamic policy conditional on cay in the CAY model delivers much better performance than

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38 We use parametric bootstrap to simulate 1000 sample paths by assuming that the CAY-VOL model is the true data generating process with known parameters reported in the last column of Table 1. Artificial data are generated by drawing randomly with replacement from the residuals of the CAY-VOL model. Each sample path has 220 observations comparable with the 55-year historical sample. For each hypothetical path, we carry out the out-of-sample experiment described in Section 5.1 and compute along each sample path the utilities for investment policies examined in Table 5 except the Bayesian policy with learning because it is very time-consuming to evaluate the out-of-sample performance of this policy along 1000 sample paths. By averaging utilities across sample paths, cay-CAY-VOL performs better than the policy conditional on cay in the CAY model and the theoretical optimal policy for the CAY-VOL model. This simulation result further confirms that cay-CAY-VOL can improve portfolio performance in the presence of estimation risk.

39 We do not report the performances of these strategies because they are similar to the performance of the IID model.
the dynamic policy relying on time-varying volatility in the VOL model and the CAPM. These findings imply that time-varying risk premia are more informative than time-varying volatility in forming portfolio weights and in making investment profits for a real-world investor who can rebalance portfolios quarterly.

Table 9 presents utility gains of hedging demands for the CAY-VOL, DY, and DYH models. The utility gains of hedging demands are computed as the CER of a dynamic policy in excess of the CER of the corresponding myopic policy for a given model. Both the DY and DYH models show that myopic behavior leads to large utility costs, consistent with the findings by Balduzzi and Lynch (1999) and Campbell and Viceira (1999). In contrast, when \( \text{cay} \) is constructed based on the full sample, the dynamic rule for the CAY-VOL model without considering parameter uncertainty performs even worse than the corresponding myopic rule because estimation risk hurts portfolio performance even more for multiperiod investments. Even if investors adopt the Bayesian approach with learning to account for parameter uncertainty in the CAY-VOL model, the utility gain of hedging demands is still less than 0.3% in terms of the annualized CER, lower than the gain of around 1% for the DY and DYH models. Note that the portfolio rules based on the dividend yield perform poorly out of sample, while portfolio strategies for the CAY and CAY-VOL models lead to good out-of-sample performance. The utility loss of applying myopic policies for the CAY and CAY-VOL models is small.

5.4 Robustness Check

Transaction costs are an important realistic consideration at the time of making investment decisions, so we also examine the performance of various portfolio policies in the presence of transaction costs. Following Balduzzi and Lynch (1999), we set the proportional transaction costs equal to 50 bp.\(^{40}\) Let \( c \) be the proportional transaction costs, and let \( x_{t-} \) be the portfolio weight before rebalancing at the end of the period \( t \), equal to \( [x_{t-1}(r_t^e + R^f)]/(x_{t-1}r_t^e + R^f) \).

\(^{40}\)We do not account for fixed transaction costs because each policy studied in this paper is exposed to fixed costs due to quarterly rebalancing. Ignoring the fixed costs would not affect the relative performance of any two policies.
The wealth in excess of the transaction costs can be written as follows: \[ W_{t+1} = W_t (1 - c|x_t - x_t^-|) (x_t r_{t+1}^e + R^f). \] (32)

All the preceding out-of-sample evaluation stays similar in the presence of transaction costs.

Mean-variance portfolio weights are used popularly in both academic research (see Fleming, Kirby, and Ostdiek, 2001; DeMiguel, Garlappi, and Uppal, 2007) and in the investment industry. We find that the performance associated with this strategy is pretty similar to that associated with the myopic strategy. In addition, the conclusions drawn in this paper stay the same when we vary investors’ relative risk aversion to be 2 or 10 and investment horizons to be two years. For the sake of conciseness, these results are not reported in this paper.

6 Conclusion

This paper studies a multiperiod portfolio choice problem encountered by an investor who can invest in one risky asset and one risk-free asset. After choosing either the two-factor model or the alternative models with simplified RGPs to specify the risky return, the investor, in turn, selects portfolio strategies to maximize power utility of her terminal wealth. This paper shows that ignoring time variation in conditional return moments leads to significant utility costs out of sample. The time-varying risk premium is more informative than time-varying volatility in forming portfolio weights and in making profits for a real-world investor who can rebalance portfolios quarterly.

There is an ongoing debate about stock return predictability. Failing to find out-of-sample return forecastability, Bossaerts and Hillion (1999), Goyal and Welch (2003), and Ang and Bekaert (2007) cast doubt on the in-sample evidence documented by some early research. In contrast, Campbell and Thompson (2005) and Cochrane (2008) argue that poor out-of-sample forecast results per se cannot be taken as evidence against predictability because there may be too little information in small samples to accurately estimate parameters in a predictive model.

\[ ^{41}\text{DeMiguel, Garlappi, and Uppal (2007) examine the effect of transaction costs on myopic portfolio performance in this way.} \]

\[ ^{42}\text{In a discrete time setting, the myopic portfolio weight is different from the mean-variance policy for the investor with power utility.} \]
and to improve return forecast. This paper examines, from the perspective of a real-world investor, the economic value of out-of-sample return forecastability and shows that the utility costs of ignoring return predictability are significant.

Appendix A

\[ A_{t+1} = V_{t+1}(W_t R^f, Z_{t+1}) = E_{t+1}[u(\hat{W}_T)] \]

\[ B_{t+1} = \partial_t V_{t+1}(W_t R^f, Z_{t+1})r^e_{t+1} = E_{t+1}\left[ \partial u(\hat{W}_T) \prod_{s=t+1}^{T-1} (x_s r^e_{s+1} + R^f) \right] r^e_{t+1} \]

\[ C_{t+1} = \frac{1}{2} \partial^2_t V_{t+1}(W_t R^f, Z_{t+1})(r^e_{t+1})^2 = \frac{1}{2} E_{t+1}\left[ \partial^2 u(\hat{W}_T) \prod_{s=t+1}^{T-1} (x_s r^e_{s+1} + R^f)^2 \right] (r^e_{t+1})^2 \]

\[ D_{t+1} = \frac{1}{6} \partial^3_t V_{t+1}(W_t R^f, Z_{t+1})(r^e_{t+1})^3 = \frac{1}{6} E_{t+1}\left[ \partial^3 u(\hat{W}_T) \prod_{s=t+1}^{T-1} (x_s r^e_{s+1} + R^f)^3 \right] (r^e_{t+1})^3 \]

\[ E_{t+1} = \frac{1}{24} \partial^4_t V_{t+1}(W_t R^f, Z_{t+1})(r^e_{t+1})^4 = \frac{1}{24} E_{t+1}\left[ \partial^4 u(\hat{W}_T) \prod_{s=t+1}^{T-1} (x_s r^e_{s+1} + R^f)^4 \right] (r^e_{t+1})^4 \]

References


Campbell, John, and Samuel Thompson, 2005, Predicting the equity premium out of sample: Can anything beat the historical average?, NBER Working Paper 11468.

Campbell, John, and Luis Viceira, 1999, Consumption and portfolio decisions when expected returns are time varying, *Quarterly Journal of Economics* 114, 433–495.


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Parker, Cameron, Efstathios Paparoditis, and Dimitris N. Politis, 2006, Unit root testing via the stationary bootstrap, *Journal of Econometrics* 133, 601–638.


The IID, CAY, VOL, CAPM, CAY-VOL models have the following general form:

\[
\log(\hat{\sigma}_{t+1}^2) = b_1 + b_2 \log(\hat{\sigma}_t^2) + b_3 \text{cay}_t + e_{\sigma,t+1}
\]

\[
c\text{ay}_{t+1} = c_1 + c_2 \text{cay}_t + c_3 \log(\hat{\sigma}_t^2) + e_{\text{cay},t+1}
\]

\[
r_{t+1}^c = \mu_t + \sigma_t e_{t+1}
\]

\[
\mu_t = d_1 + d_2 \text{cay}_t + d_3 \sigma_t^2
\]

\[
\sigma_t^2 = \exp[b_1 + b_2 \log(\hat{\sigma}_t^2) + b_3 \text{cay}_t + 0.5\text{var}(e_{\sigma})].
\]

The CAY-VOL model nests all other models. The parameters of these models are estimated using GMM based on the quarterly data spanning the period from January 1952 to December 2006. Newey-West corrected t-statistics are reported in parentheses.

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<th>CAPM</th>
<th>CAY-VOL</th>
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Table 2: Estimation of Return Models Based on the Dividend Yield

The DY and DYH models have the following general form:

\[ dy_{t+1} = k_1 + k_2 dy_t + e_{dy,t+1} \]
\[ r_{t+1}^e = \mu_t + \sigma_t \epsilon_{t+1} \]
\[ \mu_t = k_3 + k_4 dy_t \]
\[ \sigma_t^2 = (k_5 + k_6 dy_t)^2 \]

The parameters of these two models are estimated using GMM based on the quarterly market excess return and the quarterly dividend yield spanning the period from January 1952 to December 2006. Newey-West corrected t-statistics are reported in parentheses.

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<tr>
<td></td>
<td>0.120</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>1.7%</td>
<td>1.7%</td>
</tr>
</tbody>
</table>
Table 3: In-Sample Portfolio Performance Comparison
This table reports in-sample annualized CERs associated with dynamic and myopic strategies for the IID, CAY, VOL, CAPM, and CAY-VOL models. The second column lists the state variables used to compute portfolio weights. All CERs are reported in a percentage unit. The utility measures are computed under the assumption that the CAY-VOL specification is the correct RGP with known parameters reported in the last column of Table 1. The investor has power utility with relative risk aversion of 5 and can rebalance her portfolios quarterly over a 5-year investment horizon.

<table>
<thead>
<tr>
<th>Model</th>
<th>State variables</th>
<th>Myopic</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>IID</td>
<td></td>
<td>5.90</td>
<td></td>
</tr>
<tr>
<td>CAY</td>
<td>$cay$</td>
<td>7.65</td>
<td>7.86</td>
</tr>
<tr>
<td>VOL</td>
<td>$\sigma^2$</td>
<td>5.81</td>
<td>5.75</td>
</tr>
<tr>
<td>CAPM</td>
<td>$\sigma^2$</td>
<td>6.33</td>
<td>6.33</td>
</tr>
<tr>
<td>CAY-VOL</td>
<td>$cay, \sigma^2$</td>
<td>7.95</td>
<td>8.06</td>
</tr>
</tbody>
</table>
Table 4: Out-of-Sample Portfolio Performance Comparison I
This table presents annualized unconditional CERs associated with dynamic and myopic strategies for the IID, CAY, VOL, CAPM, CAY-VOL, DY, and DYH models. The second column lists the state variables used to compute portfolio weights. All CERs are reported in a percentage unit. Panel A reports the case in which \( cay \) is constructed based on the full sample, whereas panel B presents the case in which \( cay \) is constructed recursively based on the data available only at the time of forecast for the CAY, CAPM, and CAY-VOL models in which \( cay \) is a state variable. In every quarter starting from the first quarter of 1982 to the first quarter of 2002, there is one identical investor who commences a 5-year investment. Investors have power utility with relative risk aversion of 5 and can rebalance their portfolios quarterly. Over investment horizons, portfolio weights are obtained by combining the current values of state variables with updated model parameters that are estimated based on the historical data from the first quarter of 1952 to the fourth quarter of the past year.

<table>
<thead>
<tr>
<th>Model</th>
<th>State variables</th>
<th>Myopic (%)</th>
<th>Dynamic (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Panel A: ( cay ) based on the full sample (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IID</td>
<td>6.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAY</td>
<td>( cay )</td>
<td>9.05</td>
<td>8.96</td>
</tr>
<tr>
<td>VOL</td>
<td>( \sigma^2 )</td>
<td>7.03</td>
<td>6.99</td>
</tr>
<tr>
<td>CAPM</td>
<td>( \sigma^2 )</td>
<td>7.04</td>
<td>7.09</td>
</tr>
<tr>
<td>CAY-VOL</td>
<td>( cay, \sigma^2 )</td>
<td>8.10</td>
<td>7.72</td>
</tr>
<tr>
<td>DY</td>
<td>( dy )</td>
<td>3.33</td>
<td>4.30</td>
</tr>
<tr>
<td>DYH</td>
<td>( dy )</td>
<td>3.34</td>
<td>4.18</td>
</tr>
<tr>
<td></td>
<td>Panel B: Recursively constructed ( cay ) (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAY</td>
<td>( cay )</td>
<td>7.27</td>
<td>7.68</td>
</tr>
<tr>
<td>CAPM</td>
<td>( \sigma^2 )</td>
<td>6.89</td>
<td>7.05</td>
</tr>
<tr>
<td>CAY-VOL</td>
<td>( cay, \sigma^2 )</td>
<td>6.61</td>
<td>6.78</td>
</tr>
</tbody>
</table>
Table 5: Out-of-Sample Portfolio Performance Comparison II

This table shows annualized unconditional CERs associated with different strategies for the IID, CAY, VOL, CAPM, and CAY-VOL models. All CERs are reported in a percentage unit. Panel A reports the case in which $cay$ is constructed based on the full sample, whereas panel B presents the case in which $cay$ is constructed recursively based on the data available only at the time of forecast for the CAY, CAPM, and CAY-VOL models in which $cay$ is a state variable. In every quarter starting from the first quarter of 1982 to the first quarter of 2002, there is one identical investor who commences a 5-year investment. Investors have power utility with relative risk aversion of 5 and can rebalance their portfolios quarterly. The learning rule for the CAY-VOL model accounts for estimation risk by learning about the parameters in the expected return based on the Bayesian approach. For the other dynamic rules, $cay$ and/or $\sigma^2$ are used as state variables to compute portfolio weights. Over investment horizons, portfolio weights are obtained by combining the current values of state variables with updated model parameters that are estimated based on the historical data from the first quarter of 1952 to the fourth quarter of the past year.

<table>
<thead>
<tr>
<th>Model</th>
<th>Myopic</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$cay$</td>
<td>$\sigma^2$</td>
</tr>
<tr>
<td>Panel A: $cay$ based on the full sample (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IID</td>
<td>6.14</td>
<td></td>
</tr>
<tr>
<td>CAY</td>
<td>9.05</td>
<td>8.96</td>
</tr>
<tr>
<td>VOL</td>
<td>7.03</td>
<td>6.99</td>
</tr>
<tr>
<td>CAPM</td>
<td>7.04</td>
<td>7.09</td>
</tr>
<tr>
<td>CAY-VOL</td>
<td>8.10</td>
<td>9.23</td>
</tr>
<tr>
<td>Panel B: Recursively constructed $cay$ (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAY</td>
<td>7.27</td>
<td>7.68</td>
</tr>
<tr>
<td>CAPM</td>
<td>6.89</td>
<td>7.05</td>
</tr>
<tr>
<td>CAY-VOL</td>
<td>6.61</td>
<td>7.84</td>
</tr>
</tbody>
</table>
This table presents the differences in annualized CERs of pairs of policies that equal the CERs of the policies in the first row minus the CERs of the policies in the first column for the case that $cay$ is constructed based on the full sample. The policy learning-CAY-VOL is the Bayesian policy with learning about the parameters of the expected return in the CAY-VOL model. The other dynamic policies are named by the model’s name proceeded by the names of state variables used to form those policies. Both asymptotic and empirical p-values for the one-sided test are reported except that only asymptotic p-values are presented for the performance comparison involving the policy learning-CAY-VOL. The empirical p-values are obtained based on 1000 sample paths generated by using block bootstrap. All the results are reported in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>$cay$-$\sigma^2$-CAY-VOL</th>
<th>$cay$-CAY-VOL</th>
<th>$\sigma^2$-CAY-VOL</th>
<th>learning-CAY-VOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$cay$-CAY</td>
<td>Δ CER</td>
<td>-1.24</td>
<td>0.27</td>
<td>-1.39</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(14.41)</td>
<td>(12.81)</td>
<td>(11.94)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[1.23]</td>
<td>[7.07]</td>
<td>[9.12]</td>
</tr>
<tr>
<td>$\sigma^2$-VOL</td>
<td>Δ CER</td>
<td>0.73</td>
<td>2.24</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(22.82)</td>
<td>(6.49)</td>
<td>(11.35)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[47.75]</td>
<td>[0.61]</td>
<td>[6.05]</td>
</tr>
<tr>
<td>$\sigma^2$-CAPM</td>
<td>Δ CER</td>
<td>0.63</td>
<td>2.14</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(19.88)</td>
<td>(8.02)</td>
<td>(4.90)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[40.27]</td>
<td>[2.87]</td>
<td>[0.51]</td>
</tr>
<tr>
<td>IID</td>
<td>Δ CER</td>
<td>1.58</td>
<td>3.09</td>
<td>1.43</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(4.84)</td>
<td>(4.19)</td>
<td>(2.25)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
</tr>
<tr>
<td>$dy$-DY</td>
<td>Δ CER</td>
<td>3.42</td>
<td>4.93</td>
<td>3.27</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(9.56)</td>
<td>(6.38)</td>
<td>(8.70)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
</tr>
<tr>
<td>$dy$-DYH</td>
<td>Δ CER</td>
<td>3.54</td>
<td>5.05</td>
<td>3.39</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(9.53)</td>
<td>(6.53)</td>
<td>(8.68)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
</tr>
</tbody>
</table>
Table 7: Utility Comparison of Exploiting Time-Varying Return Moments II

This table presents the differences in annualized CERs of pairs of policies that equal the CERs of the policies in the first row minus the CERs of the policies in the first column for the case that $cay$ is constructed recursively. The policy learning-CAY-VOL is the Bayesian policy with learning about the parameters of the expected return in the CAY-VOL model. The other dynamic policies are named by the model’s name proceeded by the names of state variables used to form those policies. Both asymptotic and empirical p-values for the one-sided test are reported except that only asymptotic p-values are presented for the performance comparison involving the policy learning-CAY-VOL. The empirical p-values are obtained based on 1000 sample paths generated by using block bootstrap. All the results are reported in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>$cay$-$\sigma^2$-CAY-VOL</th>
<th>$cay$-CAY-VOL</th>
<th>$\sigma^2$-CAY-VOL</th>
<th>learning-CAY-VOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$cay$-CAY</td>
<td>$\Delta$ CER</td>
<td>-0.90</td>
<td>0.16</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(19.22)</td>
<td>(18.52)</td>
<td>(49.89)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[21.79]</td>
<td>[21.49]</td>
<td>[29.43]</td>
</tr>
<tr>
<td>$\sigma^2$-VOL</td>
<td>$\Delta$ CER</td>
<td>-0.21</td>
<td>0.85</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(35.42)</td>
<td>(18.58)</td>
<td>(10.08)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[53.05]</td>
<td>[37.88]</td>
<td>[2.95]</td>
</tr>
<tr>
<td>$\sigma^2$-CAPM</td>
<td>$\Delta$ CER</td>
<td>-0.27</td>
<td>0.79</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(30.84)</td>
<td>(20.25)</td>
<td>(12.57)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[55.60]</td>
<td>[35.03]</td>
<td>[3.77]</td>
</tr>
<tr>
<td>IID</td>
<td>$\Delta$ CER</td>
<td>0.64</td>
<td>1.70</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(17.33)</td>
<td>(10.64)</td>
<td>(2.32)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[9.37]</td>
<td>[4.58]</td>
<td>[0.00]</td>
</tr>
<tr>
<td>$dy$-DY</td>
<td>$\Delta$ CER</td>
<td>2.48</td>
<td>3.54</td>
<td>3.39</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(9.29)</td>
<td>(5.76)</td>
<td>(8.41)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[1.22]</td>
<td>[0.00]</td>
<td>[0.00]</td>
</tr>
<tr>
<td>$dy$-DYH</td>
<td>$\Delta$ CER</td>
<td>2.60</td>
<td>3.66</td>
<td>3.51</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(9.12)</td>
<td>(5.87)</td>
<td>(8.41)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[0.00]</td>
<td>[0.00]</td>
<td>[0.00]</td>
</tr>
</tbody>
</table>
Table 8: Relative Importance of the First and Second Conditional Moments

This table presents the differences in annualized CERs of pairs of policies that equal the CERs of the policies in the first row minus the CERs of the policies in the second row. The dynamic policies are named by the model’s name proceeded by the names of state variables used to form these policies. Panel A reports the case in which \(cay\) is constructed based on the full sample and panel B for the case of recursively constructed \(cay\). Both asymptotic and empirical p-values for the one-sided test are reported. The empirical p-values are obtained based on 1000 sample paths generated by using block bootstrap. All results are reported in percentage points.

<table>
<thead>
<tr>
<th>Panel A: (cay) is constructed based on the full sample</th>
<th>(cay)-CAY-VOL</th>
<th>(cay)-CAY</th>
<th>(cay)-CAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta) CER</td>
<td>1.66</td>
<td>1.97</td>
<td>1.87</td>
</tr>
<tr>
<td>asy. p-value</td>
<td>(11.58)</td>
<td>(6.12)</td>
<td>(7.98)</td>
</tr>
<tr>
<td>emp. p-value</td>
<td>[7.58]</td>
<td>[0.61]</td>
<td>[2.77]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: (cay) is constructed recursively</th>
<th>(\sigma^2)-CAY-VOL</th>
<th>(\sigma^2)-VOL</th>
<th>(\sigma^2)-CAPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta) CER</td>
<td>0.15</td>
<td>0.69</td>
<td>0.63</td>
</tr>
<tr>
<td>asy. p-value</td>
<td>(44.91)</td>
<td>(25.06)</td>
<td>(26.83)</td>
</tr>
<tr>
<td>emp. p-value</td>
<td>[69.55]</td>
<td>[48.37]</td>
<td>[44.81]</td>
</tr>
</tbody>
</table>
Table 9: Importance of Hedging Demands

This table presents annualized CERs of the dynamic and myopic policies and the differences in the CERs of the dynamic policies over those of the myopic policies for the CAY-VOL, DY, and DYH models. Both asymptotic and empirical p-values for the one-sided test for the null that $\Delta CER = 0$ are reported except that only asymptotic p-values are presented for the performance comparison involving the policies with learning for the CAY-VOL model. The empirical p-values are obtained based on 1000 sample paths generated by using block bootstrap. All results are reported in percentage points.

<table>
<thead>
<tr>
<th>Models</th>
<th>Full sample constructed cay</th>
<th>Recursively constructed cay</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAY-VOL</td>
<td>myopic CER</td>
<td>8.10</td>
</tr>
<tr>
<td></td>
<td>dynamic CER</td>
<td>7.72</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ CER</td>
<td>-0.38</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(16.41)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[27.46]</td>
</tr>
<tr>
<td>CAY-VOL with learning</td>
<td>myopic CER</td>
<td>8.18</td>
</tr>
<tr>
<td></td>
<td>dynamic CER</td>
<td>8.27</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ CER</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(12.55)</td>
</tr>
<tr>
<td>DY</td>
<td>myopic CER</td>
<td>3.33</td>
</tr>
<tr>
<td></td>
<td>dynamic CER</td>
<td>4.30</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ CER</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(4.06)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[0.51]</td>
</tr>
<tr>
<td>DYH</td>
<td>myopic CER</td>
<td>3.34</td>
</tr>
<tr>
<td></td>
<td>dynamic CER</td>
<td>4.18</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ CER</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>asy. p-value</td>
<td>(4.33)</td>
</tr>
<tr>
<td></td>
<td>emp. p-value</td>
<td>[0.92]</td>
</tr>
</tbody>
</table>
Figure 1: This figure exhibits total, myopic, and hedging portfolio weights, represented by the dash-dot, dashed, and solid lines respectively, with respect to initial values of single state variables at the beginning of a 5-year investment horizon.
Figure 2: This figure describes portfolio weights conditional on \( cay \) and time-varying stock variance for the CAY-VOL model at the beginning of a 5-year investment horizon. Dash-dot, dashed, and solid lines represent optimal, myopic, and hedging portfolio weights, respectively. Left to right, the first (second) row shows how these weights are related to \( cay \) (conditional variance) when the conditional variance (\( cay \)) is one standard deviation below, at, or one standard deviation above the sample mean. For a clear view, the horizontal axes cover 90% in-sample realizations of \( cay \) or conditional stock variance.
Figure 3: This figure plots the horizon effect of hedging demands induced by $cay$ and by conditional volatility in the first and second rows, respectively, with short-sale and borrowing constraints imposed. This figure presents the horizon effect for nine different initial states of the economy. Left to right, this figure exhibits the horizon effect of the hedging demands when the variance level at the beginning of a 5-year investment horizon is one standard deviation below, at, and one standard deviation above the sample mean. The solid, dashed, and dash-dot lines depict the horizon effect of the hedging demands when the initial $cay$ is one standard deviation below, at, and one standard deviation above the sample mean, respectively.
Figure 4: This figure plots the horizon effect of hedging demands induced by $cay$ and by conditional volatility in the first and second rows, respectively, without short-sale and borrowing constraints imposed. This figure presents the horizon effect for nine different initial states of the economy. Left to right, this figure exhibits the horizon effect of the hedging demands when the variance level at the beginning of a 5-year investment horizon is one standard deviation below, at, and one standard deviation above the sample mean. The solid, dashed, and dash-dot lines depict the horizon effect of the hedging demands when the initial $cay$ is one standard deviation below, at, and one standard deviation above the sample mean, respectively.