Fiscal Space under Demographic Shift*

(Preli­minary and In­com­plete)

Christine Ma† Chung Tran‡

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Abstract

In this paper we analyze the implications of demographic shift for an economy’s fiscal sustainability through lens of fiscal space. We formulate a stochastic dynamic general equilibrium, overlapping generations model to measure fiscal space in terms of budgetary room between the current and maximum tax revenues (the peaks of Laffer curves). We calibrate the model to Japan and US data and conduct a quantitative analysis. Our findings are summarized as follows. First, the size of fiscal space varies greatly over time and across countries, depending on the underlying demographic structures. Second, the fiscal space and limits will shrink sharply by the mid of 21st century as their population ageing accelerates. In particular, our model calibrated to Japan indicates that the increase in dependency ratio from around 40 percent in 2010 to over 70 percent in 2040 will leads to a contraction in the capital-labor fiscal space by around 36 percent. The fiscal limit will be reached when factoring in the increased fiscal cost of age-related commitments.

JEL Classification: E62, H20, H60, J11

Keywords: Demography, Laffer Curve, Fiscal Limit, Sustainability, Overlapping Generations, Dynamic General Equilibrium.

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*We appreciate comments from seminar participants at Keio University, Bank of Japan and Australian National University.
†Deloitte Access Economics
‡Research School of Economics, Australian National University, ACT 2600, Australia. Telephone: 61 6125 5638, E-mail: chung.tran@anu.edu.au.
1 Introduction

The issue of fiscal sustainability has taken on special importance in the aftermath of the global financial crisis in advanced economies. There have been sharp increases in debt to GDP ratio because of the worldwide financial crisis and the prolonged and deep recession that generated automatic budget deficits and induced many countries to implement sizeable fiscal stimulus packages. Unfortunately, the fiscal situations in advanced economies are heading to even more trouble. The medium- and long-term projections of debt to GDP ratio indicates the fiscal situation will be even worse with rising public debt is a prominent worry in all advanced economies. It has been argued that the ageing demographic trends will deepen fiscal stress in years to come. Providing funding to meet the need of age-related government spending will be even more of a pressing challenge. The advanced economies that have promised substantially more benefits of ageing-related public programs than they have made provisions to finance are soon heading into an era of long-run fiscal stress.

Every economy faces a fiscal limit, the point at which a government can no longer raise taxes to cover its spending commitments as well as to stabilize government debt. At the fiscal limit a government has no choice but to default on its outstanding fiscal obligations. In a fundamentals-based economy model, fiscal limit arises from the peak of the Laffer curve, that depends on the economic fundamentals: endowments, preferences, technology and the state of fiscal policy. In an ageing economy, the demographic structure as a part of economic fundamentals shifts over time and so does its fiscal limit. The shift in demographic structure has impacts on both the revenue and expenditure sides of government budget. As the aged population works fewer hours the tax base and revenue shrink for a given tax rate. On other hand, a larger old-age group increases pressure on government expenditure as more retirees demand more old-age benefits. Understanding how demographic shift underpins an economy’s fiscal limit requires explicit modeling of underlying demographic factors together with other economic factors.

What is the fiscal limit to that the government can raise tax revenues to finance its old-age benefits programs and to service debt obligations under demographic shift? What is a budgetary room that government can maneuver fiscal policy? This paper addresses these questions through lens of fiscal space, defined as budgetary room between the current revenue level and the maximum tax revenue level (the peak of the Laffer curve).

To that end, we formulate an incomplete market, overlapping generations model with uninsurable idiosyncratic risk. The model consists of heterogeneous households, a perfect competitive representative firm and government. We first calibrate our benchmark model to the Japanese economy. Our model is capable to match key patterns of life-cycle behavior and essential features of the Japanese macroeconomy. We conduct a number of experiments
to quantify fiscal space and the effects of demographic shift on fiscal space for Japan. Next, we extend our framework to other advanced economies. Our results are summarized as follows.

Firstly, we use the model to construct Laffer curves for labour, capital and consumption, and the Laffer hill and quantify gross and net fiscal space for Japan in 2010. We find that the labour and capital Laffer curves have a single peak, while the consumption Laffer curve has no peak and monotonically increases as the tax rate increases. For both labour and capital, the benchmark Japanese economy lies to the left of the revenue maximizing peak, and the government can increase any of the tax rates to increase revenue. By altering one tax rate while keeping the other two constant, the benchmark economy can increase current taxation revenue by 43 percent and 17 percent, for labour and capital, respectively. While the Laffer curve alters one tax rate while keeping the other two constant, the capital and labour income taxes are varied jointly in the Laffer hill.

Next, we quantify the effects of demographic shift on the size and shape of fiscal space, using demographic structure in 2040. Note that, we use two metrics to measure fiscal space: the gross one measuring the government’s capacity to generate tax revenue; the net one measuring the government’s capacity to raise tax revenue after accounting for increases in pension benefits. We find that such shift in demographic structure shrink the gross fiscal space by shifting the Laffer curves for labour, capital, and consumption downward. More specifically, the gross fiscal space for capital taxation revenue is the most sensitive to ageing. The gross fiscal space for capital taxation will shrink by 37.0 percent compared to the benchmark fiscal space. The fiscal space for labor taxation is the next sensitive one, with a decrease by 33.84 percent over the same period. The fiscal space for consumption taxation shrinks by only 10.67 percent. While the fiscal space is diminished, the fiscal space sill exists for all three taxes. The government has the flexibility to maintain current levels of taxation revenue by increasing any one of the tax rates. Altering a single tax rate (either labour or capital), the benchmark economy can increase current tax revenues by an additional 51.3 percent when the labour income tax rate is set to 0.68. By raising labour and capital tax rates simultaneously, the government can raise revenue by an additional 87.86 percent compared to the current revenue.

We construct the net fiscal space when we account for the increased government outlay due to public pension commitments. We find that the net fiscal spaces will shrink more than its gross counterparts. That is, when altering a single tax, the net fiscal spaces in 2040 will be lower by 59.0 percent, 110.9 percent and 12.98 percent for labour, capital and consumption, relatively. The net fiscal space for capital will disappear completely, and a maximum of 95.93 percent of current revenue can be maintained at a capital income tax rate of 70 percent. To maintain current revenues, the government can either raise the
labour or consumption tax, or vary the labour and capital taxes jointly. In the second case, a maximum additional 36 percent of current uncommitted revenue can still be generated.

We quantify the relative importance of the fertility and mortality rates in contributing to the decrease in fiscal space. We find that over the 1980 to 2010 period, the increase in survival probabilities and decrease in fertility rate have contributed equally in ageing the population and decreasing fiscal space. However, the contribution will be asymmetrical over the 2010 to 2040 period, with fertility as the main driver for further decreases in the fiscal space. We also note that a purely fertility rate driven change in the dependency ratio will shift the fiscal space to a greater extent than a purely mortality rate driven change in the dependency ratio of the same size.

Lastly, we calibrate our model to other advanced economies including US, France. We find that fiscal space is greatly dependent of the underlying demographic structure. Moreover, we argue that such contraction of fiscal space is country-specific since the demographic structure varies greatly across advanced countries.

**Related literature.** We place our paper in the context of existing literature. Although there is little doubt that every economy has a fiscal limit, the literature does not have a clear understanding of how to quantify that limit. There is an growing literature attempting to quantify fiscal limit and space.

? use the notion of the debt limits above which the debt becomes unsustainable to define fiscal limit. They estimate the debt limits using a reduced-form model for policy reaction function. They construct fiscal space in terms of a distance between the current debt levels and the debt limits. Similarly, ? estimate a country’s debt limit using a stochastic ability-to-pay model of sovereign default. They use data from 23 advanced economies to estimate the responses of primary surpluses to debt levels and compute a debt limit for each country that is fully determined by the risk-free interest rate, the recovery rate, and the support of the shock to primary balances. They define fiscal space in terms of the difference between the long-run average debt ratio and the debt limit. Notice that, the empirically-based approach to calculating fiscal limit and space is grounded in the historical data. Their calculations rely on the underlying assumption that the government always follows its historically estimated rule as well as there is no structural change in the economic environment. Any changes in policy rules as well as economic fundamentals would alter the country’s fiscal limit, destabilizing the backward measure of fiscal space.

? argue that a country’s fiscal limit varies systematically with the economic environment, including the specification of policy behavior. They formulate a real business cycle model and maps the economic environment into a distribution for the maximum sustainable debt-GDP ratio. They calibrate their model to Greek and Swedish data to illustrate how the framework can be used to study actual fiscal reforms undertaken by developed
economies facing sovereign risk pressures. ? extends this approach into a perpetual youth model and examines how intergenerational redistributions of wealth, the average duration of government debt, and entitlement reform impact the consequences of explosive government transfers. These studies focus on interactions between monetary and fiscal policies. They point out that future projected fiscal deficits and fiscal stress are driven by the growth in entitlement spending. However, demographic factors that switch promised government transfers from a stable to an explosive are not included in the economic environment. Their model is not able to map out an explicit link between demographic structures and fiscal limit, which is the main focus of our paper.

Our paper contributes to a growing body of literature using the neoclassical growth models to quantify Laffer curves. ? characterize Laffer curves for the US and the EU 14 and the classic Laffer curve shape exists for labour and capital income tax rates in a infinitely lived representative agent model. Their estimates indicate that the peak of the labor income tax Laffer curve in both regions is located between 50% and 70% tax depending on parameter values. Since Trabandt and Uhlig (2011) there has been renewed interest in the Laffer curves. ? examines the effects of ageing on the revenue generating capacities of the G-7 nations, using the Trabandt and Uhlig framework. He considers a notion of fiscal space defined as a distance between the current tax revenue level and the peak of the Laffer curve, which is equivalent to our concept of gross fiscal space. However, the micro-foundation of demographic shift is assumed away because of the infinitely lived representative agent model approach. In fact, population ageing is artificially approximated by an exogenous shift in the disutility from supplying labour over time. ? develop a two-country model with cross-country tax externality to quantify Laffer curves for eurozone countries. They focus on examining the positive and normative effects of alternative tax strategies that countries could follow to restore fiscal solvency in response to debt shocks. An important limitation of these studies is that they abstract from modelling demographic structures. Differently, we construct a full life-cycle model that accounts explicitly for demographic factors behind changes in demographic structures. This modelling approach allows us to transparently model the ageing process and its impact on household life-cycle behaviour and the underlying tax bases; and disaggregate the effects of changes in the survival probabilities and fertility rate on the fiscal space.

? quantify Laffer curves in an infinitely lived heterogeneous agent model and show that the Laffer curve depends on the level of outstanding government debt. ? extends Trabandt and Uhlig (2011)'s analysis to a quantitative life cycle model with realistically calibrated wage heterogeneity and risk, extensive margin labor supply choice as well as endogenous human capital accumulation. They find that household heterogeneity and the degree of tax progressivity matter for the level and location of the peak of the Laffer curve. Notice
that, Holter et al (2014) do not consider the impact of changing demographic structure on the shape of Laffer curves. We share a similar modelling approach but focus on mapping out the link between ageing and the size and shape of Laffer curves.

Since ?, there is a vast literature that uses overlapping generations (OLG) models featured with inter- and intra-generational heterogeneity and population dynamics study the dynamic effects of policy policy. Recently, that literature is extended to quantifying the consequences of population ageing and fiscal adjustment in advanced economies (e.g., see ? and ?). Notice that, most of these studies focus on quantifying the expenditure-side effects of population ageing as well as the effects of counterfactual fiscal adjustments. In this paper, we bridge that branch of the literature to the literature on Laffer curves and fiscal limit. We basically base on the Laffer curve idea to define a fiscal limit point and measure fiscal space. This extension allows us to analyze the fiscal challenges caused by demographic shift through lens of fiscal space and to quantify the relationship between underlying demographic factors and fiscal limit. We are able to examine explicitly how demographic shift leads an ageing economy into an era of fiscal stress.

The remaining paper is organized as follows. Section 2 provides an overview of the model. Section 3 outlines the calibration process to match for the benchmark Japanese economy in 2010. Section 4.1 outlines the definitions for fiscal space and results in the benchmark economy. Section 4 quantifies the effects of ageing on household life-cycle behavior, the macroeconomic aggregates and fiscal space. Section 5 offers a sensitivity analysis, and isolate the effects of mortality and fertility changes on the fiscal space. Lastly, section 7 offers a conclusion.

2 Model

We formulate a stochastic dynamic general equilibrium overlapping generations model, which consists of heterogeneous households, a perfect competitive representative firm, and a government with full commitment technology.

2.1 Demographics

In each discrete time period $t$, the economy is populated by $J$ overlapping generations of households of generations $j = 1, ..., J$. Each period, a new cohort of households of the generation is born. Each period, households of a particular generation share a common chance of dying before reaching the next period. The probability of surviving to generation $j + 1$ conditional on belonging to generation $j$ is denoted by $sp_j$. We denote the size of the generation $j$ cohort at the beginning time $t$ as $P_j$. The cohort share of the generation
$j$ households at time $t$ is given by $\mu_j = \frac{P_j}{\sum_{j=1}^J P_j}$. Demographic structure is driven by two factors: (i) the age-dependent survival probability ($sp_j$), and (ii) the growth rate of the new generation ($g^n$). When the demographic pattern is stationary, as assumed here, the population share of the cohort age $j$ is constant at any point in time and can be recursively defined as $\mu_j = \mu_{j-1} sp_j / (1 + g^n)$. The share of agents who do not survive to age $j$ is $\tilde{\mu}_j = \mu_{j-1} (1 - sp_j) / (1 + g^n)$.

2.2 Preferences

All households have identical lifetime preferences over consumption $c_j \geq 0$ and leisure $l_j$, where household leisure time per period for household $j$ is constrained by $0 \leq l_j \leq 1$. Preferences are time-separable with a constant subjective discount factor $\beta$ and are given by the expected utility function

$$E \left[ \sum_{j=1}^J \beta^j u(c_j, l_j) \right].$$

(1)

2.3 Endowments

In each period of life households are endowed with 1 unit of labor time that has labor efficiency (or working ability) denoted by $e_j$. The efficiency unit $e_j$ is skill and age dependent and follows a Markov switching process with $\pi_{j} (e_{j+1} | e_j)$ denoting the conditional probability that a person of working ability $e_j$ at age $j$ will have working ability $e_{j+1}$ when at age $j + 1$. According to this specification, agents have working abilities that vary by age and change stochastically over the life cycle; they therefore face idiosyncratic earnings risk, which is assumed to be non-insurable.

Households devote $l_j$ units of time to leisure and supply $n_j = (1 - l_j)$ units of time to the labor market. The quantity of effective labour supplied is given by $h_j = n_j e_j = (1 - l_j) e_j$, and labor earnings are $w_t h_j$. For the retired periods $j = J^w + 1, ..., J$, households are retired and consume $l_j = 1$ units of leisure time, and do not have labor earnings.

We assume households enter the economy with no assets, and $a_1 = 0$. We let $a_j$ denote asset holdings of a typical agent at age $j$. We let $x_j = \{a_j, e_j\}$ denote the state variable of a typical household at age $j$ and $\mu(x_j)$ denote the measure of households in state $x_j$.

2.4 Technology

The production sector is made up of a large number of competitive firms, and can be proxied by a single producer that maximizes profits. It produces a single output ($Y_t$) each
period from two inputs, capital \((K_t)\) and effective labour \((H_t)\), based on a constant returns to scale production function \(Y_t = A_t F(K_t, H_t)\) where \(A_t\) is the total factor productivity and grows at a constant rate \(g\). The firm is a price taker in the input markets for capital and labour, and aims to maximize its profit given the rental rate \((q_t)\) and market wage rate \((w_t)\) by choosing \(K_t\) and \(H_t\) such that

\[
\max_{K_t, H_t} \{ A_t F(K_t, H_t) - q_t K_t - w_t H_t \} \tag{2}
\]

### 2.5 Fiscal policy

The government runs a social security system and two other spending programs.

**Social security system.** The government provides the retirement benefits \(p_j\) to all retiring households at age \(j = J^w+1, \ldots, J\). The social security benefits are given by a replacement rate \(Ψ\) and an average life-time labor earning \(wH_{J^w}\) at time \(t\), so that \(p_j = Ψ wH_{J^w}\). The total social security payment for all retirees at time \(t\) is \(SS_t = \sum_{j=J^w+1}^{J} p_j \mu_j(x_j)\).

The social security system is partially funded by social security tax revenue and partially funded by the general government budget. Let \(θ^{ss}\) denote a fraction of the total social security payment funded by the general government budget. Social security tax \(τ^{ss}\) adjusts to clear the rest of the social security payment

\[
τ^{ss} \sum_{j=J^w}^{J} \sum_{x_j} w_t h_j \mu_j(x_j) = (1 - θ^{ss}) SS_t. \tag{3}
\]

We restrict \(θ^{ss} \in [0, 1]\). Note that, when \(θ^{ss} = 0\), the social security system is fully self-financed.

**Government budget.** The government also has two other spending programs: general government purchases \(G_t\) and transfers to the households \(Tr_t\). The government collects tax revenues through taxes on consumption \(τ^c\), labour income \(τ^l\) and capital income \(τ^k\) to finance its expenditures. The total tax revenue is given by

\[
Tax_t = τ^c \sum_{j=1}^{J} \sum_{x_j} c_j \mu_j(x_j) + τ^l \sum_{j=1}^{J^w} \sum_{x_j} w_t h_j \mu_j(x_j) + τ^k r_t \sum_{j=1}^{J} \sum_{x_j} a_j \mu_j(x_j),
\]

where \(w_t\) is the market wage rate and \(r_t\) is the market interest rate.

The government maintains budget balance each period, using a combination of the taxation revenue and issuance of new debt \(D_{t+1}\) to fund interest and principle payments on existing debt \((1+r^d_t)D_t\) with \(r^d_t\) is the interest rate for government debt. The government
inter-temporal budget is given by
\[(1 + g^n)(1 + g)D_{t+1} + Tax_t = (1 + r_t^d)D_t + G_t + \theta^{ss}SS_t + Tr_t. \quad (4)\]

### 2.6 Market structure

Markets are incomplete and households cannot insure against the idiosyncratic labor income and mortality risks by trading state contingent assets. They can, however, hold one-period riskless assets to imperfectly self-insure against idiosyncratic risks. We assume that agents are not allowed to borrow against future income, implying asset holdings are non-negative, i.e., \(a_j \geq 0\) for all \(j\).

The economy is closed, and the domestic interest rate is determined endogenously. It is related to the rental price of capital by \(r_t = q_t - \delta\), where \(q_t\) is determined by the demand and supply for capital in the economy, and \(\delta\) is the depreciation rate of capital.

### 2.7 Household problem

In this model, households are heterogeneous with respect to their age, working ability and asset holdings. Over their lifetime, households have different sources of income. First, as a worker with age between 1 and \(J^w\), households supply labor to the firm in return for wage income, \(w_t h_j\). Labor income is subjected to a social security tax (\(\tau^{ss}\)) and labor income tax (\(\tau^l\)). Households exit the labor market and retire at age \(j > J^w\). Households’ savings are rented out to the firms in the form of capital the following period. It earns interest at the rate \(r_t\) and is taxed at the rate \(\tau^k\). Henceforth, the post-tax return is \(R_t^k = 1 + (1 - \tau^k)r_t\). During the retirement time, households receive a public pension benefit (\(p_j\)) from the government. Households also receive an equal share of lump-sum transfers from the government (\(tr_j\)). Finally, as there are no annuity markets, the savings (including interest return) of households who die each period are shared out equally amongst the remaining households as accidental bequests (\(b_j\)).

At the beginning of age \(j\) the household realizes its individual state \(x_j\) and chooses its optimal consumption, \(c_j\), leisure time, \(l_j\), or working hours, \((1 - l_j)\), and the end-of-period asset holdings, \(a_{j+1}\), taking the transition law for working ability, \(\pi_j\) (\(e_{j+1}|e_j\)), conditional survival probabilities, \(sp_j\), the wage and interest rates, and government tax and pension policies as given. Formally, the Bellman equation for a household of age \(j\) is given by
\[V_j(x_j) = \max_{c_j, l_j, a_{j+1}} \{u(c_j, l_j) + sp_j\beta E[V_{j+1}(x_{j+1})]\} \quad (5)\]
subject to the budget constraint

\[
(1 + g) a_{j+1} + (1 + \tau) c_j = \begin{cases} 
R^k_a j + (1 - \tau^l - \tau^s) w l h_j + tr_j + b_j & \text{if } j = 1, \ldots, J^w \\
R^k_a j + p_j + tr_j + b_j & \text{if } j > J^w,
\end{cases}
\]

where \( V_j(x_j) \), the value function of a household at age \( j \) conditional on the given state variable \( x_j \) and \( E[V_{j+1}(x_{j+1})] \) is the expected value function. Additional constraints are \( a_1 = 0, a_{J+1} = 0, a_j \geq 0 \) and \( 0 < l_j \leq 1 \).

### 2.8 Equilibrium

Given a set of exogenous for demographic parameters \( \{s p_j\}_{j=1}^J \) and \( \{g^n\} \), exogenous growth rate \( \{g\} \) and fiscal policy variables \( \{\Psi, \theta^s s, \tau^k, \tau^l, G, D, v, tr, r^d\} \), a competitive equilibrium consists of a collection of individual household decisions \( \{c_j(x_j), l_j(x_j), a_{j+1}(x_j)\}_{j=1}^J \) for each state vector \( x_j \), factor prices \( \{w, r\} \), consumption tax \( \{\tau^l\} \), the measure of individual state \( \{\mu(x_j)\} \) such that

(a) the households solve the household problem (5);

(b) the firm chooses labour and capital inputs to solve the profit maximization problem (2);

(c) factor prices are determined competitively, i.e., \( w = F_L(K, L), q = F_K(K, L) \) and \( r = q - \delta \); and the domestic markets for capital and labor clear

\[
K = \sum_{j \in J} \sum_{x_j} a_j(x_j) \mu_j(x_j) + B - D, \\
H = \sum_{j \in J} \sum_{x_j} (1 - l_j) e_j(x_j) \mu_j(x_j),
\]

where \( B = \sum_{j \in J} \sum_{x_j} a_j(x_j) \bar{\mu}_j(x_j) \) is the total amount of assets left by all the deceased agents;

(d) the labor tax \( \{\tau^l\} \) adjusts, so that government budget constraint defined in Eq. (4) is satisfied;

(f) the aggregate resource constraint is given by \( Y = C + I + G \), where \( C = \sum_{j \in J} \sum_{x_j} c_j(x_j) \mu_j(x_j) \) and \( I = \sum_{j \in J} \sum_{x_j} a_{j+1}(x_j) \mu_j(x_j) \).
3 Calibration

We choose Japan as a benchmark case for quantitative analysis. We calibrate our benchmark model to match the Japanese economy in an artificial steady state in 2010. In this section we describe the parameterization and calibration of the benchmark model.

We source the values of model parameters from (i) the previous literature for specifying preferences; (ii) the macro data on government tax and fiscal policy, and population dynamics. We calibrate some structural parameters and fiscal policy variables to replicate life-cycle profiles of labour supply and asset holdings and targeted macroeconomic aggregates in the base year. The values of key parameters of the benchmark model in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Comments/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>J =16</td>
<td>Maximum lifetime periods</td>
<td>Equivalent to 80 years</td>
</tr>
<tr>
<td>J^w = 9</td>
<td>Maximum working periods</td>
<td>Equivalent to 45 years</td>
</tr>
<tr>
<td>σ = 2</td>
<td>Inverse of IES</td>
<td>Literature</td>
</tr>
<tr>
<td>φ = 1</td>
<td>Frisch labour supply elasticity</td>
<td>Literature</td>
</tr>
<tr>
<td>κ = 3.3</td>
<td>Weight of labour</td>
<td>Calibrated to match labour supply</td>
</tr>
<tr>
<td>β = 0.998</td>
<td>Time discount factor</td>
<td>Calibrated to match K/Y</td>
</tr>
<tr>
<td>τ</td>
<td>Age-specific labour productivity</td>
<td>Braun (2008)</td>
</tr>
<tr>
<td>z</td>
<td>Stochastic component of labor productivity</td>
<td>Lise et al (2014)</td>
</tr>
<tr>
<td>A = 1</td>
<td>Total productivity factor</td>
<td>–</td>
</tr>
<tr>
<td>α = 0.406</td>
<td>Capital share in production</td>
<td>Data (40.6%)</td>
</tr>
<tr>
<td>δ = 0.082</td>
<td>Annual depreciation rate</td>
<td>Data (8.2%)</td>
</tr>
<tr>
<td>θ^αs = 0.413</td>
<td>Govt coverage of pension benefits</td>
<td>Data</td>
</tr>
<tr>
<td>ψ = 0.33</td>
<td>Pension replacement rate</td>
<td>Calibrated</td>
</tr>
<tr>
<td>G/Y = 0.20</td>
<td>Government purchase to GDP ratio</td>
<td>Data (20%)</td>
</tr>
<tr>
<td>D/Y = 1.10</td>
<td>Government debt to GNP ratio</td>
<td>Data (110%)</td>
</tr>
<tr>
<td>τ^c = 0.05</td>
<td>Consumption tax</td>
<td>Data (5%)</td>
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<tr>
<td>τ^l = 0.25</td>
<td>Labour income tax</td>
<td>Data (14%)</td>
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<tr>
<td>τ^h = 0.40</td>
<td>Capital income tax</td>
<td>Data (27%)</td>
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<tr>
<td>τ_T = 0</td>
<td>Transfers</td>
<td>Balances Laffer curve</td>
</tr>
<tr>
<td>τ^d = 0.01</td>
<td>Interest rate on government bonds</td>
<td>Data (1%)</td>
</tr>
</tbody>
</table>

Table 1: Model parameter values for demographics, preferences, technology and fiscal policy.
3.1 Demographics

The model economy is populated by 16 overlapping generations of households in each discrete time period $t$, with each period lasting for 5 years. Households become economically active at age 20, and the $j = 1$ generation correspond to ages 20 to 24, $j = 2$ to 25 to 29 and so forth, with $J = 16$ – the oldest generation – corresponding to ages 95 to 99. We base on the data from the Japanese National Institute of Population and Social Security Research (IPSS) to construct the demographic structures for Japan. In our benchmark calibration, we use the actual age-distribution in 2010. In addition, we use actual 5 x 5 (age interval × year interval) life tables (for both sexes) from IPSS mortality database to construct the conditional survival probabilities. We use the demographic structure of 2010 to solve the optimization problem and to compute aggregate statistics.

3.2 Preferences

The instantaneous utility from consumption and leisure is given by

$$u(c_j, l_j) = \frac{1}{1 - \sigma} \left\{ [c_j]^{1-\sigma} \left[ 1 - \kappa (1 - \sigma)(1 - l_j)^{1+\frac{1}{\sigma}} \right]^\sigma - 1 \right\}, \quad (6)$$

where $\kappa$ represents the disutility from work and $1/\sigma$ is the inter-temporal elasticity of substitution. This functional form of Constant Frisch Elasticity (CFE) preferences is specified in ?.

? estimated a mean of $\sigma$ at 2.041 in their dynamic stochastic general equilibrium model of the Japanese economy. ? estimated the Frisch elasticity ($\phi$) for Japan on the extensive and intensive margins combined at between 0.7 and 1.0 for both sexes over the 1990 period. These estimates are in the value range used in the literature (e.g., see ? and Trabandt and Uhlig (2011)).

We calibrate the value of parameter $\kappa$ so that the average hours worked per working age person ($\bar{n}$) as a fraction of total time is 0.30. The value of parameter $\beta$ is set to match capital output ratio ($\frac{K}{Y}$) of 3.

3.3 Endowments

The labor productivity, $e_j$, of an age $j$ household in the model economy evolves over the life cycle according to $\ln e_j = \ln \bar{e}_j + \ln \tilde{e}_j$ for $j = 21, ..., 65$. The evolution of labor productivity has two components: deterministic one $\bar{e}_j$ and stochastic one $\tilde{e}_j$.

The deterministic component $\{\bar{e}_j\}$ is based on the estimates of the age-profile for Japan in ?. That age-profile of time-invariant labour productivity is constructed from Japanese
data on employment, wages, and weekly hours from 1990 to 2000. The age-specific labor productivities are values over five-year age groups, starting from age 20−24, to ages 65 and over. We set $\bar{e}_j = 0$ for $j = J^u + 1, \ldots, J$. The results plotted in Figure 1. The life-cycle profile is hump-shaped, reflecting the productivity gains as households gain experience, before declining at the end of the working life.

![Age-dependent labor productivities](image)

**Figure 1**: Age-specific labour productivities in the benchmark model

The idiosyncratic component $\tilde{z}_j$ of labor productivity is specified as a first-order autoregressive process in log as

$$\ln \tilde{z}_j = \rho \ln \tilde{z}_{j-1} + \epsilon_j,$$

where the temporary shock, $\epsilon_j$, is normally distributed. We set the persistence parameter $\rho = 0.97$ and the variance of the white noise $\sigma^2_\epsilon = 0.03$, which lie in the range of estimates in Lise et al. (2014). We approximate this continuous process with a three-state, first-order discrete Markov process.

### 3.4 Technology

We assume the production function has the Cobb-Douglas functional form

$$Y = AK^\alpha H^{1-\alpha}. \quad (7)$$

We set the capital share $\alpha$ at 0.4 and the depreciation $\delta$ at 0.082, using estimates from ?. These are also close to the values from Hayashi and Prescott (2002). We set $A$ grow at a constant rate.
3.5 Fiscal policy

Social security. The replacement rate $\Psi$ is set at 0.33 to match the size of the social security system as a proportion of output in 2010. The fraction of social security payment contributed by the general government budget is determined by

$$\theta^{ss} = 1 - \frac{\text{total pension contribution}}{\text{total pension transfers}}$$

(8)

As in we choose $\theta^{ss} = 0.413$. The social security tax rate is adjusted to keep the social security fund in balance. In our benchmark model, the equilibrium social security tax rate is around 10%.

Other government expenditures and debt. Government expenditures including the spending for healthcare and long-term care are 20% of aggregate output according to the National Accounts of Japan (SNA) in 2010. We set $G/Y$ at 20% to match that size.

The net government debt to GDP ratio $D/Y$ is calculated using the net debt to GDP ratio from the IMF. The government debt to GDP is set at 110%. The average number of years to maturity of outstanding government bonds is about 7 years and the average real interest rate on 7 year government bond is 1.0% in 2000 – 2010. The interest rates on government bonds ($r^d$) is set at 0.01, matching the implied yield on 10-year Japanese government bonds. This is set exogenously as they are significantly lower compared to the endogenous interest rate on private capital ($r$).

Taxes. The consumption tax is set at 5% in the initial steady state. Capital income tax is set at 30%, which is in the range of estimates of effective tax rates on capital income, for example, in . The labor income tax rate that clears the government budget constraint is 26% in the benchmark calibration. The combining rate of the labor income tax rate and the social security tax rate is around 36%, which is very close to the labor tax rate in Kitao (2015).

3.6 Benchmark model performance

In this section, we present the calibration results of the benchmark model based on 2010 demographics, and discuss how well the model matches the data in describing the Japanese economy.

The results for household assets, wages and labour supply are summarised in Figure 2. The first panel depicts the life-cycle asset holdings by households, relative to the asset holdings at age 50. The hump-shape is consistent the life-cycle hypothesis, where households are expected to build up their assets over their working life and run down the savings over retirement. We match the benchmark model to data from the 2009 National Survey of
Family Income and Expenditure (NSFIE), looking at the average total household savings categorised by the age of the household head. Our model is able to replicate the accumulation of asset holdings over the households’ working life, with households maximising their asset holdings at age 60.

The second panel presents the average number of labour hours supplied by households of different ages per week. We compare this to data from the 2010 Labour Force Survey, using the average hours of hours worked by employed persons of different age groups across all industries. As we do not distinguish between employed and unemployed households in our model, the hours worked are adjusted by the employment rate within each particular age group. It matches the general shape, with households supplying less hours at the younger ages, before supplying more during the latter stages, before supplying less again as the households near retirement age.

The last panel presents the wages earning profile of households of different age groups, relative to households of age 50. Our model can match the pattern of average wages and salaries income over the life cycle for workers from the 2009 NSFIE. The model generates the hump-shape of the wages. However, whereas wages is maximised for households of age 50 in the data, it is maximised at aged 45 in the model. Furthermore, the drop off in wages income in the model is both earlier and steeper. This is especially apparent for ages 65 and over where we assumed exogenous retirement and the wage income is zero. This is in contrast to the data, where the surveyed households by definition continue to work in those periods and earn wages income.

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1 Average hours worked per week per household (per generation) = \( \frac{\text{Total number employed}}{\text{Total population}} \times \) Average hours worked per week for each employed person. Further, for the model Labour supply, average hours per week is derived by multiplying the result by \( 14 \times 7 \), where 14 is the number of total free hours we assumed households have per day from the calibration section.

2 The model uses \( \epsilon_j w(1 - l_j) \) to proxy for the wages income.
4 Quantitative analysis

In this section, we discuss how we measure fiscal space and then quantify how demographic shift influences the size of fiscal space.

4.1 Fiscal space

In our model, higher distorting taxes diminish incentives to work and save, so there is some set of tax rates that maximize tax revenue and place the economy at the peak of its Laffer curves. Laffer curve reasoning provides an economic fiscal limit.\(^3\) The peak of Laffer curves naturally defines fiscal space in terms of budgetary room between the current tax revenue level and the maximum tax revenue level (fiscal limit). Intuitively, fiscal space measures how flexible a government can be in terms of adjusting its spending choices. Holding government spending constant, fiscal space describes the government’s fiscal flexibility to change its fiscal policy without compromising fiscal sustainability or the level of debt.\(^4\)

One-dimensional fiscal space. The current tax revenue (\(\overline{\text{Tax}}\)) is computed directly from this equation: \(\overline{\text{Tax}} = \bar{\tau}_l \bar{w} H + \bar{\tau}_k \bar{r} K + \bar{\tau}_c C\), where \(\bar{\tau}_l\), \(\bar{\tau}_k\), and \(\bar{\tau}_c\) are the labor, capital and consumption tax rates, respectively, and \(\bar{w}\) is the wage rate, \(\bar{r}\) is the interest rate, and \(H\), \(K\) and \(C\) are aggregate human capital, physical capital and consumption in the benchmark economy.

There are three tax instruments that could be used to raise tax revenue in the model. We assume that the government is allowed to vary only one tax policy at a time. That is, we compute a Laffer curve by allowing only one tax rate of interest to vary, while holding the other tax rates constant at their benchmark levels. This allows us to down the peak of Laffer curve and the maximum tax revenue. We can have three Laffer curves for labor, capital and consumption taxes.

The fiscal space (FS) is measured by distance between the maximum tax revenue and the benchmark tax revenue (\(\overline{\text{Tax}}\)). Given the benchmark tax rates \(\{\bar{\tau}_l, \bar{\tau}_k, \bar{\tau}_c\}\), the fiscal

\(^3\)We abstract from political economy arguments that are more likely to determine fiscal limits in democratic societies.

\(^4\)Alternatively, fiscal space can be defined in terms of a distance between the current debt levels and the debt limits above which the debt becomes unsustainable (e.g. see Heller (2005) and Ostry et al. (2010)). Our definition of fiscal space is similar to Park (2012).
space for labor, capital and consumption taxes are given by

\[
\text{Labor FS} = \max_{\tau^l} \left\{ \tau^l w H + \tau^k r K + \tau^c C \right\} - Tax,
\]

\[
\text{Capital FS} = \max_{\tau^k} \left\{ \tau^l w H + \tau^k r K + \tau^c C \right\} - T tax,
\]

\[
\text{Consumption FS} = \max_{\tau^c} \left\{ \tau^l w H + \tau^k r K + \tau^c C \right\} - T ax,
\]

respectively. Notice that, in our baseline analysis, we let the government adjust general government purchase \((G)\) to keep its budget in balance. We call it the \(g\)-Laffer curve.  

We present the fiscal spaces for the labour, capital and consumption Laffer curves in the 2010 benchmark model. The results are given in Figure 3 (a) to (c) below, with the level of tax revenue raised under the debt-specification in 2010 at 2010 tax rates normalised to 100. The labour and capital Laffer curves have the classic single-peaked concave shape, while the consumption Laffer curve does not have a peak and is an increasing function of the tax rate. This is consistent with the results from ?’s representative agent model.

The differing shapes arise due to the interaction between the tax rate and tax base. The graphs for the decomposition can be found in Appendix ???. As the labour tax rate increases, the labour tax base decreases almost linearly as the hours worked decrease. The multiplication of two linear factors with opposing signs gives rise to the inverse-U shape. For capital, the capital tax base is mostly flat at the lower tax rates, before decreasing

\[\frac{\text{multiplication of two linear factors with opposing signs gives rise to the inverse-U shape.}}{\text{We have a number of other options to keep the government budget in balance: general government purchases \((G)\), debt \((D)\), transfers \((tr)\) or public pension \((p)\). We will consider these types of Laffer curve in section 5.}}\]

\[\frac{\text{We define the labour tax base in time} \ t \ \text{as:} \ (1 - \tau^s s(\tau)) \times w_l(\tau) \times H_l(\tau), \ \text{where the social security contribution rate, wages and effective labour supply are functions of the tax rate combinations} \ \tau = \{\tau^l, \tau^k, \tau^c\}}{\text{6We define the labour tax base in time} \ t \ \text{as:} \ (1 - \tau^s s(\tau)) \times w_l(\tau) \times H_l(\tau), \ \text{where the social security contribution rate, wages and effective labour supply are functions of the tax rate combinations} \ \tau = \{\tau^l, \tau^k, \tau^c\}}\]
abruptly at the higher rates.\textsuperscript{7} This gives rise to the asymmetrical shape. However, unlike in the representative agent’s model by ?, and ?
, the capital Laffer curve is not flat to the left of the peak. The difference arises due to the labour tax revenue collected while the capital tax rate is changed. Whereas in the representative agent model, the labour tax revenue decreases, it is flatter in our OLG model as households supply more labour in response to the decrease in wages resultant from an increase in the capital tax rate. This means that rather than the decrease in labour revenue cancelling out the increase in capital revenue at the lower tax rates (which would produce a constant level of total revenue), the total revenue increases. Lastly, the shape of the consumption Laffer curve arises as both labour and capital tax bases are unaffected as the consumption tax rate increases.\textsuperscript{8}

In the benchmark, the economy lies to the left of the peak for both the labour and capital Laffer curves, and the government can raise additional revenue by raising either of the rates. The gross fiscal space for the labour g-Laffer curve is 77.53 percent of current revenue levels, and can be achieved by setting \( \tau^l = 0.63 \). This implies that the government can raise a total of 177.53 percent of current revenues by only altering only the single tax for labour. Raising the capital income tax rate is less viable as the gross fiscal space for the capital g-Laffer curve is 51.75 percent, achieved when \( \tau^k = 0.86 \). Lastly, while the consumption curve does not have a peak, we note that by setting \( \tau^c = 0.90 \), 279.25 percent of current revenues can be raised.

**Two-dimensional fiscal space.** While the Laffer curve alters one tax rate while keeping the other two constant, the capital and labour income taxes are varied jointly in the Laffer hill. We now consider a two-dimensional measure of fiscal space, based on the assumption that the government is allowed to vary two tax rates at a time. A Laffer hill is characterised by jointly varying two tax rates, while keeping the last one fixed.

In our analysis, we fix the consumption tax to calculate the labour-capital Laffer hill, whose peak will give the \( \tau^l \) and \( \tau^k \) combination that raises the maximum level of revenue. The capital and labor fiscal spaces can similarly be written by allowing the tax rate of interest to vary instead. The total gross fiscal space at time \( t \) compared to the benchmark revenue in time \( s \) \((\text{Tax}_s)\) can be written as

\[
\text{Labor-Capital FS} = \max_{\tau^l,\tau^k} \left[ \{ \tau^l wH + \tau^k rK + \tau^c C \} - \text{Tax}_s \right]
\]

The result for the benchmark economy are depicted in the the panel. The contour lines depict the different tax rate combinations that raise the same level of revenue, relative

\textsuperscript{7}We define the capital tax base in time \( t \) as: \( r_t(\tau) \times K_t(\tau) \), where the pre-tax interest rate and capital stock are functions of the tax rate combinations \( \tau = \{ \tau^l, \tau^k, \tau^c \} \)

\textsuperscript{8}We define the consumption tax base in time \( t \) as: \( C_t(\tau) \), where consumption is a function of the tax rate combinations \( \tau = \{ \tau^l, \tau^k, \tau^c \} \)
Figure 4: g-Laffer hill and fiscal space in 2010

to the benchmark revenue level that is set at 100. The dotted lines mark the benchmark taxation rates. As the peak of the hill lies in the north-east quadrant, the government can maximise the level of taxation revenue by increasing both the capital and labour tax rates. The total gross fiscal space in the 2010 is 87.86 percent of current revenue levels, and is achieved by setting $\tau^l = 0.63$ and $\tau^k = 0.55$. We note that although the individual capital Laffer curve peaks at $\tau^k = 0.86$, the total amount of revenue can be increased by lowering the capital tax rate and encourage additional output.

The results of the benchmark Laffer hill in the OLG model differ qualitatively from the representative agents’ model. While our findings suggest both taxes should be increased to maximise revenue, ? found that for the US and other G7 countries (except Germany), total taxation revenue could be increased by raising the labour tax rate while lowering the capital tax rate. This discrepancy likely arises from both the different shape of the capital Laffer curve, and the differing assumption on where Japan is currently positioned on the curve. As the capital Laffer curve is mostly flat to the left of the peak in the representative agent model capital Laffer curve, lowering capital income tax would have minimal impact on revenue collected, while simultaneously reduce distortions in the economy and increasing the tax bases for labour and consumption. This allows the government to raise more revenue at a lower capital tax rate. By contrast, lowering the capital tax rate in our model would decrease revenue more, as the slope is steeper to the left of the peak. Second, there are also differences in tax rates used, with Park setting the current capital tax rate for Japan at 0.42, versus 0.27 in our model. This means that the distance between the current
tax revenue and the peak of the capital Laffer curve is higher in our model, providing more incentive to raise capital tax relative to current levels.

**Maximum debt to GDP.** We now analyze the relationship between the government capacity to raise tax revenue and debt level. We let \( Tax = \tau^l w_H + \tau^K r K + \tau^c C \) and \( Spend = G + \theta^{ss} SS + Tr \) denote total tax revenue total and government spending, respectively. Primary fiscal surplus (deficit) is given by \( SPL = Tax - Spend. \) Given the government primary surplus we can derive the maximum debt level that the government can sustain in the steady state as

\[
D_{\text{max}} = \frac{SPL}{\rho_d}.
\]

This equation indicates that The maximum debt to GDP is given by

### 4.2 Demographic shift and fiscal space

In this section, we consider two alternative demographic structures for Japan: one in 1980 and one in 2040. We compare the corresponding fiscal space to that in the 2010 benchmark case. The experiments allows us to quantify the mapping from demographic shift to fiscal space. Notice that, all other non-demographic variables are kept constant at their benchmark levels to isolate the effects of a demographic shift on the economy. As this includes all fiscal policy variables, this model cannot account for the preventative or adaptive policy responses to the ageing problem, such as legislation on retirement age and pension age eligibility.

**Demographic structures.** We consider two alternative demographic structures in 1980 and 2040. We use the relevant sequence of survival and fertility rates from demographic data from IPSS to construct these two demographic structures. Figure presents the age distributions and conditional survival probabilities used in our analysis.

Our model reproduces the age-dependency ratios of 1980, 2010 and 2040.

**Macroeconomic aggregates.** We start with the impacts of the demographic shift on the macroeconomic variables. The results are aggregated in Table ?? below. The 2010 results from the model are given, while most of the 1980 and 2040 figures are reported as percentage changes from their 2010 (per capita) levels, with the exception of the tax rate changes, which are expressed in percentage point changes instead.

Household behaviour also changes as a result of the improvement in survival probabilities over the 1980 to 2040 period. Despite the decrease in overall levels, savings and asset holdings remain high for longer and dissaving in retirement occurs at a slower rate than in 1980. In 2010, household savings surpass the 1980 levels at age 80, while in 2040, household savings surpass the 1980 levels at age 85. As survival probability improvements continue to occur at the higher ages between 2010 and 20140, dissaving for households
aged 85 and above occurs even slower, and catches up to 2010 levels by age 95. This also implies that households are consuming more in retirement.

Household labour supply behaviour is also affected by the decrease in disposable wages and households increase their labour supply, especially at the later working years. From 1980 to 2010, the labour supply falls during the middle period from ages 30 to 39, before rising for the remainder working periods from ages 40 to 64. This can be explained by the interaction of the income and substitution effects resultant from the fall in disposable wages as a result of ageing. In the younger periods when the decrease in wages is relatively larger once we account for the lower age-specific labour productivity, the substitution effect dominates over the income effect, and households consequently work less. In the older periods when the decrease in wages is relatively smaller due to the higher age-specific productivity, the income effect dominates and households are willing to work more. From 2010 to 2040, as the disposable wages fall further, the substitution effect dominates, and the labour supply falls for all ages. Although, we note that households still supply more labour than in 1980 for ages 40 to 64.

The labour tax base was 17.67 percent higher in 1980, and will be 18.48 percent lower compared to the benchmark level. This arises from the decrease in the labour supply ($N_t$) and effective labour supply ($H_t$). As the household’s labour supply is mostly unchanged on the individual level, this decrease is mostly driven by the lower working age cohort share as the dependency ratio rises. The per capital labour supply was 19.02 percent higher in 1980, and will be 15.24 percent lower in 2040 compared to 2010 levels. The effective labour supply changes to a greater extent, arising from demographic and behavioural changes. The age-specific labour productivity is asymmetrical, peaking at age 50 with the decrease over ages 50 to 64 sharper than the rise from ages 20 to 50 (see Figure 1). While ageing leads
<table>
<thead>
<tr>
<th>Variable</th>
<th>1980</th>
<th>2010 (level)</th>
<th>2040</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour tax base %</td>
<td>17.67</td>
<td>1.09</td>
<td>-18.48</td>
</tr>
<tr>
<td>Labour supply %</td>
<td>19.02</td>
<td>0.20</td>
<td>-15.24</td>
</tr>
<tr>
<td>Effective labour supply %</td>
<td>20.10</td>
<td>0.21</td>
<td>-15.66</td>
</tr>
<tr>
<td>Wage rate %</td>
<td>-6.42</td>
<td>0.42</td>
<td>1.7</td>
</tr>
<tr>
<td>Capital tax base %</td>
<td>20.27</td>
<td>0.03</td>
<td>-15.66</td>
</tr>
<tr>
<td>Capital stock %</td>
<td>1.06</td>
<td>0.09</td>
<td>-11.64</td>
</tr>
<tr>
<td>Interest rate %</td>
<td>17.10</td>
<td>0.047</td>
<td>-4.17</td>
</tr>
<tr>
<td>Consumption (tax base) %</td>
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<td>-9.91</td>
</tr>
<tr>
<td>Social security system %</td>
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<td>29.48</td>
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<tr>
<td>Pension benefit %</td>
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<td>0.044</td>
<td>0.89</td>
</tr>
<tr>
<td>Pension contribution rate %</td>
<td>percentage point</td>
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<td>9.77</td>
</tr>
<tr>
<td>Labour income tax rate %</td>
<td>percentage point</td>
<td>-5.46</td>
<td>20.43</td>
</tr>
<tr>
<td>Output (GDP) %</td>
<td>11.37</td>
<td>0.15</td>
<td>-13.80</td>
</tr>
</tbody>
</table>

Notes: All variables other than the wage and interest rate, and pension contribution and labour income tax rates, are expressed in per capita terms.

Table 2: The effects of shifting demographic dynamics on macroeconomic variables

To greater labour supply at age 50, it also increases labour supply at the beginning and end of working ages (20 to 25, and 50 to 65) where households are relatively unproductive compared to ages 30 to 40. As the cohort share is concentrated between ages 20 to 50 in 1980, it captures the additional effective labour supply of the 30 to 40 households. In 2040, as cohort shifts to the age 50 to 80 age range, it captures the lower effective supply of the 50 to 65 age range.

The capital tax base also shrinks as a result of demographic changes, with it higher by 20.27 percent in 1980, and lower by 15.66 percent in 2040. This arises from the fall in capital stock resultant from both behaviour and level changes. This is consistent with the life-cycle hypothesis, with the proportion of working age savers decreasing, and retired age dissavers increasing as a result of ageing. This is compounded by the decrease in disposable wages as the population ages, and households’ savings levels fall. The asymmetrical decrease from 1980 to 2010 and 2010 to 2040 arises from the different levels of asset held over the life cycle. From 1980 to 2010, the effects of ageing was masked as in 2010, the most populous cohorts were concentrated in the high-assets age range between ages 50 and 70. However, with the shift to 2040 demographics, the 60 to 80 age range will be more populous, and as they lie to the right of the asset holdings peak, total capital stock within the economy will fall to a greater extent. While the labour supply decreases proportionally less than the labour tax base, the capital tax base decreases more than the capital stock. This arises from the simultaneous fall in interest rates (and rise in wage rates) driven by capital deepening.
Consumption is also negatively affected by ageing, with the consumption tax base 20.27 percent higher in 1980, and 15.66 percent lower in 2040. The asymmetrical decrease results from the life-cycle behaviour of households, with the 1980 to 2010 effects softened by the increase in households with high disposable incomes. As the proportion of low income retired households continue to increase, the consumption tax base will decrease at a faster rate.

Despite a decrease in the pension benefits received by each retired household in 2040, the effects of the demographic shift will dominate, and the social security system will expand by 29.48 percent as a result of ageing. To support the growing social security system, both $\tau^a$ and $\tau^l$ has to be increased for debt and government spending levels to remain constant. At the aggregate level, the ageing population has a negative effect on output, and GDP per capita will be 13.80 percent lower in 2040.

**Contraction in fiscal space.** We examine how the changes in demographic shift – at both the individual household and aggregate level – affect the one-dimensional fiscal spaces.

The labor, capital and consumption g-Laffer curves for the benchmark 2010 case, and alternative ageing scenarios in 1980 and 2040 are plotted in Figure 6 (a) to (c). The benchmark revenue levels under the g-Laffer curve specification is set as the baseline at 100. Normalising the current fiscal space as 100 percent, the resultant percentage changes in the gross fiscal spaces are given in (d) to (f). Due to the decrease in the tax bases described in the previous section, all three of the Laffer curves shift down as a result of the demographic shift, and the individual gross fiscal spaces shrink. The amount of taxation revenue generated for each tax rate is now lower at every tax rate.

![Figure 6: The effects of shifting demographics on Labor g-Laffer curves](image)

In absolute terms, the maximum labour revenue decreases by the most as a result of
ageing. In 1980, 207.42 percent of the baseline revenue can be raised; by 2040, this will decrease to 151.29 percent. Capital will see a smaller absolute change in the maximum revenue raised, with $\tau^k = 0.83$ in 1980 raising 170.74 percent of current revenues, compared to $\tau^k = 0.86$ raising 132.62 percent in 2040. Increasing the consumption tax rate to the maximum 0.90 used in the model, 294.2 percent and 260.13 percent of current revenue can be raised in 1980 and 2040 respectively.

When comparing the relative changes in the fiscal space compared to the current fiscal space, capital is the most sensitive to demographic changes, with gross fiscal space lower by 36.98 percent in 2040. This is followed closely by labour, with gross fiscal space lower by 33.84 percent. Consumption is the least sensitive, and gross fiscal space is still at 89.3 percent of the current level by 2040. The sensitivity of capital is again driven by the compounded fall of both the capital stock and interest rate. This is most apparent when we look at 1980 compared to 2010, when capital stock was only 1.06 percent higher than 2010 levels, with the fiscal space decrease largely driven by the 17.10 percent decrease in interest rates.

![Figures](image1.png)

**Figure 7:** The effects of shifting demographics on Capital g-Laffer curves

Looking at the slopes of the fiscal space decreases, we can quantify the relative movements of the fiscal space over the next 30 year period compared to the last 30 year period. While the labour and capital fiscal spaces are linear, the consumption fiscal space is kinked and will decrease at a faster rate over the next 30 years. Lastly, we note that while the fiscal space will shrink as a result of the ageing, it will remain positive for all three taxes. Consequently in 2040, the Japanese government will be able to maintain the 2010 levels of taxation revenue by increasing either one of the labour, capital, and consumption tax rates.

**Two-dimensional fiscal space.** We now pay attention to the impact of the demo-
graphic shift on the two-dimensional fiscal space when both the labour and capital income taxes are allowed to change. The g-Laffer hills for 2010, 1980, and 2040 are depicted in Figure ??.

First, we note the contour lines raising a particular revenue level will shift north-east as a result of ageing. This means that higher labour and capital tax rates are needed to generate the same level of income as before. For instance, at the benchmark tax rates, 115.91 percent of current revenue could be raised in 1980, and only 90.13 percent will be raised in 2040. The maximum revenue collected is affected to a greater extent, at 219.05 percent and 160.63 percent of current revenue levels respectively. The corresponding revenue maximising tax rates for labour and capital are largely unchanged, at 0.63 and 0.58 in 1980: 0.63 and 0.55 in 2010 and 2040.

Figure 8: The effects of shifting demographic dynamics on the g-Laffer hill

Looking at the fiscal space in the last panel, the fiscal space was 35.5 percent greater in 1980, and will shrink by a further 31 percent in 2040. There is a linear relationship between the dependency ratio and the change in fiscal space relative to the benchmark, with a 10 percentage point increase in the dependency ratio is associated with an approximate 12.2 percent decrease in the fiscal space.

4.3 The role of demographic factors

In the analysis so far, we have assumed both the age-specific survival probabilities and fertility rate change according to either their actual or projected paths. This allowed us to quantify the total effect of the two demographic factors on the fiscal space. To isolate and
quantify the individual effects, we perform the following decomposition exercise. We keep the 2010 economy based on the actual time series of survival probabilities and fertility rates up until 2010 as the benchmark. This is compared to four counter-scenarios based on the following demographic changes: (i) a “high fertility” case, where the survival probabilities are kept at the benchmark values, and the fertility rate revert to their time sequence up until 1980; (ii) a “low fertility” case with 2010 survival probabilities and net reproduction rate at their 2040 levels; (iii) a “high mortality” case with benchmark fertility rates and 1980 survival probabilities; and (iv) a “low mortality” case with benchmark fertility rates and 2040 survival probabilities. The effects of the alternative population dynamics on the population composition are plotted in Figure 9. For ease of comparison, we have also included the benchmark 2010 case as the baseline.

Both high fertility and high mortality rates increase the proportion of young households relative to the baseline, while low fertility and mortality rates decrease the proportion. Over the 1980 to 2040 period, the mortality rate decrease is the single biggest driver of the demographic composition, with the dependency at 0.33 in the high mortality case compared to 0.51 in the baseline. This is followed closely by the fertility rate decrease, with the dependency ratio at 0.37 in the high fertility case. As the demographic changes slow down over the 2010 to 2040 period, so does the relative change in cohort sizes. This is especially true for the mortality rate (see Figure ??), and the dependency ratio will only rise to 60 in the low mortality case. This compares to 0.64 for the low fertility case.

To quantify the isolated effects of changes in the fertility rate on the gross fiscal space, we compare the g-Laffer curve under the baseline economy with the high fertility and low fertility cases. Similarly, we compare the g-Laffer curve under the baseline economy with the high mortality and low mortality cases to isolate the effects of the changes in
Notes: The fiscal space here refers to the individual gross fiscal spaces for labour, capital and consumption when changing only the relevant tax rate and keeping the other two tax rates constant at their benchmark levels. Government debt is fixed at the benchmark level.

survival probabilities. The results are summarised in Figure ?? (a) to (c), and (d) to (f) respectively.

All three Laffer curves shift down as either the fertility rate or mortality rate decreases. This is consistent with our intuition as both demographic movements age the population by increasing the dependency ratio. Using the size of the effect on the fiscal space as a measure of the relative sizes of the demographic trends, we can compare the relative sizes of the fertility trends with the mortality trends, as well as comment on the relative sizes of each trend over different time periods. Over the 1980 to 2010 period, the size of the isolate fertility and mortality changes on the gross fiscal spaces are similar. In the high fertility counter-scenario where the fertility rate decrease does not occur, the gross labour fiscal space would be 95.33 percent of current revenue levels, compared to 94.58 percent for the high mortality counter-scenario where the survival probability improvements do not occur. We note that the change in fertility has a bigger effect on the fiscal space despite affecting the population composition (through the dependency ratio) to a lesser degree. This is reversed for the capital Laffer curve, with the fiscal space under the high mortality case slightly higher than the high fertility case, at 64.38 and 63.73 percent respectively. Similarly, the consumption fiscal space at $\tau_c = 0.9$ would have been 192.03 percent and
190.93 percent, under the high mortality and high fertility cases.

Over the coming 2010 to 2040 period, the changes in both the fertility and mortality rates will slow down, as measured by a smaller shift in the Laffer curves between the high fertility (mortality) and the baseline case, compared to the baseline and low fertility (mortality) case. For labour, the fiscal space under the high fertility case would have been 22.96 percent larger, compared to 20.35 percent lower for the low fertility case when compared against the baseline fiscal space. Similarly for capital and consumption, the fiscal spaces under the low fertility case will decrease less, by 20.83 and 5.52 percent respectively. This compares to an increase of 23.15 percent and 6.52 percent under the high fertility case.

Further, while the relative scale of the fertility and mortality changes were similar over the 1980 to 2010 period, this is not the case for the 2010 to 2040 period, with further mortality rate improvements contributing less to the fiscal space decrease experienced by the economy in 2040. Under the low mortality case, the fiscal space for labour, capital and consumption are 9.73 percent, 11.86 percent, and 2.80 percent lower than the baseline levels, which is approximately half the size of the decreases under the low fertility case. Further, as fertility rates are projected to remain depressed into 2060 – with the 0 to 14 age group shrinking by 1 percent per annum, while the survival probabilities remain largely unchanged, we expect that future decreases in the fiscal space will be increasingly driven by the fertility rate.

As it is not surprising that a larger change in the scale of the demographic factors would lead to a corresponding larger movement in the Laffer curves, we quantify the relative importance of the factors on a per unit basis, with the dependency ratio used as a general measure of the level of ageing. We plot the fiscal space of the various scenarios against to the dependency ratio that they produce, with the results summarised in Figure ???. The “fertility only” case refers to the the low and high fertility scenarios, the “mortality only” case refers to the the low and high fertility scenarios, and the “total” case refers to the actual population dynamics we originally used, with 1980 survival probabilities matched with 1980 fertility rates, and 2040 survival probabilities matched with 2040 fertility rates.

Firstly, the effects are mostly linear and the capital tax revenue is the most sensitive to changes in the dependency ratio, with a 10 percentage point increase in the dependency ratio associated with an approximate 14 percent fall in the gross fiscal space. Labour is the second, with a 13 percent fall, while consumption is the least, with a 4 percent fall at the $\tau^c = 0.9$ level.

There are also some minor qualitative differences on how the mortality and fertility rates affect the respective Laffer curves on a per unit basis. As we have commented earlier, the labour Laffer curve is relative more sensitive to changes in the fertility rate. This
Figure 11: Gross fiscal space and the dependency ratio

is depicted in the first panel by the position of the “fertility only” points relative to the benchmark 2010 point and the trend line. For the high fertility point to the left of the benchmark, the point lies above the trend line, indicating that for a given decrease in the dependency ratio driven purely by a fertility rate increase, the fiscal space would increases more than the trend suggests. Similarly for the low fertility point to the right of the benchmark, the point lies below the trend line, indicating that for a given increase in the dependency ratio driven by purely by a fertility rate decrease, the fiscal space decreases more than the trend suggests. In comparison, the low mortality point lies above the trend line, while the high mortality point lies below it, indicating a lesser impact on the fiscal space than suggested by the movements in the dependency ratio. Looking at the capital and consumption spaces, the relative importance of fertility on a per unit basis also holds true, abiet to a lesser extent.

5 Extension and sensitivity analysis

In this section, we conduct sensitivity analysis on the results by altering the specifications and parameter values for the preferences. Lastly, we examine how the isolated changes in the survival probability and fertility rates contribute to the movements in the Laffer curves and the shift in fiscal space.

5.1 Fiscal space with d-, tr-, and p-Laffer curves

tr-Laffer curve. For the tr-Laffer curve, we assume that additional revenue is redistributed evenly to all households in the form of a lump-sum transfer (tr). For a certain
combination of tax rates $\bar{\tau}^c, \bar{\tau}^k, \bar{\tau}^l$, the transfer would be characterised by
\[
tr = \frac{\bar{\tau}^c C + \bar{\tau}^k rK + \bar{\tau}^l wH - \bar{\tau}^s wH - \bar{\tau}^s SSS}{\sum_{j=1}^J P_j}
\]  

**d-Laffer curve.** For the d-Laffer curve, we relax the assumption that debt is fixed at $\bar{D}$ and the government uses the additional tax revenue to repay outstanding government debt. The new maximum sustainable level of debt in period $t$ is given by
\[
D = \frac{\bar{\tau}^c C + \bar{\tau}^k rK + \bar{\tau}^l wH - \bar{\tau}^s wH - \bar{\tau}^s SSS}{r^d}
\]  

**pb-Laffer curve.** For the pb-Laffer curve, the government pre-commits to the benchmark level of pension benefits ($\bar{pb}$) and social security contribution rate ($\bar{\tau}^{ss}$) for future periods. This is a reasonable assumption as governments are often politically constrained and cannot adjust age-related expenditure or contribution rates. The general government then covers the shortfall in the social security system by changing its contribution rate $\theta_t^{ss}$, which is now given endogenously by
\[
\theta_t^{ss} = \frac{\bar{pb} \sum_{j=J^w+1}^J P_{j,t} - \bar{\tau}^{ss} wH}{\bar{pb} \sum_{j=J^w+1}^J P_{j,t}}
\]  
The level of government spending and debt then changes endogenously to maintain budget balance.

In our model, the d-Laffer and g-Laffer curves are equivalent as the interest rate on government bonds ($r^d$) is exogenous, and either the debt or government spending is fixed at the benchmark levels. It becomes a matter of balancing the government’s budget condition by adjusting either the sustainable debt level or government spending, and revenue levels are unaffected. The g-Laffer (and d-Laffer) curves have higher peaks in both the labour and capital curves, while a lower peak for the consumption curve compared to the tr-Laffer curves. The difference arises from the additional revenue that is fed back into the economy through transfers. The transfers increase the households’ income, and as a result, households consume more leisure and consumption, while saving less. This lowers the labour and capital tax bases and the entire curve shifts down as a result. Conversely, the additional consumption level increases consumption tax revenue under the tr-Laffer specification. The effects become more pronounced at the higher tax rates as the size of the transfers increase. For the remaining analysis on the gross fiscal space, we use the g-
Laffer curves as a government interested in increasing revenue would prefer the specification that raises more revenue.

5.2 Net fiscal space

Gross fiscal space not account the expenditure-side pressures and purely captures the unused revenue generating capacity. As this could possibly underestimate the true degree to which fiscal manoeuvrability is compromised as a result of ageing, we use concept of net fiscal space (NFS) – defined as the gross fiscal space minus the general government’s spending on the social security system – to capture the pension-related expenditure pressures resulting from a demographic shift. We also distinguish between the individual labour, capital, and consumption fiscal space (calculated from the Laffer curves), and the total fiscal space (calculated from the Laffer hills). We use the tr-Laffer, d-Laffer and g-Laffer curves to examine the effects of ageing on gross fiscal space, and the pb-Laffer curve to examine the effects on the net fiscal space.

**Net fiscal space.** The labour net fiscal space at time \( t \) compared to the benchmark uncommitted tax revenue in time \( s \) (\( \text{UTax}_s \)) given the benchmark tax rates can be written as

\[
\text{labour NFS} = \max_{\tau_l} \left\{ \tau_l(1 - \tau^{ss})wH + \bar{\tau}^{k}rK + \bar{\tau}^{c}C - \theta^{ss}\bar{p}b \sum_{j=J^w+1}^{J} P_j - (\text{UTax}_s) \right\}
\]

where \( \text{UTax}_s = \tau^l w_s H_s + \bar{\tau}^{k} r_s K_s + \bar{\tau}^{c} C_s - \theta^{ss}\bar{p}b \sum_{j=J^w+1}^{J} P_j \)

The capital and consumption net fiscal spaces can similarly be written.

Lastly, the total net fiscal space at time \( t \) compared to the benchmark uncommitted revenue level can be written as

\[
\text{total NFS} = \max_{\tau^l} \left\{ \tau^l(1 - \tau^{ss})wH + \bar{\tau}^{k}rK + \bar{\tau}^{c}C - \theta^{ss}\bar{p}b \sum_{j=J^w+1}^{J} P_j - (\text{UTax}_s) \right\}
\]

where both the labour and capital tax rates are allowed to vary and only the consumption tax rate is kept constant.

For the benchmark economy, the pb-Laffer curves for labour, capital and consumption are plotted in Figure 12 (a) to (c), and the pb-Laffer hill is plotted in (d). We take the 2010 total tax revenue minus current government contribution to the social security system as our benchmark and set it to 100. This is also referred to as the benchmark “uncommitted” tax revenue.
As with the g-Laffer curves, the single maximum revenue-raising peak exists for the labour and capital pb-Laffer curves. However, the peak for labour arises not from the trade off between the labour tax revenue and labour tax base (see Appendix ??). Instead, labour tax revenue now decreases monotonically as the labour income tax rate increases, and the concave shape arises from the increase in capital revenue. This is as we have fixed the social security contribution rate $\tau^{ss}$ and total (effective) labour supply remains constant as the tax rate increases. While the revenue maximising labour tax rate is similar under both the g and pb specifications, the capital tax rate is significantly lower in the pb case. This arises as both the capital and labour tax revenue decreases faster at the higher capital tax rates.

**Ageing and net fiscal space.** The previous section found that while ageing will shrink the gross fiscal space, the Japanese government in 2040 can maintain current total revenue levels by increasing either the labour or capital tax rate. However, the gross fiscal space only measures the ability to generate additional revenue, and ignores the additional expenditure pressures associated with ageing. We account for the general government’s social security system commitments and describe the effects of the demographic shift on the net fiscal space. The pb-Laffer curves for labour, capital and consumption in 1980, 2010 and 2040 are plotted in Figure 13 (a) to (c), and the corresponding net fiscal spaces
are plotted in (d) to (f). The total tax revenue raised (with $\tau^l = 0.14$, $\tau^k = 0.27$, and $\tau^c = 0.05$) minus the government contribution to the social security system in the benchmark 2010 economy is set at 100. This measures the uncommitted total tax revenue, or the net total tax revenue. As in the previous section, the Laffer curves for labour, capital and consumption shift downward and the net fiscal spaces decreases as a result of ageing. Accounting for the expenditure constraints leads to a greater shift in the Laffer curves compared to the gross g-Laffer counterparts. This arises as in the 1980 pre-ageing economy, (i) there is a smaller proportion of retired households; and (ii) the government’s contribution to the social security system ($\theta^{ss}$) is smaller than the benchmark levels as we fixed the contribution rate at 2010 levels, which is higher than was needed in the 1980 economy to maintain budget balancedness (see Table ??). Therefore, the net fiscal space is greater than the gross fiscal space in 1980. Similarly, the net fiscal space in 2040 will be smaller as both the contribution rate and the total size of the social security system increase from both the population shift and the fixed pension benefits.

Altering the single tax rate in 1980, the labour tax rate can raise 234.69 percent of current uncommitted revenue levels. By 2040, this will only be 131.26 percent. For capital, it is respectively 234.69 percent and 95.93 percent; and for consumption, it is 381.90 percent and 316.75 percent at $\tau^c = 0.9$. Once again, capital is the most sensitive to the demographic shift, with the fiscal space shrinking by 110.90 percent compared to the benchmark fiscal space. When we account for the pension-related spending commitments that constrain the government’s fiscal space from the expenditure side, the Japanese government will no longer be able to maintain current levels of revenue by increasing capital income tax alone. The labour tax rate is the next sensitive, with fiscal space in 2040 shrinking by 59 percent. While the government can either maintain current revenue levels by setting $\tau^l = 0.3$ in 2040, when we consider that ageing is projected to continue into 2060 – with the dependency ratio rising to 84 percent – relying on the age-sensitive labour income tax alone is also problematic. This is especially the case as the labour net fiscal space is projected to hit zero at a dependency ratio of 0.9.\footnote{This is derived from plotting the movements in the individual fiscal spaces against the dependency ratio. The results can be found in the section ??, and will be explained in detail in the relevant sections.} When we consider the age-sensitivity of the capital and labour Laffer curves, the consumption tax is the only tax that is able to generate a high level of uncommitted revenue that is insensitive to demographic shifts. It would be possible to maintain current revenue by $\tau^c = 0.15$ in 2040. However, it is important to consider how realistic the model consumption Laffer curve is, as it seems infeasible that the government would be able to raise revenues without limitation by increasing the tax rate.

As raising the capital tax rate alone is no longer viable, we also compare the pb-Laffer
hills in 2010, 1980 and 2040 to quantify the demographic shift on the net fiscal space when altering two taxes. The results are summarised below in Figure ??.

While the revenue maximising tax rates remain largely unchanged, the maximum (uncommitted) revenue is lower compared to g-Laffer counterpart in the relatively aged 2010 and 2040 cases, while higher for the 1980 pre-ageing one. Maintaining the benchmark tax rates, 151.88 percent and 65.93 percent of the benchmark revenue can be raised in 1980 and 2040 respectively. We note that this is significantly lower than the gross counterpart, which can still maintain 90.13 percent of current total revenue in 2010. This difference arises from the qualitative difference in how the Laffer curves are shifted by ageing. Whereas the g-Laffer curves’ shift were less pronounced at the lower rates, the pb-Laffer curves display a parallel shift at all tax rates as the government has committed to a set deduction in pension contributions that have to be paid at any tax rate.

From 2010 to 2040, the net fiscal space will shrink by 57.22 percent, and the government can raise a maximum of 136.30 percent of current revenue levels by setting $\tau^l = 0.60$ and $\tau^k = 0.55$. The net fiscal space also decreases linearly as a function of the dependency ratio, and a 10 percentage point increase in the dependency ratio is associated with an 11 percent fall in the net fiscal space. Recalling that the gross fiscal space falls by 12 percent, there is no significant qualitative difference in the relationship between the gross and net fiscal spaces and the dependency ratio. The large decrease in the fiscal space arises as a level phenomena due to our assumption that the other fiscal policies are kept fixed.

In summary, we conducted experiments by changing the survival probability rates and fertility rates from the benchmark levels – using the actual time path leading up to 2010, to a “pre-ageing” 1980 scenario with lower survival probability rates and higher fertility rates, and a “post-ageing” 2040 scenario with higher survival probability rates and lower fertility rates. We find that the fiscal space for both the individual Laffer curves – when altering one tax –and the Laffer hill – when altering two taxes – will shrink as a result of ageing. However, the gross fiscal space will still be positive for all three individual Laffer curves and the government will be able to choose the tax to raise. In comparison, the net fiscal space will decrease to a greater extent, and no longer exist for the capital Laffer curve. Instead the government will need to raise either the labour or consumption taxes, or the capital and labour tax rates simultaneously to maintain current revenue levels. We note that our measure of net fiscal space is unable to capture all of the expenditure-side pressures associated with ageing as it do not include medical and other indirect age-related expenditures. Therefore, it is likely that the actual net fiscal space will be even smaller in 2040.
Notes: The fiscal space here refers to the individual net fiscal spaces for labour, capital and consumption when altering only the relevant tax rate. The per household pension benefits and pension contribution rates are fixed at the benchmark levels.

Figure 13: The effects of shifting demographic dynamics on pb-Laffer curves

5.3 Alternative projections

Lastly, we examine at how isolated changes in the fertility and mortality rates affect the net fiscal spaces for labour, capital and consumption. The results for the “fertility only” changes are plotted in Figure ?? (a) to (c), and the “mortality only changes” are plotted (d) to (f). Whereas in the g-Laffer curves, both mortality and fertility trends contribute equally to a decrease in the gross fiscal space over the 1980 to 2010 period; the decrease in the mortality rates contributes to a bigger extent than the decrease in fertility rates for all three taxes. Comparing the high fertility case with the baseline, the net fiscal space for labour, capital and consumption would be 41.39 percent, 74.80 percent, and 8.69 percent higher respectively. In contrast, the corresponding fiscal spaces are 47.47 percent, 81.93 percent, and 9.18 percent higher under the high mortality case. Whereas a high fertility rate increases the size of the younger cohorts to the older cohorts at a constant proportion, a high mortality rate – especially one with the survival probability increases concentrated at the high ages – disproportionally decreases the size of the older cohorts. The government’s social security contributions consequently decrease, and this expenditure side movement drives the difference in the gross and net fiscal spaces.

Like the gross fiscal space, the net fiscal space decrease over the 2010 to 2040 period
Notes: The fiscal space here refers to the total net fiscal space when jointly changing $\tau^l$ and $\tau^k$, while $\tau^c$ is kept at the benchmark level. The per household pension benefits and pension contribution rates are fixed at the benchmark levels.

will slow down for both fertility and mortality changes. Further, the changes will be increasingly driven by the decrease in fertility rates. Over the period, the net fiscal spaces for labour, capital and consumption in the low mortality case will shrink by 18.18 percent, 37.12 percent, and 3.98 percent respectively. While in the low fertility case, they will correspondingly shrink by 36.51 percent, 68.19 percent, and 7.75 percent, contributing twice as much as the mortality changes. Lastly, recalling that the net fiscal space no longer existed for capital g-Laffer curve in 2040, we note that if either the mortality improvements or fertility decreases did not occur, the fiscal space would still remain positive.

We plot the net fiscal space against the dependency ratio in Figure 16 to examine the per unit effects of a change in the fertility and mortality rates. The capital fiscal space is the most sensitive to changes in the dependency ratio, with an approximate 43 percent decrease in the net fiscal space for a 10 percentage point increase in the dependency ratio. For labour, the decrease is 23 percent, while for consumption, it is 5 percent. While all three fiscal spaces decrease at a faster rate than the gross counterparts, with the rate for
Figure 15: pb-Laffer curves under alternate fertility and mortality assumptions

Notes: The fiscal space here refers to the individual net fiscal spaces for labour, capital and consumption when altering only the relevant tax rate. The per household pension benefits and pension contribution rates are fixed at the benchmark levels.

labour nearly double, and the rate for capital nearly triple. Lastly, despite the bigger contribution of the mortality rate in shifting the fiscal space from 1980 to 2010, when we examine the effects on a per unit basis, we see that changes induced by the fertility rate shift the fiscal spaces more than what is suggested by the trend, while changes induced by the mortality rate shift it by less.

We compared the benchmark economy to four alternative ageing scenarios driven purely by either changes in the fertility rate, or changes in the mortality rate. We note that while both the fertility and mortality improvement over the 1980 to 2010 period contributed significantly to the decrease in gross and net fiscal spaces, the decrease over the 2010 to 2040 period will be increasingly driven by the fertility rate. When we compare the per unit contribution of the demographic changes, we find that for a given effect on the dependency ratio, the fertility rate driven case will experience a larger shift in the fiscal space than the mortality rate driven counterpart.

We note that while the net fiscal space accounts for the pension benefits contribution by the government, it does not include other age-related expenses – such as medical and age-care costs – that are also set to increase with ageing. Further, when we consider the political limitations in raising taxes to the corresponding revenue maximising levels (the
2040 uncommitted revenue maximised when the labour and capital tax rates are 0.60 and 0.55 respectively), the actual degree of fiscal manoeuvrability is likely much lower.

5.4 Alternative preferences

Following ?, we calibrate and compare the benchmark preferences specified above with two alternative specifications to test the robustness of our results. In the first case, we set the inverse of the IES ($\sigma$) to 1 as this reflects the recent shift in the consensus view and estimations. For instance, ? estimates the mean of the IES $1/\sigma = 1.25$ for Japan, in contrast to the higher value found by ?. We then set the Frisch labour supply elasticity $\phi = 3$ and labour supply is more elastic to changes in the wage rate. With these alternative CFE parameters, the preferences in Equation 6 can be written as

\[
\begin{align*}
    u(c_j, l_j) &= \log(c_j) - \kappa(1 - l_j)^{1 + \frac{1}{\phi}} \\
    u(c_j, l_j) &= \log(c_j) - \kappa(1 - l_j)^{1 + \frac{1}{\phi}}
\end{align*}
\] (12)

In the second case, we use Cobb-Douglas (C-D) preferences as it is one of the most commonly used functional forms and an appropriate benchmark. In particular, we use the following preferences:

\[
\begin{align*}
    u_{c_j, l_j} &= \rho \log(c_j) + (1 - \rho) \log(l_j)
\end{align*}
\] (13)

where $\rho$ is the leisure-consumption preference parameter. Under Cobb-Douglas preferences, the Frisch elasticity is given by $1/(n_j) - 1$, where $n_j = 1 - l_j$ and is no longer constant. Instead, the elasticity decreases as the labour supply increases (leisure decreases).

The budget constraint and other competitive equilibrium conditions of the model hold as before. We calibrate the models for the benchmark economy, and the results for the deep parameters under both the original and the alternative preference specifications are
<table>
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<tr>
<th>Variable</th>
<th>Description</th>
<th>Comments</th>
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</thead>
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<td></td>
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<td>$\bar{n} = 0.3$</td>
</tr>
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<td>Weight of labour</td>
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<tr>
<td>$\beta = 0.975$</td>
<td>Time discount factor</td>
<td>$SS/Y = 0.10$</td>
</tr>
<tr>
<td>$\psi = 0.33$</td>
<td>Pension replacement rate</td>
<td></td>
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<tr>
<td><strong>CFE preferences (Alternative)</strong></td>
<td></td>
<td></td>
</tr>
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<td>$\kappa = 3.53$</td>
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<td></td>
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<tr>
<td><strong>Cobb-Douglas preferences</strong></td>
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<td></td>
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<tr>
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<td>Weight of consumption</td>
<td>$\bar{n} = 0.3$</td>
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<td>$K/Y = 3.05$</td>
</tr>
<tr>
<td>$\psi = 0.33$</td>
<td>Pension replacement rate</td>
<td>$SS/Y = 0.10$</td>
</tr>
</tbody>
</table>

Table 3: Model parameter values for alternative preferences

given in Table 3. All other parameters for demographics, endowments, technology, and fiscal policy are kept the same as in Table 1.

First, we note that the common parameters for the alternative CFE and Cobb-Douglas cases are equal, $\beta = 0.962$ and $\psi = 0.33$. This similarity stems from the separability of the consumption and leisure in the utility function, and the common $log(c_j)$ term from $\sigma = 1$. When we compare the alternative CFE with the benchmark CFE case, the time discount factor is lower, with $\beta = 0.962$, while the disutility from work is higher, with $\kappa = 3.53$.

We then calculate the respective g-Laffer curves in the 2010 benchmark case under the two alternative parameter specifications. The results are plotted below in Figure 17 with the current revenue levels under the benchmark CFE preferences set to 100. First, we note that due to the similarity in the form and calibration results between the alternative CFE and Cobb-Douglas preferences, the Laffer curves for all three taxes are very similar. We also note that while all three specifications produce similar results at the lower tax rates, there are differences at the higher tax rates for labour and capital. In the labour curve, the slope of the Cobb-Douglas Laffer curve is greater than in both CFE preferences at the higher tax rates. This results from the changing Frisch elasticity, with households decreasing their labour supply less in response to increasing tax rates at the higher end. Consequently, total tax revenue lower falls slower in the Cobb-Douglas case. Further, for the labour Laffer curve, the maximum revenue raised is higher in the alternative CFE and Cobb-Douglas preferences than the benchmark CFE case. This is qualitatively different to the result produced in the representative-agent framework, where the labour Laffer
Figure 17: Comparing the 2010 g-Laffer curves under alternative preferences

curves under the benchmark CFE preferences had a higher peak for labour. This results from the difference in $\beta$ value needed to calibrate the economy in the OLG framework. As $\beta$ increases, the competitive interest rate decreases. This means that the tax revenue collected from capital is lower at every level, leading to a lower peak for the benchmark CFE case. In the capital Laffer curve, the benchmark CFE raises less at lower capital tax rates (compared to the Cobb-Douglas case), but has a higher peak at a higher tax rate. In contrast, the capital curves under the benchmark and alternative preferences in the representative-agent model are almost identical. The difference arises again from the calibrated $\beta$ value, with a higher $\beta$ implying households are patient and willing save more even as the return on capital decreases. This leads to a slower decrease in the tax base and the the total taxation revenue level is higher. The consumption Laffer curves for all three specifications are very similar, especially between the two CFE specifications. The Cobb-Douglas preferences produces the highest taxation revenue at the higher tax rates.

When we examine the pb-Laffer curves under the alternative preferences in Figure 18, the differences between the benchmark case and the alternative preferences, and also the alternative CFE and Cobb-Douglas preferences become more apparent for labour. For the labour pb-Laffer curve, the Cobb-Douglas preferences now generates significantly more revenue than both the alternative CFE and benchmark CFE preferences, with the net fiscal space for labour at 110.15 percent, 96.72 percent and 76.23 percent respectively. For capital, the Cobb-Douglas preferences now generates the most revenue, followed by the benchmark and then alternative CFE cases. For consumption, the differences remain minimal.

Overall, we find while altering the preferences does lead to differences, they capture the main idiosyncratic features of the Laffer curves within the OLG framework, including the asymmetric change in capital revenue, and the monotonically increasing consumption Laffer curve.
6 International comparison

7 Conclusion

Ageing will negatively affect a government’s fiscal balance through both expenditure and revenue channels, with spending on social security expected to increase, while the labour, capital and consumption tax bases shrink. To measure the government’s ability to meet the fiscal demands of ageing, we use the twin concepts of (i) gross fiscal space, which focuses on the government’s unused revenue generating capacities, and (ii) net fiscal space, which also accounts for the government’s age-related expenditure commitments.

We quantify the effects of ageing on the fiscal space through experiments that (i) alter one tax rate while keeping the other two fixed, or (ii) alter two tax rates – labour and capital – while keeping the consumption tax rate fixed. Altering one tax, all three Laffer curves shift downwards as a result of ageing. Capital is the most sensitive to ageing, with a 10 percentage point increase in the dependency ratio is associated with a 14 percent decrease in the gross fiscal space relative to the benchmark fiscal space. This sensitivity arises as ageing causes both a decrease in the capital stock and a decrease in the return on capital. Labour is the next sensitive, with fiscal space decreasing by 13 percent, with the decrease in effective labour supply partially softened by an increase in the wage rate. Consumption is the least sensitive, with fiscal space decreasing by only 4 percent at $\tau^c = 0.9$. Over the 2010 to 2040 period, the gross fiscal spaces will be lower by 36.98 percent, 33.85 percent, and 10.67 percent respectively. We find that despite the shrinking fiscal space, the government would be able to maintain 2010 levels of taxation revenue by increasing either one of its tax rates. Keeping the consumption tax fixed at 5 percent, the government can maximise total tax revenue by increasing both the labour and capital tax rates. The gross fiscal space is at 187.86 percent of current revenue levels in 2010 and will fall to 160.63
percent in 2040.

As the decrease in tax revenue alone does not capture the expenditure-side fiscal constraints resultant from demographic changes, we then examined the net fiscal space. We find that the net fiscal space is also more sensitive to demographic shifts and changes in the dependency ratio. Again, capital fiscal space is the most sensitive, with a 10 percentage point increase in the dependency ratio associated with an approximate 43 percent decrease in the net fiscal space. For labour and consumption, it is 23 percent and 5 percent respectively. If the government keeps current pension benefits and contribution rates constant, by 2040, the net fiscal space will only be 31.26 percent of current uncommitted revenue levels for labour, and -4.07 percent for capital. This means that raising the capital tax alone will be unable to maintain the current fiscal manoeuvrability. If both capital and consumption taxes were altered, the total net fiscal space will fall to 60.63 percent of current revenues.

Our analysis on the individual population dynamics finds that while both the improvements in life expectancy and decreases in fertility rates have contributed in shrinking the fiscal spaces over the 1980 to 2010 period, future fiscal space movements will be predominantly driven by the fertility rate, as mortality improvements are slowing down while fertility rates remain below replacement rate. When we account for the size of the changes in the demographic variables, we find that there is a linear relationship between the dependency ratio and the fiscal space decrease, with a dependency ratio change driven purely by fertility rate changes affecting the fiscal space more than one driven purely by mortality rate changes.

While we have quantified the maximum revenue that can be raised and used this as a measure of fiscal space, we note there are several limitations to this analysis. Firstly, we have kept policies constant in the simulations and altered the population dynamics only. This does not account for the corrective policy reactions to the ageing trend – including increasing the retirement age, labour force participation and pension reform – that are currently under consideration. Secondly, while the peaks of the Laffer curves offer the theoretical maximum revenue that can be raised, the practical “effective” maximum that is politically feasible is likely much lower than the theoretical maximum. Furthermore, as the peak of the Laffer curve does not necessarily maximise the welfare of the population, and the results do not imply that raising labour and capital tax rates would be optimal.
8 Appendix

8.1 An algorithm to solve the model

We use dynamic programming to solve the model in MATLAB. The general procedure can be summarised as follows:

1. Discretise the state space of assets as \([a_0, \ldots, a_{\text{max}}]\), the grids for leisure as \([l_{\text{min}}, \ldots, l_{\text{max}}]\) (where \(l_{\text{min}} \geq 0\) and \(l_{\text{max}} \leq 1\)), and the taxation rate as \([0, \ldots, \tau_{\text{max}}]\) (where \(\tau_{\text{max}} \leq 1\)).

2. Choose a combination of \(\tau^l\), \(\tau^k\), and \(\tau^c\) by either setting 2 constant and looping 1 over the taxation grid, or setting 1 constant and looping 2 over the taxation grid.

3. Guess for initial wage rate \(w\), post-tax interest rate \(R^k\), pension benefit \(pb\), bequest \(b\) and capital stock \(K\).

4. Work backwards from \(J\) to period 1 to obtain optimal decision rules for consumption, savings, labour supply, and the value and marginal value functions of the household.

5. Iterate forwards to obtain the vector of optimal consumption, savings, and labour supply choices for the households across different generations, using \(a_1 = 0\) and following the optimal decision rules.

6. Work out the distribution of households within the economy \(\mu_j\) using the exogenous fertility rates and survival probabilities, before aggregating for the capital and effective labour in the economy; Work out the new wage rate and interest rates from the market clearing conditions, new bequest, and pension values; Balance government and social security budgets to determine endogenous fiscal variables.

7. Check the relative changes in the aggregate variables after each iteration and stop the algorithm when the change is sufficiently small. Otherwise, repeat from steps 3 to 6.

8.2 Tax revenue decomposition and tax bases
Figure 19: Tax revenue decomposition and tax bases for benchmark g-Laffer curve

Figure 20: Tax revenue decomposition and tax bases for benchmark pb-Laffer curve