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**EVALUATING POLICY: WELFARE WEIGHTS  
AND VALUE JUDGEMENTS**

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# Evaluating Policy: Welfare Weights and Value Judgements

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## Abstract

This paper is concerned with the use of social welfare functions in evaluating changes. In particular, it considers suggestions that welfare weights to be used in comparing the gains and losses of different individuals (or other appropriate units of analysis), and a social time preference rate for use in cost benefit evaluation, can be estimated either from consumers' behaviour or from the judgements implicit in tax policy. It is suggested that results are highly sensitive to the context and model specification assumed. More importantly, the argument that an estimated elasticity of marginal utility or time preference rate *should* be used in policy evaluations fails to recognise that fundamental value judgements are involved. Various estimates may be of interest, but they cannot be used by economists to impose value judgements. The main contribution economists can make is to examine the implications of adopting a range of alternative value judgements.

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# 1 Introduction

Any attempt to answer the question, ‘when is a change an improvement?’, faces the fundamental difficulty that it cannot avoid the use of value judgements. Hence complete agreement in any particular context – say the effect of a proposed change to a tax and transfer system – is most unlikely, even if there are no losers.<sup>1</sup> The approach adopted in economics is to specify explicit value judgements in a formal manner, using a social evaluation function or, following Samuelson (1947), a ‘Social Welfare Function’. Crucially, this function formally expresses the value judgements of a fictional judge or policy maker. It is not, despite the use of the term ‘social’, intended to represent any kind of aggregate or representative views of society.<sup>2</sup> Indeed the judge is considered to be an independent person who is not affected by the outcomes.

Social welfare functions are used in static contexts, involving a distribution over income units, and in dynamic contexts, involving a distribution over time periods. In attaching ‘welfare weights’ to each unit or time period, the concept of the elasticity of the marginal valuation of income (or consumption, depending on the welfare metric adopted) plays an important role. Many contributions to the literature appear to regard this elasticity as something that can be objectively measured. For reviews of various approaches to measuring or estimating this elasticity, see Pearce and Ulph (1998), Cowell and Gardiner (1999) and Evans (2005).

The aim of the present paper is to review a number of frameworks in which the elasticity of marginal valuation is central. Stress is placed on the need to distinguish these contexts and models clearly, in order to avoid the possible inappropriate ‘transfer’ of a value from one context to another. In particular, it is argued that the central concept of the elasticity cannot be measured objectively but involves value judgements. Hence the role of economists is not to propose the use of particular values, which is equivalent to imposing

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<sup>1</sup>The Pareto criterion has little practical use as it refuses to pass judgement where losers exist, and is certainly not a value-free criterion. There are also well-known problems with the use of ‘potential Pareto improvements’.

<sup>2</sup>The extreme case of a ‘representative agent’, mentioned below, is the exception where the welfare function corresponds to the utility function of the fictional representative.

judgements, but to examine the implications of adopting alternative value judgements. The view put forward here is in line with that of Robbins (1935, p.148) when he argued, ‘between the generalisations of positive and normative studies there is a logical gulf fixed which no ingenuity can disguise and no juxtaposition in space or time bridge over’. This contrast with those, such as Evans (2005), who suggested that the UK Treasury recommendation of a value for the elasticity of around unity, when forming distributional welfare weights or computing a social time preference rate in cost-benefit studies, is too low. Similarly, Pearce and Ulph (1998, p. 282) argued that the UK Treasury recommended discount rate at the time was too high. They argued, ‘we find it impossible to support the continued use of rates in the region of 6% for the UK. Such rates are far too high’. In each of these examples, the view was presented as a finding, rather than a value judgement.

First, section 2 briefly introduces the form of social welfare function and welfare weights widely adopted in the literature on policy evaluation. Some simple properties of welfare functions are typically specified, with the hope that while they cannot be expected to represent any kind of consensus, they are at least likely to appeal to a large number of people. It is in this spirit that the form of evaluation function widely adopted reflects adherence to value judgements such as the ‘principle of transfers’ (whereby a transfer from a richer to a poorer person is judged to produce an ‘improvement’<sup>3</sup>) as well as being individualistic, additive and Paretean. The first problem is to select a welfare metric and a unit of analysis, with both choices involving value judgements. An additive social welfare function is typically formed as an appropriate weighted sum. Thus the problem is to specify precisely how those weights, referred to as ‘welfare weights’, are formed.<sup>4</sup>

Section 3 examines the estimation of a value of the elasticity that is thought to be implicit in an income tax structure. Attempts to impute such

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<sup>3</sup>This is conditional on the transfer being such that the transferee does not become richer than the transferor.

<sup>4</sup>For ‘classical utilitarians’, the evaluation criterion was simply the sum, over all individuals, of utilities. The welfare weights were thus all unity. However, substantial differences of viewpoint exist concerning both the welfare metric to be used and the value judgement that the distribution among individuals is irrelevant, such that only the aggregate matters.

a value include Mera (1969), Stern (1977), Christiansen and Jansen (1978), Moreh (1981), Brent (1984), Cowell and Gardiner (1999) and Evans (2005). This approach, based on an assumption of equal absolute sacrifice as a policy objective, necessarily produces for a progressive tax a value of the elasticity of marginal valuation in excess of unity. Section 3 considers whether this approach can actually provide reliable evidence of implicit judgements, or could legitimately be used as the foundation of an argument in favour of using those values. The sacrifice approach is contrasted in section 4 with a standard optimal linear tax model. This gives progressive taxes with values of the elasticity well below unity, demonstrating the sensitivity of any implied value to the precise objective assumed on the part of policy makers and to the model which is assumed to generate outcomes.

Section 5 briefly gives some idea of the implications, in terms of value judgements, of adopting alternative values of the elasticity. This involves the idea, familiar from the literature on inequality measurement, of the ‘leaky bucket’ experiment, which makes explicit the tolerance of losses when making income transfers between individuals. The main conclusions are in section 6, where the fundamental difference between what ‘is’ and what ‘should’ be is stressed. Given the need to make value judgements, economists have no special qualifications or authority to impose their own judgements on others, and cannot use ‘estimates’ as support for their views. Economists have an obligation to make value judgements explicit, and their role is to examine the implications of adopting a range of such judgements. Having been presented with alternative results, readers can then form their own opinions. Ultimately, the paper aims to make clear some important distinctions which are often confused in the literature.

## **2 Social Evaluations**

As mentioned in section 1, a standard approach to the evaluation of a policy or other change is to use a social welfare, or social evaluation, function. This is often defined as individualistic, additive and Paretean in form. This section discusses the form of social welfare function commonly used in the evaluation

literature and the associated elasticity of marginal valuation. First, single-period comparisons are discussed, and this is followed by a brief examination of the multi-period context.

## 2.1 Single-Period Comparisons

Define the contribution to social welfare of unit  $h$  as  $W(x_h)$ , where  $x_h$  is some measure relating to that unit. A prior decision must be made regarding the unit of analysis: this important issue is not discussed here, and for simplicity the unit is referred to below as the individual. The definition of  $x$ , the ‘welfare metric’, is problematic and usually depends on the context. It is variously defined as income, consumption, utility, or money metric utility, where each concept may, in addition, be expressed in ‘adult equivalent’ terms, and such equivalence scales themselves involve difficult value judgements. Social welfare,  $W_S$ , is thus defined as:

$$W_S = \sum_{h=1}^H W(x_h) \quad (1)$$

and any change, which may or may not be induced by a policy variable or variables (such as an income tax structure or a set of indirect tax rates), gives rise to a total change in social welfare of:

$$dW_S = \sum_{h=1}^H \frac{\partial W(x_h)}{\partial x_h} dx_h \quad (2)$$

and letting  $v_h = \frac{\partial W(x_h)}{\partial x_h}$  denote the ‘marginal valuation’, the contribution to the change in  $W_S$  of a change in  $x_h$ , (2) becomes:

$$dW_S = \sum_{h=1}^H v_h dx_h \quad (3)$$

The term  $v_h$  represents the ‘welfare weight’ attached to the  $h$ th individual. An aversion to inequality on the part of the ‘policy maker’ is specified by an assumption that  $W(\cdot)$  is concave, so that it satisfies the ‘principle of transfers’. A measure of relative inequality aversion,  $R$ , is therefore based on

the concavity measure:

$$R = -\frac{xd^2W(x)/dx^2}{dW(x)/dx} \quad (4)$$

This is equivalent to the ‘elasticity of marginal valuation’,  $(\frac{dv}{dx})(\frac{x}{v})$ . This elasticity plays an important role in what follows. Importantly, the term ‘marginal valuation’ is used here rather than using ‘marginal utility’, since  $W(x)$  is not a utility function. The function  $W(x)$  actually represents, as mentioned above, the contribution to ‘social welfare’ (that is to the social evaluation function) of the  $h$ th person, however  $x$  is defined. In some contexts (such as the optimal tax literature)  $x$  actually represents utility (or some money metric measure of utility), and in the marginal indirect tax reform literature it represents indirect utility.<sup>5</sup>

If  $W$  is specified, as in the vast majority of studies, as:

$$W(x) = \frac{x^{1-\varepsilon_s}}{1-\varepsilon_s} \quad (5)$$

for  $\varepsilon_s \neq 1$ , and  $W(x) = \log x$ , where  $\varepsilon_s = 1$ . In this case, substitution in (4) gives  $R = \varepsilon_s$  and  $\varepsilon_s$  thus reflects a constant degree of relative inequality aversion of the judge or policy maker.

Importantly,  $\varepsilon_s$  is not an objective measure relating to individuals in society, but reflects the subjective value judgements of a fictional policy maker who is evaluating the effects of alternative policies or outcomes. There is therefore no reason why a value of  $\varepsilon_s$ , to be imposed by economists in making

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<sup>5</sup>In the literature on marginal tax reform,  $W$  is defined in terms of (indirect) utilities,  $V_h$ , so that  $W = \sum_{h=1}^H W(V_h)$  and the effect of a change in the price of good  $i$ ,  $p_i$ , say arising from a tax change, is:

$$\frac{\partial W}{\partial p_i} = \sum_{h=1}^H \frac{\partial W(V_h)}{\partial V_h} \frac{\partial V_h}{\partial p_i}$$

From Roy’s Identity,  $x_{hi} = -(\partial V_h / \partial p_i) / (\partial V_h / \partial m_h)$ , where  $m$  is total expenditure, and:

$$\frac{\partial W}{\partial p_i} = \sum_{h=1}^H \left( \frac{\partial W}{\partial V_h} \frac{\partial V_h}{\partial m_h} \right) x_{hi}$$

In specifying the term in brackets,  $W$  is usually re-interpreted in terms of total expenditures, with  $W(m_h) = m_h^{1-\varepsilon} / (1-\varepsilon)$  and so the term in brackets becomes  $m_h^{-\varepsilon}$ .

comparisons, could be estimated using information from studies of household budgets. This suggestion is sometimes made even though the demand studies use quite different utility functions. For example, elasticities obtained on the basis of the linear expenditure system (LES) are discussed by Evans (2005, pp. 204-206). The use of the LES necessarily produces a value of  $\varepsilon$  well above unity, since it can be interpreted as the ratio of total expenditure to supernumerary expenditure, that is, expenditure above a ‘committed’ amount.

## 2.2 Multi-Period Contexts

Evaluations are also made in a multi-period context. Consider first a single individual where  $C_t$  represents consumption (or some other suitable metric) in period  $t$ . An additive utility function defined over  $T$  periods, where  $\rho$  is the ‘pure time preference’ rate of the individual is thus:

$$U_T = \sum_{t=1}^T \left( \frac{1}{1+\rho} \right)^{t-1} U(C_t) \quad (6)$$

Consider periods 1 and 2. The pure time preference, or impatience, rate measures the extent to which the slope of an indifference curve, at a point where  $C_1 = C_2$ , deviates from a downward sloping (from left to right) 45 degree line.<sup>6</sup> It reflects impatience, or an ‘aversion to waiting’ on the part of the individual, whereby faced with a constant consumption stream the individual is prepared to give up more than one unit of  $C_2$  in order to obtain one more unit of  $C_1$ . For any combination of  $C_1$  and  $C_2$ , the marginal rate of substitution between consumption in the two periods,  $MRS_{c_1,c_2}$ , is the absolute value of the slope of the individual’s indifference curve, and is thus  $\frac{\partial U_T / \partial C_1}{\partial U_T / \partial C_2}$ .

The discount rate,  $r$ , at any combination of  $C_1$  and  $C_2$ , is defined as:

$$1 + r = MRS_{c_1,c_2} \quad (7)$$

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<sup>6</sup>The terminology is not universally accepted. For example, Pearce and Ulph (1998), refer to this simply as the ‘rate of time preference (the rate at which *utility* is discounted’, and decompose it into a ‘pure rate’ and a term reflecting the rate of growth of life chances. They refer to what is below called the ‘social time preference rate’ as the ‘consumption rate of interest’.

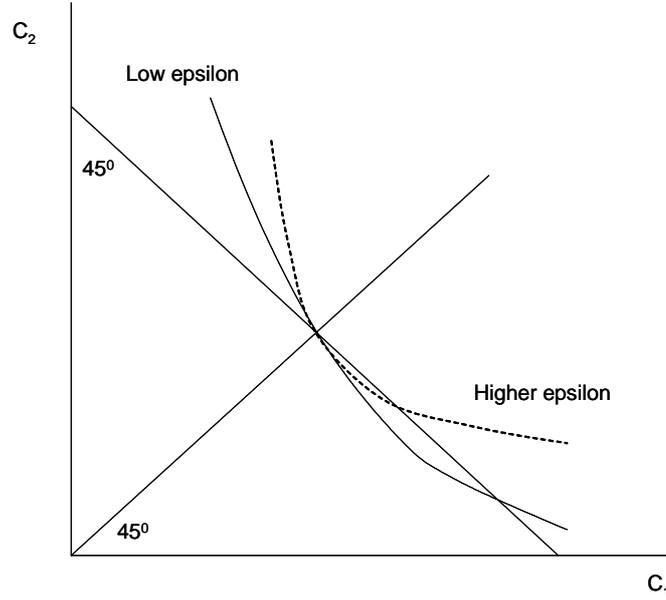


Figure 1: Time Preference

Supposing that  $U(C_t)$  is the iso-elastic form:

$$U(C_t) = \frac{C_t^{1-\varepsilon_p}}{1-\varepsilon_p} \quad (8)$$

where  $\varepsilon_p$  is the absolute value of the individual's elasticity of the marginal utility of consumption.<sup>7</sup> Then:

$$MRS_{c_1, c_2} = \left( \frac{C_2}{C_1} \right)^{\varepsilon_p} (1 + \rho) \quad (9)$$

An example is shown in Figure 1. At the point of intersection with the 45 degree line from the origin, along which consumption is equal in both periods, the solid indifference curve shown is steeper than the downward sloping 45 degree line, indicating a degree of pure time preference. The convexity of the indifference curve is affected by the value of  $\varepsilon_p$ , so that the solid curve reflects a lower value than the broken curve. If  $\varepsilon_p = 0$ , the indifference curves are straight lines and the individual's optimal position would be a

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<sup>7</sup>It is also the absolute value of the elasticity of the marginal rate of substitution with respect to the ratio of consumption levels.

corner solution, consuming everything either in period 1 or 2, depending on whether the market rate of interest (assuming equal borrowing and lending rates) is less than or greater than the pure time preference rate. In general the individual's optimal position is a tangency where the market rate of interest equals the time preference rate.

A convenient expression for the discount rate can be obtained using an approximation which holds when the various rates are small. Let  $g$  denote the growth rate of consumption, so that  $\frac{C_2}{C_1} = 1 + g$  and:

$$r = (1 + g)^{\varepsilon_p} (1 + \rho) - 1 \quad (10)$$

Expanding  $(1 + g)^{\varepsilon_p}$  and neglecting squared and higher-order powers gives  $(1 + g)^{\varepsilon_p} \approx 1 + \varepsilon_p g$ .<sup>8</sup> Hence  $r = (1 + \varepsilon_p g) (1 + \rho) - 1$ , and making the further assumption that  $\varepsilon_p g \rho \approx 0$ , the individual's discount rate is:

$$r = \rho + \varepsilon_p g \quad (11)$$

Hence the discount rate is equal to the pure time preference, or impatience, rate plus the product of the growth rate of consumption and the individual's (absolute) elasticity of the marginal utility of consumption. The growth rate affects the difference between consumption in the present and future, and  $\varepsilon_p$  reflects an aversion to inequality between periods (not impatience to consume in the present), and the combination of these involves an addition to pure time preference.

The above applies to an individual person; hence the  $p$  subscript. However, there is a substantial literature which is based on the concept of a 'representative individual'. In this case, optimal plans are unambiguously based on the preferences of this representative individual. Typically, such an individual is assumed to exist without consideration of aggregation requirements.<sup>9</sup>

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<sup>8</sup>Writing  $(1 + g)^\varepsilon = \alpha_0 + \alpha_1 g$ , so that setting  $g = 0$ ,  $\alpha_0 = 1$ . Further  $d(1 + g)^\varepsilon / dg = \varepsilon (1 + g)^{\varepsilon-1} = \alpha_1$ , and setting  $g = 0$ ,  $\alpha_1 = \varepsilon$ . Alternatively take logarithms of  $1 + r$  and use, for each relevant term, the approximation  $\log(1 + y) = y$ .

<sup>9</sup>It is known that quasi-homothetic (Gorman) preferences are required for aggregates to be interpreted as arising from the preferences of a representative individual. In this case demands are linear, with a common slope, although intercepts may differ to allow for, say, demographic factors.

In the absence of a representative individual, optimal plans may be regarded instead as being based on a social welfare function, whereby the  $C$ s now represent aggregates and:

$$W_T = \sum_{t=1}^T \left( \frac{1}{1 + \rho_m} \right)^{t-1} W(C_t) \quad (12)$$

In this case,  $W(C_t)$  represents the contribution of period  $t$ 's aggregate consumption to the evaluation function, and  $\rho_m$  represents the pure time preference, or impatience, rate of the judge. The corresponding iso-elastic function is thus:

$$W(C_t) = \frac{C_t^{1-\varepsilon_m}}{1-\varepsilon_m} \quad (13)$$

In this context the discount rate is commonly referred to as the 'social time preference rate',  $r_m$ , and is given, where  $g_m$  is the aggregate growth rate of consumption, by:

$$r_m = \rho_m + \varepsilon_m g_m \quad (14)$$

Estimates of  $\varepsilon_p$  have been obtained from studies of saving behaviour over time, for a sample of individuals.<sup>10</sup> However, in the general situation where there are many individuals and an evaluation is required in terms of a social welfare function, the term  $\varepsilon_m$  represents the value judgements of a judge or policy maker. Therefore, there is no reason to impose  $\varepsilon_m = \varepsilon_p$ ; the former involves a value judgement and there is no logical connection between the two rates.<sup>11</sup>

In thinking about the appropriate values for  $\varepsilon_m$  in this context, quite different considerations apply compared with the case of single-period distributional judgements involving inequality aversion,  $\varepsilon_s$ , discussed earlier. The term  $\varepsilon_m$  is, in the multi-period framework, more accurately interpreted in

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<sup>10</sup>On the approach, with a review of alternative estimates, see Pearce and Ulph (1998).

<sup>11</sup>Marina and Scaramozzino (1999, p.6) provided an interesting analysis of growth in an overlapping generations framework. They stated that, 'a social rate of pure time preference is justifiable on purely ethical grounds'. A clearer statement of what the authors showed is that *if* the objective of maximising average steady-state consumption per capita is adopted, then *an implication* of this *ethical value judgement*, combined with a model containing productivity and population growth, is that positive time preference exists that does not reflect myopia.

terms of an aversion on the part of the judge towards variability – inequality between periods rather than inequality between persons. Yet unfortunately these terms are often conflated in the literature, where discussion proceeds as if  $\varepsilon_m$ ,  $\varepsilon_p$  and  $\varepsilon_s$  were the same thing.

### 3 Taxation and Equal Absolute Sacrifice

In using  $\varepsilon$  values to evaluate social welfare functions, or carry out cost-benefit evaluations, it is important to ensure that they are within a range that is considered appropriate by a reasonable number of users of the results. Hence questionnaire studies have been designed to elicit information about individuals' value judgements. Nevertheless, those conducting the surveys do not suggest that they can produce any single value that should be used in policy evaluations.<sup>12</sup>

Alternatively, several studies have attempted to estimate the implicit value judgements revealed by tax and transfer policies, and have suggested that they provide a guide to  $\varepsilon$  values which should be applied in policy evaluations.<sup>13</sup> These attempts, such as Stern (1977), Cowell and Gardiner (1999) and Evans (2005), are examined in this section. The approach is based on the principle of equal absolute sacrifice. It also takes the view that incomes are exogenously given, rather than arising from labour supply behaviour (subject to endowments and education which give rise to individual productivities).

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<sup>12</sup>It was in this spirit that the questionnaire study of Amiel *et al.* (1999) was carried out; there was no pretence that resulting values represent estimates which 'should' be used in social evaluations. A substantial number of respondents did not adhere to the constant relative inequality aversion form. In addition, Amiel and Cowell (1992, 1994) have found that a large number of respondents do not actually share the value judgements that are explicit in the most common forms of social welfare function used in evaluation work. This presents a challenge to produce alternative flexible specifications. Early questionnaire studies were carried out by Glesjer *et al.* (1977), and Gevers *et al.* (1979), although no attempt was made to estimate precise specifications of distributional preferences.

<sup>13</sup>A different view is that such estimates may be useful in checking whether there is in fact any correspondence between policies and basic value judgements of policy makers. Given the complexities involved in tax policy design, it may be useful to know if a particular structure is associated with implicit judgements that may be very different from those actually held (though seldom made explicit). This view was expressed by van de Ven and Creedy (2005) when examining adult equivalence scales implicit in tax and transfer systems.

It seems likely that the approach has been chosen largely for its simplicity.<sup>14</sup>

Suppose  $x$  represents ‘income’ and the tax function is  $T(x)$ . Equal absolute sacrifice requires, for all  $x$ , that the absolute difference between pre-tax and post-tax utility is the same for all individuals. Hence:

$$U(x) - U(x - T(x)) = k \quad (15)$$

where  $U(\cdot)$  represents a utility function which is considered to be the same for all individuals. The parameter  $k$  depends on the amount of revenue per person. A standard assumption is that  $U$  takes the iso-elastic form discussed above, whereby  $U(x) = x^{1-\varepsilon_\tau} / (1 - \varepsilon_\tau)$  for  $\varepsilon_\tau \neq 1$ . Young (1987) actually showed that this form is required if an indexation requirement is imposed on the tax structure in addition to equal sacrifice. But of course fiscal drag is a common, indeed almost universal, feature of income tax structures.

The method is therefore to estimate the value of  $\varepsilon_\tau$ , where it is simply assumed that policies are actually based on a view by the fictional policy-maker that all utility functions take identical iso-elastic functional forms, combined with equal absolute sacrifice as an objective.<sup>15</sup> The combination of equal absolute sacrifice with the iso-elastic function gives, from (15) above:

$$\frac{x^{1-\varepsilon_\tau}}{1 - \varepsilon_\tau} - \frac{(x - T(x))^{1-\varepsilon_\tau}}{1 - \varepsilon_\tau} = k \quad (16)$$

Differentiation and simplification gives, as in Evans (2005, p.207), the result that:

$$\log(1 - T'(x)) = \varepsilon_\tau \log\left(1 - \frac{T(x)}{x}\right) \quad (17)$$

where  $T'(x)$  and  $T(x)/x$  are marginal and average tax rates. Alternative approaches have been used given the expression in (17). It has been used to carry out ordinary least squares regressions using tax functions,

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<sup>14</sup>Those using the approach to ‘estimate’  $\varepsilon$  have tended to ignore the objections raised by Edgeworth and others concerning the various interpretations of sacrifice theories. This does not apply to those, such as Richter (1983) and Young (1987) who were interested only in deriving the implications of various axioms.

<sup>15</sup>A somewhat different view would replace  $U(x)$  with  $W(x)$ . Thus, as with inequality measurement, a judgement is made regarding the welfare metric, and then a view is taken about variations in  $x$ . This judgement is quite separate from the way individuals may themselves view such variations.

so that  $\varepsilon_\tau$  and its standard error are obtained as a regression coefficient.<sup>16</sup> In practice there are also serious issues relating to the definition of  $x$ , but these need not be considered here. Alternatively, (17) is rearranged to get  $\varepsilon_\tau = \log(1 - MTR)/\log(1 - ATR)$ , and different  $\varepsilon_\tau$  are obtained and compared at different income levels.

The first point to stress regarding this approach is that it automatically produces a value of  $\varepsilon_\tau$  in excess of unity for a progressive tax system, for which the marginal tax rate exceeds the average tax rate.<sup>17</sup> The values of  $\varepsilon_\tau$  obtained in this way are thus severely constrained by the specification of the objective of equal absolute sacrifice.

It is thus important to ask whether it is sensible to model the tax structure as if it arose from equal absolute sacrifice. Rather than simply estimating  $\varepsilon_\tau$ , using an untested assumption about objectives, consider the nature of the implied tax function. For  $\varepsilon_\tau > 1$ , equation (16) can be rearranged as:<sup>18</sup>

$$T(x) = x - \{x^{1-\varepsilon_\tau} - k(1 - \varepsilon_\tau)\}^{1/(1-\varepsilon_\tau)} \quad (18)$$

The marginal tax rate is:

$$\frac{dT(x)}{dx} = 1 - x^{-\varepsilon_\tau} \{x^{1-\varepsilon_\tau} - k(1 - \varepsilon_\tau)\}^{\frac{\varepsilon_\tau}{1-\varepsilon_\tau}} \quad (19)$$

Hence the relevant question, before attempting to infer inequality aversion, is to ask whether (18) is a reasonable approximation to actual tax structures. The coefficient  $k$  is determined by the amount of revenue raised by the tax. Suppose that  $x$  follows a lognormal distribution with mean and variance of logarithms of  $\mu = 10$  and  $\sigma^2 = 0.5$  respectively. These values imply an arithmetic mean income of \$28,282. Suppose it is required to raise revenue per person of \$10,000 and that  $\varepsilon_\tau = 1.5$ . Using a numerical iterative search procedure, it is found that this requires  $k = 0.0025$ .<sup>19</sup> This gives the

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<sup>16</sup>There is some difference of opinion over whether to include a constant in the regression. Compare Cowell and Gardiner (1999) and Evans (2005), who also use different income measures.

<sup>17</sup>This was discussed by Edgeworth (1897) and formally shown by Samuelson (1947).

<sup>18</sup>In stating this result, Young (1987, p. 212) rewrote  $-k(1 - \varepsilon)$  as  $\lambda^{1-\varepsilon}$ , so that the tax function compares with a constant elasticity of substitution form.

<sup>19</sup>This is based on a simulated population obtained from 5000 random draws from the assumed income distribution.

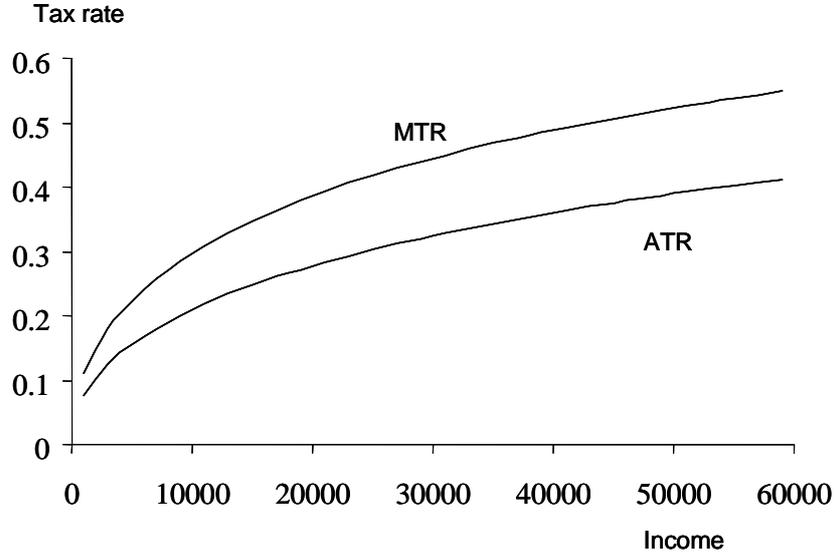


Figure 2: Marginal and Average Tax Rates: Equal Absolute Sacrifice

schedules in Figure 2. Of course, in practice tax functions are multi-step functions with ranges where the marginal rate is constant. In structures like that in the UK, there is a ‘standard rate’ which applies over a wide range of taxable income; the above function obviously has difficulty capturing this range. But it is probably sufficiently flexible to give a reasonable approximation above the standard rate.

However, other flexible tax functions can easily be produced with variants of the basic model. The objective of equal absolute sacrifice may, for example, be combined with a welfare function displaying constant absolute inequality aversion, such that:

$$W = 1 - \exp(-\alpha x) \quad (20)$$

This has constant absolute aversion of  $-(d^2W(x)/dx^2)/(dW(x)/dx) = \alpha$ .<sup>20</sup> Substitution of this form gives a tax function of the form:

$$T(x) = x + \frac{1}{\alpha} \log \{k + e^{-\alpha x}\} \quad (21)$$

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<sup>20</sup>Here the slopes of social indifference curves relating to any two individuals are constant for a given absolute difference between incomes of the two people, contrasting with the case of relative aversion where the slope is constant for a given ratio of incomes.

The marginal tax rate is therefore:

$$\frac{dT(x)}{dx} = 1 - (1 + ke^{\alpha x})^{-1} \quad (22)$$

This is quite flexible and can be made to display rate schedules similar to those above. Indeed, several other forms are candidates to approximate tax schedules. Dalton (1954, pp.68-70) discussed several examples using alternative utility functions and sacrifice principles, and showed that if equal absolute sacrifice produces progression, equal proportional sacrifice produces a more progressive tax structure.<sup>21</sup>

Hence some scepticism must be attached to the usefulness of estimates obtained on the basis of an arbitrary utility function and sacrifice principle. Furthermore, any such estimates of  $\varepsilon_\tau$  cannot be interpreted as providing values which ‘should’ be used in any particular policy evaluation, that is, where values of  $\varepsilon_m$  or  $\varepsilon_s$  are imposed. There is no alternative to simply accepting that value judgements are required and the best attitude of professional economists is to report a range of results based on alternative value judgements, rather than suggesting that particular values can be attributed to ‘society’ and duly estimated objectively.

All tax functions such as those considered in this section have in common, since they are based on a form of sacrifice principle, the limitation that they apply onto to positive taxes. They can thus relate at best to a small component of a much broader set of taxes and transfers. The following section discusses the optimal tax approach which allows for a transfer payment, equivalent to a negative income tax.

## 4 Maximising a SWF

This section contrasts the minimum sacrifice approach with the standard form of the optimal linear income tax problem, where it is required to select the values of a social dividend,  $a$ , and constant marginal tax rate,  $t$ , in

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<sup>21</sup>In an early study, Preinreich (1948) considered the form of the utility schedule consistent with the US tax legislation, without imposing a specific functional form over the whole income range. He assumed equal proportional sacrifice.

order to maximise a social welfare function  $W$ . In a pure transfer system this must satisfy the government's budget constraint that  $a = t\bar{y}$ , where  $\bar{y}$  is arithmetic mean earnings. At the same time each individual maximises utility  $U_h = U_h(c_h, \ell_h)$  where  $c$  and  $\ell$  respectively denote consumption (net income) and leisure, expressed as a proportion of total time available. If the price of consumption is normalised to unity and the wage obtained by person  $h$  is  $w_h$ , each individual's budget constraint is expressed as  $c_h = w_h(1-t)(1-\ell_h) + a$ . In view of the government's budget constraint, there is only one degree of freedom in the choice of tax parameters. In specifying the extent of heterogeneity in the model, it is usual to assume that there is an exogenous distribution of wage rates,  $w_h$ , but that all individuals have the same tastes and face the same tax rates and commodity prices.

Typically the welfare function to be maximised is expressed in terms of utilities, so that  $W_U = W(U_1, \dots, U_H)$ . However, results are not invariant with respect to the cardinalisation used, so that an alternative is to use a money metric welfare measure that is not subject to this limitation. In general, where the direct utility function is written as  $U = U(x)$ , where  $x$  is a vector of consumption levels, and the indirect form is written as  $V = V(p, y)$ , where  $y$  is income (the budget) and  $p$  is a vector of prices, a money metric utility measure,  $y_E$ , is defined by:

$$V(p_r, y_E) = V(p, y) \quad (23)$$

The expenditure function  $E(p, U)$ , gives the minimum expenditure required to achieve utility level  $U$  at prices,  $p$ . Money metric utility can therefore be expressed as:

$$y_E = E(p_r, V(p, y)) \quad (24)$$

and the social welfare function becomes  $W_E = W(y_{E,1}, \dots, Y_{E,N})$ .<sup>22</sup>

Comparisons can be made using simulation methods, based on the constant inequality aversion welfare function discussed earlier. Suppose that

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<sup>22</sup>In the standard two-good Cobb-Douglas case of consumption of  $x_1$ , and of  $x_2$  of the goods at given prices  $p_1$  and  $p_2$  respectively,  $U = x_1^\alpha x_2^{1-\alpha}$ , the expenditure function is:

$$E(p, U) = U \left( \frac{p_1}{\alpha} \right)^\alpha \left( \frac{p_2}{1-\alpha} \right)^{1-\alpha}$$

wage rates follow the lognormal distribution with variance of logarithms of 0.5. Furthermore, assume that  $\alpha = 0.6$ . Assume that individuals' utility functions are Cobb-Douglas, where:

$$U_h = c_h^\alpha \ell_h^{1-\alpha} \quad (25)$$

and all individuals have the same value of  $\alpha$ .<sup>23</sup> In computing an optimal tax rate, the first step is to solve for the transfer payment,  $a$ , given values of  $t$ , using the government's budget constraint,  $a = t\bar{y}$ , for a pure transfer scheme. Since  $\bar{y}$  depends on the transfer,  $a$ , this constraint must be solved iteratively.<sup>24</sup>

A simulated population of 5000 individuals was generated in order to calculate the optimum tax rates shown in Table 1.<sup>25</sup> The optimal tax rate is reported for variations in values of the relative inequality aversion parameter,  $\varepsilon_o$ , using the two social welfare functions:

$$W_U = W(U_1, \dots, U_N) = \sum_{h=1}^H \frac{U_h^{1-\varepsilon_o}}{1-\varepsilon_o} \quad (26)$$

and:

$$W_E = W(y_{E,1}, \dots, y_{E,N}) = \sum_{h=1}^H \frac{y_{E,h}^{1-\varepsilon_o}}{1-\varepsilon_o} \quad (27)$$

For infinitely large  $\varepsilon_o$ , the solution approaches the maxi-min, which maximises the welfare associated with the poorest person, so that in each case the optimal marginal rate approaches the value that maximises the threshold,  $a$ . This rate is found to be given by  $t = 0.64$ .

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and money metric utility is:

$$y_E = y \left( \frac{p_{r1}}{p_1} \right)^\alpha \left( \frac{p_{r2}}{p_1} \right)^{1-\alpha}$$

<sup>23</sup>The Cobb-Douglas case has a constant elasticity of substitution between leisure and consumption of unity, so the optimal linear tax rates are relatively high.

<sup>24</sup>The approach uses the result that  $\bar{y} = \alpha\bar{w}G_\xi(w_L)$ , where  $\bar{w}$  is the arithmetic mean wage rate. Further,  $G_\xi(w_L) = \{1 - F_1(w_L)\} - (\xi/\alpha\bar{w})\{1 - F(w_L)\}$   $F(w)$  is the distribution function and  $F_1(w)$  is the first moment distribution; for details of these functions, see Creedy (1996). For those who work, earnings are given by  $y = \alpha w - \xi$ , with  $\xi = a(1 - \alpha)/(1 - t)$ .

<sup>25</sup>In each case, the value of  $t$  was varied in steps of 0.005, and a grid search was carried out to find  $t$  giving maximum social welfare.

Table 1: Optimal Linear Tax Rates

$\varepsilon_o$	Optimal tax rate using:	
	$W_U$	$W_E$
0	0.305	0.005
0.2	0.335	0.170
0.4	0.355	0.260
0.6	0.375	0.325
0.8	0.405	0.375
1.0	0.410	0.410
1.2	0.420	0.440
1.4	0.430	0.460
1.6	0.440	0.480
10.0	0.570	0.635

The optimal rate is clearly more sensitive to  $\varepsilon_o$  when a money metric utility measure is used in the welfare function, with the rates being equal where  $\varepsilon_o = 1$ . It is equally clear from the table that the framework gives relatively high tax rates even for low values of inequality aversion. Those examining optimal tax models have not suggested that actual tax policies are driven by such a framework, or that policies should be dictated by such results. They are in the spirit of examining the implications of alternative objectives. Not surprisingly, the optimal tax framework has not been used as the basis of attempts to obtain implicit value judgements – indeed it demonstrates the considerable complexity involved in the link between value judgements and the tax structure. It also clearly shows that the imposition of a model, such as equal absolute sacrifice, to estimate a value of  $\varepsilon$  that is necessarily restricted to the range  $\varepsilon > 1$  is a rather artificial and highly restrictive case.

## 5 Interpreting Orders of Magnitude

In examining the implications of alternative value judgements, using an iso-elastic weighting function with different values of  $\varepsilon$ , it is of course important to appreciate the precise nature of the comparisons being made. When the

link between this type of social welfare function and a measure of inequality was introduced by Atkinson (1970), he recognised the difficulty of forming views about the orders of magnitude of  $\varepsilon$  using the welfare function  $W = \sum_{h=1}^H \frac{y_h^{1-\varepsilon}}{1-\varepsilon}$ . In order to help interpretation, he used the idea of a ‘leaky bucket’ experiment, which considers the extent to which a judge is prepared to tolerate some loss in making a transfer from one person to another.<sup>26</sup>

Consider two individuals, so that from the welfare function, setting the total differential equal to zero gives:

$$-\left. \frac{dy_1}{dy_2} \right|_W = \left( \frac{y_1}{y_2} \right)^\varepsilon \quad (28)$$

The welfare function is thus homothetic, as the slopes of social indifference curves are the same along any ray drawn through the origin. Consider two individuals and, using discrete changes, suppose a dollar is taken from the richest, such that  $\Delta y_2 = -1$ . The amount to be given to the other individual to keep social welfare unchanged is thus:

$$\Delta y_1 = \left( \frac{y_1}{y_2} \right)^\varepsilon \quad (29)$$

For example, if  $y_2 = 2y_1$  and  $\varepsilon = 1.5$ , it is necessary to give person 1 only 35 cents – a leak of 65 cents from the original dollar taken from person 2 is tolerated. If  $\varepsilon = 1$ , a leak of 50 cents is tolerated.

This type of experiment, and thus the sensitivity of the tolerance for a leaking bucket, is well-known in the literature on inequality measurement. But in other contexts in which the same kind of iso-elastic function is used, relatively large values of  $\varepsilon$  are often adopted without, it seems, consideration of such implications.<sup>27</sup> For example, in the intertemporal literature, a value of  $\varepsilon = 2$  is often used. Suppose that total income (or consumption) in the first period is 100 and this grows at a rate of 0.02 per period. In period 10 it is thus 119.5, and a judge with  $\varepsilon = 2$  would be prepared to take a dollar from period 10, and give only \$0.70 to period 1. By period 20 total income

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<sup>26</sup>Okun (1975) examined a slightly different kind of leaky bucket experiment involving transfers between groups of individuals.

<sup>27</sup>However, it is discussed by Pearce and Ulph (1998, pp.280-281).

would be 145.7, and the same judge would reduce period 20's income by \$1 while adding only \$0.47 to the first period. The social time preference rate is thereby increased significantly above the pure time preference rate.

The leaky bucket experiment therefore provides a useful illustration of the implications, in terms of value judgements, of adopting particular values of  $\varepsilon$  in any policy evaluation.

## 6 Conclusions

This paper has been concerned with the use of social welfare functions in evaluating actual or potential changes resulting from policies or other factors affecting a well-defined group of individuals. In particular, it has considered suggestions regarding the welfare weights to be used in comparing the gains and losses of different individuals (or other appropriate units of analysis), and a social time preference rate for use in cost benefit evaluation. While these variables essentially reflect value judgements, some authors have argued that they can be estimated either from consumers' behaviour or from the judgements implicit in tax policy. It was instead suggested here that results are highly sensitive to the context and model specification assumed.

More importantly, the argument that an estimated elasticity of marginal utility or social time preference rate *should* be used in policy evaluations fails to recognise that fundamental value judgements are involved. The various estimates and models may be of interest, but they cannot be used by economists to *impose* value judgements. The main contribution economists can make is to examine the implications of adopting a range of alternative value judgements.

This argument is of course not new. Indeed it was stated most forcefully and eloquently by Robbins (1935) in his important book on the *Nature and Significance of Economic Science*. He argued, 'propositions involving the verb "ought" are different in kind from propositions involving the verb "is". And it is difficult to see what possible good can be served by not keeping them separate, or failing to recognise their essential difference' (1935, p.149). It seems worthwhile to repeat these warnings of Robbins, along with his view

that:

All this is not to say that economist may not assume as postulates different judgments of value, and then on the assumption that these are valid enquire what judgment is to be passed upon particular proposals for action. On the contrary, as we shall see, it is just in the light that it casts upon the significance and consistency of different ultimate valuations that the utility of economics consists. (1935, p.149)

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