A Computational General Equilibrium Approach to Sectoral Analysis for Tax Potential: An Application to Pakistan

Andrew Feltenstein and Musharraf R. Cyan
Georgia State University
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ABSTRACT

This study develops a dynamic general equilibrium model, applied to Pakistani data, in which optimizing agents evade taxes by operating in the underground economy. The cost to firms of evading taxes is that they find themselves subject to credit rationing from banks. Our model simulations show that in the absence of budgetary flexibility to adjust expenditures, raising tax rates too high drives firms into the underground economy, thereby reducing the tax base. Aggregate investment in the economy is lowered because of credit rationing. Taxes that are too low eliminate the underground economy, but result in unsustainable budget and trade deficits. Thus, the optimal rate of taxation, from a macroeconomic point of view, may lead to some underground activity. We note, in particular, that incorporating a VAT without any other tax reductions greatly reduces the tax compliance of the service sector.

We have applied our model to Pakistan, and have calibrated our model to an 8 year period from 2004-2011. We note that it gives a reasonable approximation of Pakistani macro data. We then use a sectoral breakdown of tax data generated by the model to estimate tax gaps on a sector by sector basis. We note that certain sectors are currently paying taxes below their potential,

1 This research was funded by World Bank project, “A Computational General Equilibrium Approach to Sectoral Analysis for Tax Potential: An Application to Pakistan,” Prepared for Federal Board of Revenue (FBR) Islamabad, Tax Administration Reforms Project (Cr. 4007-PAK; Gr.TF:054392).
while others may be above their tax potential. These sectoral gap estimates may be used as indicators of where greater tax enforcement efforts should be directed.

I. INTRODUCTION

In many developing and transition countries, economic activity in the underground economy is estimated in excess of 40 percent of GDP (Schneider and Enste 2000; Friedman et. al 2000). This diversion into unofficial activity undermines the tax base and can significantly affect public finances and the quality of public administration (Loayza 1996; Johnson et al. 1997; Dessy and Pallage 2003). The illegal nature of underground activity also constrains private investment and growth. One important cost imposed by the inability to enforce legal contracts is the limited access to formal credit markets.

We develop a simple intertemporal general equilibrium model with heterogeneous agents, multiple production activities and credit rationing to explain the prevalence of a large underground economy and corresponding gap between potential and actual taxes collected. Our model is then applied to Pakistan. In particular, we explore the link between tax rates, access to credit, and the extent of tax evasion, and examine the consequences of the underground economy for public finances and aggregate economic performance. Entry and exit into the underground economy is derived as part of optimizing behavior that depends on taxes and interest rates. Firms

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2 As in Braun and Loayza (1993), “the underground economy is defined as a set of economic units which do not comply with one or more government imposed taxes and regulations, but whose production is considered legal”. We do not explicitly model regulations, assuming they are uniform across sectors.

3 Access to credit could also be provided by microcredit institutions. However, these operations remain very small and do not offer a substitute to small firms at the threshold of informal sector. The microcredit operations target the poor households who do not own assets and accept social collateral in place of physical collateral. For example, in 2003 there were 240,000 active borrowers; the number increased to 1.27 million in 2007 under government policy but the gross loan portfolio increased from Rs. 2.3 billion in 2004 to 12.7 billion in 2007, with loan size increasing from Rs. 6,629 in 2004 to Rs. 10,000 in 2007 (Rauf and Mahmoud, 2009; p.100). The average loan size of around $100 does not offer a substitute financing option to the firms on the margin.
operating underground are subject to credit rationing by banks which reduce loans in relation to the firm’s nonpayment of taxes.

Since the size of the underground economy in the paper depends upon both endogenous and exogenous variables, our framework has scope for policy changes. In particular, we address the issue of policy responses towards the emergence of tax-avoiding activity and emphasize the ambiguous effects of taxation by means of numerical simulations of a computational general equilibrium (CGE) model for Pakistan. Economic reform will depend upon policies that reduce the various forms of tax evasion.

Section II provides a brief overview of our modeling of the underground economy. Section III presents our dynamic CGE model. Section IV discusses the parameterization of the model and presents an initial calibration exercise. Section V discusses extensions to further disaggregation of the tax gap analysis.

II. MACROECONOMIC BACKGROUND

The cost of operating in the underground economy is modeled in terms of the inability to borrow from the official banking system. Banks in the model are assumed not to have perfect information about the firm’s true ownership of assets and its associated true tax obligation. We assume that due to collateral requirements, credit is provided only in relation to the firm’s implied ownership of assets, which is determined from its actual tax payment. The idea here is that in the face of default, banks can only seize those assets that have been officially declared by the firm. Hence, the higher the extent of tax evasion, the lower the implied value of firm assets, and the lower the amount of credit provided by the banking system. Our approach has some similarity to Kiyotaki and Moore (1997) who model credit limits on loans. These limits are determined by estimates of collateral which, in turn, are determined by estimates of durable asset
holdings by borrowers. Here, tax payments are used to estimate the value of the durable asset of the borrower, as the asset cannot be directly observed.

We assume that firms can operate partially in the formal and partially in the underground economy. That part of their operation that takes place in the legal economy pays taxes and can borrow from the banking system. That part that is underground does not pay taxes and cannot borrow. Admittedly this distinction is artificial, but captures some of the benefits and costs of operating in the underground economy discussed in the literature. In reality, the underground firm may still be able to finance its investment needs by relying on trade credits or borrowing from secondary lenders who charge higher than market interest rates and are willing to incur high risks. More specifically, there is an alternative source of financing in Pakistan, as well as many other developing countries that may be viewed as being between borrowing from the formal banking sector and entering the underground economy. This source is micro-financing in which micro-lenders offer lower rates than undocumented money lenders, but higher than the rates offered by the formal banking system. A more complete version of our model might have investors borrowing from the micro-lenders and paying higher rates, but avoiding the credit rationing they face from formal banks who examine their tax returns. This incorporation of diverse lenders is, however, beyond the scope of our research at this point.

Our approach also assumes that firms can evade taxes without any real risk of detection or punishment. Shleifer and Vishny (1993) point out that where public pressure on corruption or the

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4 Huq and Sultan (1991) note that in Bangladesh, while borrowing rates from commercial banks were around 12 percent, firms dependent on noninstitutional sources to meet their financing needs paid rates between 48 to 100 percent.

5 See, for example, Haq and Safavian (2012) for an examination of how micro-credit in Pakistan expands borrowing opportunities for many people who would otherwise be excluded from the formal banking system, yet still excludes large numbers of eligible women borrowers.
enforcement ability of the government is relatively weak - as is the case in many developing
countries - this is in fact a fitting assumption. Of course the specific case of Pakistan may well
deviate from this general assumption, as variable enforcement of tax collection in different
sectors of the economy may occur. Such differential levels of enforcement could impact revenue
collections across sectors, independently of access to credit issues. However we lack
information on differential tax enforcement, and hence have not attempted to incorporate it in
our stylized model.

III. A DYNAMIC GENERAL EQUILIBRIUM MODEL

We have developed the formal structure of a dynamic general equilibrium model that
dendogenously generates an underground economy. Much of the structure of our model is
designed to permit numerical implementation for Pakistan. Our model has $n$ discrete time
periods. All agents optimize in each period over a 2 period time horizon, with perfect foresight
over those 2 periods, and with expectations for the future after those periods being formed by an
adaptive process formed by past errors. That is, in period $t$ they optimize given prices for periods
$t$ and $t + 1$ which are known perfectly, and with expectations for prices for the future after
period $t + 1$. The expectations for the future after $t + 1$ are formed by a weighted average of
historical prices, combined with the agent’s past errors in predicting those prices. When period
$t + 2$ arrives, agents re-optimize for periods $t + 2$ and $t + 3$, based on new information about
period $t + 2$ and past errors. We should note that we could have any perfect foresight time
horizon, not just 2 periods. Thus, at one extreme, the agents could have perfect foresight
expectations for the entire time period of the model. The problem with a perfect foresight model
in our context is that it not possible to generate unexpected outcomes. Thus, for example, a firm
would never find that it had underinvested because it had evaded taxes more than was optimal,
based on the imperfectly anticipated future. Accordingly, it would not be possible to generate the sort of fluctuations that our simulations will indicate.

Our approach is related to Gordon and Li (2009). Here the government is able to tax a firm only if that firm uses the banking system. When the firm uses a bank, it is assumed that the bank has access to the firm’s balance sheet, which it records. The bank then makes this balance sheet information available to the government, which is then able to collect taxes, in particular sales taxes, based upon its knowledge of the firm’s balance sheet.

We use a dynamic approach in which both firms and banks optimize and in which the benefits to a firm of accessing the banking system are endogenous. Our approach is related to Dabla-Norris and Feltenstein (2005). Here a firm compares the return to capital with the marginal tax rate on capital income. If the return is greater than the tax rate, then the firm pays the full capital tax. If it is less than the tax rate, then the firm reduces its tax payments proportionally. Hence the firm enters the underground economy gradually, as the gap between tax rates and returns to capital increases. At the same time banks use a firm’s capital tax payments, combined with the capital tax rate to obtain an estimate of the firm’s minimum capital value. This is thus the bank’s estimate of the firm’s collateral, and hence reflects a minimum estimate of the value of assets that the bank can seize if the loan fails. This approach is motivated by the collateral constraints in Kiyotaki and Moore (1997). The collateral is represented by the bank’s estimate of the borrowing firm’s minimum capital which is, in turn, estimated by examining the firm’s tax return. We should note that we are thus focusing on only a single type of tax evasion, namely, evasion of the corporate income tax. As we shall see, indirect tax rates can change rates of evasion of the corporate income tax by changing the rate.
of return to capital. We do not, however, consider direct evasion of sales or value added taxes, for example.

Our approach has the key feature that tax evasion is based upon optimizing behavior by firms, rather than upon some exogenous firm characteristics. In particular, enterprises, as well as individuals, will balance their need to invest by borrowing from the banking system with their desire to reduce their tax obligations. This optimizing behavior is, of course, forward looking. We do not incorporate reputational issues into tax evasion behavior. That is, if a firm evades taxes in the current period then it faces credit rationing from banks. However, if in the future it returns to fully paying its taxes, then banks will not restrict credit to it, based upon its earlier behavior. The detailed technical discussion for production, banking, consumption, government and foreign sector is provided in Appendix 1.

IV. SIMULATIONS

In this section we carry out numerical simulations, based upon the model whose technical structure is given in Appendix 1. The model is designed to give some qualitative notion of the implications for the economy of tax evasion and entry into the underground economy. Our goal is to calibrate the model to the dynamic path of the Pakistan macro economy, based upon the most recent available sources for the economy’s technological and policy parameters.

We use an input-output (IO) matrix given in Ahmad, Barrett and Coady (1985)\(^6\), in which an 87 sector matrix is derived to represent Pakistan’s technology for 1981. This has been updated

\(^6\)Unfortunately there is no up to date input-output matrix that is currently available for Pakistan. We have been informed that one is being developed, but it is not complete and we do not have access to it. Of course the structure of the Pakistan economy may well have changed since 1990, but we do not have evidence to support or reject such a conclusion. Once the new input-output matrix becomes available, then it can easily be substituted for the old matrix and the simulations and gap estimates can be re-calculated.
for 1989/90, and we use the coefficients in this updated matrix. This matrix is aggregated by adding rows and columns to the 27 sector matrix used for this study. Sectoral value addeds are taken from the national income accounts for 2004 expanded to correspond to the 27 sector IO matrix. We use 2004 as a starting point as our 8 year dynamic simulation is from 2004 – 2011. The production coefficients in sectoral value added functions are Cobb-Douglas and are taken from the IO matrix.

The model incorporates personal and corporate income taxes, sales taxes, and import tariffs. Our source for all tax rates is http://www.taxrates.cc/html/pakistan-tax-rates.html. For the personal income tax we use the various slabs from 0 to 20 percent. For the corporate tax rate we use 35% of net taxable income of a company. For nonresidents, a 15% rate is levied on the gross amount of royalties or technical service fees, and 30% for other payments under the presumptive tax regime. The standard rate of the sales tax in Pakistan is 16 percent. Note that these are statutory rather than effective rates. The model generates endogenous effective tax rates, which are different from rates generated by single equation estimates.

We should add a caveat to our specification of the tax system. In practice, a large part of the Pakistani service sector is exempt from taxation, as it has an income of below Rs 200,000. Accordingly, these sectors do not pay taxes yet are not in the underground economy, as we define it in this paper. We have not accounted for this exemption threshold in our modeling because of the aggregate structure we employ. We have only a single sector, “Services”, and
hence cannot distinguish between large and small enterprises.\textsuperscript{7} Accordingly, our simulations may overestimate the extent of tax evasion, as services represent 53.4 percent of the Pakistani economy and we attribute all non-payment of taxes in that sector to evasion.\textsuperscript{8}

Exchange Rates time series are taken from the Statistics and DWH Department, the State Bank of Pakistan. We use the annual average US dollar foreign exchange rates for the years 2003-2010, as we wish to generate a dynamic macroeconomic path for these years. We assume that the structure of financing of the government budget deficit is an exogenous policy instrument, and we take the 2003-2010 shares from the data source TABLE 4.2, SUMMARY OF PUBLIC FINANCE (CONSOLIDATED FEDERAL AND PROVINCIAL). We make a similar exogeneity assumption for public and private capital inflows, which are taken from the TABLE 8.1 of the same source. Our source for the historical series of expenditure by the consolidated public sector is \url{http://www.sbp.org.pk/departments/stats/PakEconomy_HandBook/Chap-3.7.pdf} where we use the shares of GDP table.

Our model incorporates behavioral demand for money that depends upon interest and inflation rates, as well as real income. We use the estimates given in Qayyum (2005). In order to use our model for counterfactual simulations, we first generate an equilibrium using benchmark policy, technological, and behavioral parameters described.\textsuperscript{9} The program used to solve for the

\textsuperscript{7} Services include retail sector as well. This is the most lightly taxes sector in Pakistan. Periodic attempts by government to tax retails have been met with resistance from well-organized traders with disproportionate influence in political parties. Only recently, in 2009, the government’s attempt to implement a VAT were thwarted by the opposition in the parliament, in itself influenced by the traders, as the proposed tax would have increased pressure on the retail sector to be documented.

\textsuperscript{8} This issue was pointed out by an anonymous referee.

\textsuperscript{9} The underlying computational general equilibrium program is written in FORTRAN 95 and is available by writing to Andrew Feltenstein (afeltenstein@gsu.edu). The dynamics of model, combined with optimizing evasion, credit rationing, and various other features make the underlying program quite
equilibrium converges to an accurate approximation of a Kakutani fixed point in usually less than 20 seconds for the 8 discrete time periods we are currently simulating. We run the macroeconomic model forward for eight years, giving tax rates and public expenditures their estimated values. We also suppose that the central bank maintains a fixed exchange rate, with the rate being fixed at the historical level of each year. Table 1 shows the results of the benchmark simulation. It may be worth making a few remarks concerning the simulated values. First, notice that our model generates moderate rates of growth in real GDP, with an average growth rate of 5.9 percent over the total 8 year period. This approximates Pakistan’s actual real growth rate over the period in question. The budget is in deficit for all but 1 year, with an average deficit of 1.1 percent of GDP. This is lower than the actual historical deficit for the period. The simulated interest rate is relatively stable, and averages 7.9 percent, which is in line with Pakistan’s interest rate. The trade deficit is relatively stable and averages 2.3 percent of GDP, which is somewhat better than the current level in Pakistan. The annual rate of inflation averages 22.3 percent, which is somewhat higher than the Pakistani average. Finally, sector 4, services and retail trade, operates significantly in the underground economy for all 8 years of the simulation, indicating considerable tax evasion in retail trade. This also possibly corresponds to the Pakistan experience. By the end of the 8 years of the simulation, the sector is under-reporting income for tax purposes by 31.5 percent. One interpretation of this result, as discussed at the end of the production section of Appendix 1, is that 31.5 percent of the employment in the sector is complex. It might well be possible to write the program using a package such as GAMS or GTAP, rather than a low level language such a FORTRAN or C, but we have not attempted to do so.

\footnote{In practice, we take 2004 as the base year. By this we mean that initial allocations of factors and financial assets are given by stocks at the end of 2003. We have data for fiscal and other policy parameters for the next 8 years, that is, through 2011.}
informal in the sense that workers’ wages are not being reported and hence their income taxes are not being withheld. \(^{11}\)

It may be useful to note the importance of the dynamic structure of our model in generating our results. In particular, the endogenously generated growth rate of 5.8 percent, given in Table 1, is itself largely a function of forward looking investment, itself being generated by Appendix equation (2). Similarly, interest and inflation rates reported in Table 1 and all subsequent simulation tables, are endogenous outcomes of intertemporal optimization. The structure of short term perfect foresight, combined with longer term adaptive updating, is important if we wish to introduce government policy shifts in the future, for example an unanticipated (by the public) tax change. In a perfect foresight model, such a change would be anticipated at the beginning of the simulation, and agents would compensate for it.

Suppose we now consider a change in the Pakistan tax code. Namely we will impose a 10 percent value added tax, leaving all other tax rate as in Table 1. Note that here, and in the following Tables, all numbers are relative to the benchmark case of Table 1. Hence, for example, GDP and price levels change from 100 in period 1. Pakistan has considered introducing a VAT, so this will be an example of the potential outcomes of such a reform. The results are given in Table 2. We note that the tax increase leads to a small decline in real GDP over the 8 years of the simulation. The budget deficit becomes a surplus and there is also a trade surplus. At the same time, there is a considerable increase in tax evasion by sector 4 which now is evading almost 75 percent of its tax liabilities. Thus simply imposing a VAT without making

\(^{11}\) We should note that in this and all subsequent simulations there we are not constraining the public and trade deficits to be dynamically sustainable. We justify this approach by noting that our simulations have an 8 year, medium term, time horizon so we do not generate a steady state.
other changes in tax policies might initially not seem to be a useful course of action. On the other hand, as an austerity measure it might seem acceptable. There has been a very slight decline in real GDP, major improvements in the budget and trade balances, and deterioration in tax evasion in only one sector.

Accordingly, let us now suppose that we maintain the 10 percent VAT, but reduce the GST from 16 to 10 percent. The aim of this exercise would be to reduce tax evasion. The results are given in Table 3. We see that real income has remained unchanged, as compared to Table 2. There has been a small improvement in tax compliance, which still remains much worse than in the base case. This outcome leads us to conclude that the VAT introduction needs to be compensated for by a more dramatic tax reduction elsewhere.

As a fourth example, suppose we now reduce the GST to 0 percent, so that the only domestic indirect tax collected is the VAT. The results of this exercise are given in Table 4. We see that Sector 4 has greatly increased its tax compliance, even as compared to the base case of Table 1. At the same time, however, the revenues lost from the other sectors of the economy, which were not previously in the underground economy and hence did not increase their tax compliance, has outweighed the gains from Sector 4. The budget deficit thus grows dramatically, leading to a sharp increase in the domestic interest rate. Real GDP falls, primarily because the rate of return to capital of Sector 2, manufacturing falls, leading to a decline in that sector’s output.

The results of Tables 2-4 indicate the sort of trade-offs that policy makers should consider when attempting to institute tax reform, not only in Pakistan but also in other countries where tax evasion is an issue. We have done sensitivity analysis of our results with respect to different variables, in particular with respect to $\alpha$, the coefficient of firm “honesty” in Appendix equation
At one extreme, if $\alpha = 0$, then the firm never evades taxes and there is no tradeoff between tax evasion and tax reform. As $\alpha$ increases, then the trade-off rapidly becomes unmanageable in the sense that relatively small increases in certain taxes cause large increases in tax evasion. Different taxes to not have the same impact upon evasion, due to their different coverage. Accordingly, as in the case of compensating for a VAT increase with a GST decrease, changing one tax may require a relatively larger opposite change in another tax to compensate. This outcome also reflects the fact that not all sectors were originally tax evaders, and hence offering them incentives via lowering another tax will not increase their compliance. Accordingly, our results are quite general in terms of the qualitative nature of the trade-offs. However, the quantitative size of the trade-offs would vary according to the underlying propensity to evade, as reflected in $\alpha$. Thus the calibration of $\alpha$ would be an important part of parameterizing a country specific model.\(^{12}\)

Let us now consider yet another proposed policy change, namely, an increased corporate tax upon the banking industry. Accordingly, we will increase the tax rate from its current 35 percent to 40 percent. All other policy parameters stay the same as in Table 1. The results of this exercise are given in Table 5. The outcomes are interesting, as we notice that there has been very little improvement in the overall budget deficit. At the same time, we see that there has been a considerable decrease in tax compliance by the Service Sector (Sector 4), of which the banking industry is a major component. Thus the increased tax rate has caused an increase in evasion that has negated the impact of the higher tax rates on the banking sector.

\(^{12}\) We would suspect, for example, that our model would be applicable to, say, Bangladesh or India, but with different values for $\alpha$. 

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As a final exercise, let us carry out a reduction in the customs tariff, reducing the rate from 25 percent to 20 percent. This is a policy that is currently under consideration. The results of this simulation are given in Table 6. As might be expected, we see a decline in import duties, as compared to Table 1. Tax evasion remains essentially unchanged, and there is a slight decline in real GDP, as compared to Table 1, due to the expected increase in imports. We thus see that this tariff change has little effect on the overall real economy. We should note that this is a small country model, so that world prices are given. Hence terms of trade effects are entirely determined by domestic price changes. In addition, domestic technology is fixed for all of the simulations, so that any improvement in domestic efficiency that might be caused by the tariff reduction is not incorporated.

We may thus draw certain conclusions from our simulations. Recall that the base simulation, given in Table 1, does not incorporate any counterfactual tax or tariff changes. All subsequent hypothetical simulations do have such changes. Hence it might be expected that none of these counterfactual simulations is unequivocally superior to the base case, as we are comparing second best equilibria. On the other hand, introduction of the VAT combined with a reduction in some other tax, for example the GST, does lead to improvements in the budget and trade balance, while having only a small negative impact upon real growth and tax compliance. Depending upon the preferences of the government, our model is a useful tool for comparing the outcomes of different policies.

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13 Out of the around 6000 items listed for custom duties, 300 items are zero rated and around 150 are taxed at 35 or higher rates. The overwhelming number of the remaining items is taxed at 5 or 10 percent. The reduction in rates applies to most of the items and this has resulted in a reduction in tax collection from 1.6 percent of GDP in 1991 to 1.0 percent in 2011.

14 It is well known that removal or reduction of a single distortion in a second best economy does not necessarily improve the efficiency of the resulting equilibrium. See Lipsey and Lancaster (1956).
Our model helps us to identify those sectors that are underperforming from a tax point of view. We therefore use the model to carry out a sectoral estimate of the tax gap. Here the predicted outcomes of the general equilibrium model for 2010, assuming full compliance, are compared with actual tax revenues collected. That is, we set the “honesty” parameter for each sector at 0. This is the parameter $\alpha$ in equation (3) in Appendix 1. The general equilibrium model then generates a path for tax collections for the 8 years of the simulation, and we choose the predicted collections for 2010. These are then compared with actual tax collections for 2010 for selected sectors, as well as for the aggregate economy and the manufacturing sector.

We use this approach since it essentially compares optimizing behavior with two different values for $\alpha$. When $\alpha = 0$ agents optimize subject to their never evading taxes. Such a value for $\alpha$ might be the result of a number of phenomena that are not explicitly included our model. If, for example, tax enforcement were absolutely efficient so that evasion would be impossible, then this would be operationally equivalent to $\alpha = 0$. If tax enforcement were less than perfectly efficient, then this could be equivalent to the optimizing behavior that would result from $\alpha > 0$. Accordingly, we claim that our value of $\alpha$, which is calibrated to generate a dynamic path for the model’s endogenous macro variables that approximates the Pakistani historical variables, reflects optimizing behavior under the existing tax enforcement regime. If tax enforcement were tightened sufficiently so that there would be no evasion, then this would be the same as setting $\alpha = 0$ and then optimizing. Thus comparing sectoral tax collections under the two regimes gives us an estimate of sectoral tax evasion. We do not attempt to explain how to generate effective tax enforcement.

The aggregate results are given in the Table 7 below. They indicate that, on the level of the overall economy, there is a tax gap of about 58 percent, while in the manufacturing sector the
gap is approximately 53 percent. As might be expected from the general equilibrium model, capital intensive sectors such as iron and steel, and oil and gas, have smaller gaps than do less capital intensive sectors such as finance and insurance, or hotels and restaurants. These calculations should help in the measurement of the overall problem, as well as to identify those sectors where improvement is most needed.

It may be useful to add a few remarks about the absolute values of these gap estimates. Recall that the sectoral definitions of the general equilibrium model are based upon the 87 sector Pakistan input-output matrix. These sectors are, in turn, based upon national income accounts value added definitions. These sectoral definitions are not exactly the same as those in the actual tax collection tables which we use for the gap estimates. Thus, for example, the national income account definition of Finance and Insurance may be broader than that used by the tax authorities. Accordingly, the general equilibrium model would generate greater expected tax revenue for Finance and Insurance, assuming perfect compliance, than would be reflected in actual tax collection data. Hence the estimated compliance gap would be relatively large, as we see in the Table. Of course the opposite could also be true, that the national income account definition could be narrower than the tax definition, leading to some under estimations of particular gaps. Accordingly, it is best to look at broad sectors, such as the overall economy, manufacturing, or retail sales, for example, for absolute values of gaps as there is a closer comparison between national income account and tax collection definitions for these categories. For more narrowly defined sectors, it is best to look at the gap estimates as reflecting relative (compared to other sectors) rather than absolute gaps.
CONCLUSION

We have constructed a dynamic general equilibrium model that incorporates endogenous tax evasion as part of intertemporal optimizing behavior by firms. We have used parameters from Pakistan to calibrate our model to an 8 year period from 2004-2011, and note that it gives a reasonable approximation of Pakistani macro data. We note that the service sector, at equilibrium, consistently evades some portion of its taxes.

We then incorporate a value added tax of 10 percent in a series of counterfactual simulations, as the introduction of such a tax is currently under consideration by Pakistan. We note that incorporating a VAT without any other tax reductions greatly reduces the tax compliance of the service sector. If, on the other hand, the VAT introduction is accompanied by abolishing the GST, then tax compliance rises, but revenues from the non-service sectors decline, interest rates rise, and real GDP growth slows.

We also use the model to carry out a sectoral estimate of the tax gap. Here the predicted outcomes of the general equilibrium model for 2010, assuming full compliance, are compared with actual tax revenues collected. That is, we set the “honesty” parameter for each sector at 0. The general equilibrium model then generates a path for tax collections for the 8 years of the simulation, and we choose the predicted collections for 2010. These are then compared with actual tax collections for selected sectors, as well as for the aggregate economy and the manufacturing sector. They indicate that, on the level of the overall economy, there is a tax gap of about 58 percent, while in the manufacturing sector the gap is approximately 53 percent. As

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15 A policy implication of sectoral gap analysis is that it directs tax administration efforts differentially toward certain sectors. While this may be revenue productive, it may also exacerbate distortions in the shape of varying tax incidence on certain agents in the economy.
might be expected from the general equilibrium model, capital intensive sectors such as iron and steel, and oil and gas, have smaller gaps than do less capital intensive sectors such as finance and insurance, or hotels and restaurants. These calculations should help in the measurement of the overall problem, as well as to identify those sectors where improvement is most needed. Our approach demonstrates a method for gap analysis which can be further developed to incorporate additional information on sectoral structures and use sectoral estimates of honesty parameters.
## Table 1. Base Case

<table>
<thead>
<tr>
<th>Period</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal GDP 1/</td>
<td>100.0</td>
<td>137.8</td>
<td>133.4</td>
<td>181.2</td>
<td>314.1</td>
<td>475.3</td>
<td>538.3</td>
<td>785.6</td>
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<tr>
<td>Real GDP 1/</td>
<td>100.0</td>
<td>117.4</td>
<td>113.5</td>
<td>119.7</td>
<td>131.3</td>
<td>142.3</td>
<td>144.3</td>
<td>149.6</td>
</tr>
<tr>
<td>Real GDP growth</td>
<td>17.4</td>
<td>-3.3</td>
<td>5.4</td>
<td>9.7</td>
<td>8.4</td>
<td>1.4</td>
<td>3.6</td>
<td></td>
</tr>
<tr>
<td>rate 3/</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation 3/</td>
<td>17.4</td>
<td>0.1</td>
<td>28.9</td>
<td>58.0</td>
<td>39.7</td>
<td>11.6</td>
<td>40.8</td>
<td></td>
</tr>
<tr>
<td>Price Level 1/</td>
<td>100.0</td>
<td>117.4</td>
<td>117.5</td>
<td>151.4</td>
<td>239.2</td>
<td>334.1</td>
<td>373.0</td>
<td>525.3</td>
</tr>
<tr>
<td>Nominal interest rate 3/</td>
<td>6.9</td>
<td>10.5</td>
<td>3.6</td>
<td>3.6</td>
<td>8.4</td>
<td>13.1</td>
<td>7.7</td>
<td>9.7</td>
</tr>
<tr>
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### Real return to Share of Sector in Legal Economy

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1/ Normalized to period 1 of the base case.
2/ As a percent of GDP.
3/ In percent.
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1/ Normalized to period 1 of the base case.  
2/ As a percent of GDP.  
3/ In percent.  
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K5 = Public administration, health, education
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1/ Normalized to period 1 of the base case.
2/ As a percent of GDP.
3/ In percent.
4/ The capital types are specific to broad sectors of the input-output matrix. The 5 capital types are:
   K1 = Mining
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   K3 = Electricity, gas, construction
   K4 = Services, retail trade
   K5 = Public administration, health, education
Table 4. Pakistan: 10% VAT, 0% GST

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Table 5. Pakistan: 40% CAPITAL TAX ON BANKS

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Table 6. Pakistan: REDUCED CUSTOMS TARIFF

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Real return to Share of Sector in Legal Economy

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<tr>
<td>K5</td>
<td>99.8</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

1/ Normalized to period 1 of the base case.
2/ As a percent of GDP.
3/ In percent.
4/ The capital types are specific to broad sectors of the input-output matrix. The 5 capital types are:
   K1 = Mining
   K2 = Manufacturing
   K3 = Electricity, gas, construction
   K4 = Services, retail trade
   K5 = Public administration, health, education
<table>
<thead>
<tr>
<th>Sector</th>
<th>Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mining &amp; Quarrying</td>
<td>-96.9</td>
</tr>
<tr>
<td><strong>Manufacturing</strong></td>
<td>-52.5</td>
</tr>
<tr>
<td>(of which)</td>
<td></td>
</tr>
<tr>
<td>1. Chemicals</td>
<td>-67.5</td>
</tr>
<tr>
<td>2. Automobiles</td>
<td>-48.3</td>
</tr>
<tr>
<td>3. Cigarette &amp; Tobacco</td>
<td>103.4</td>
</tr>
<tr>
<td>4. Iron and Steel</td>
<td>-10.5</td>
</tr>
<tr>
<td>5. Oil and gas</td>
<td>-25.7</td>
</tr>
<tr>
<td>6. Paper &amp; Paper Board</td>
<td>-53.2</td>
</tr>
<tr>
<td>7. Textile</td>
<td>-59.2</td>
</tr>
<tr>
<td>8. Edible Oil</td>
<td>75.2</td>
</tr>
<tr>
<td>9. Cement</td>
<td>-49.0</td>
</tr>
<tr>
<td>10. Sugar</td>
<td>-91.2</td>
</tr>
<tr>
<td>11. Pharmaceuticals</td>
<td>-46.9</td>
</tr>
<tr>
<td>12. Fertilizer</td>
<td>-23.0</td>
</tr>
<tr>
<td>Telecom</td>
<td>-81.3</td>
</tr>
<tr>
<td>Wholesale and Retail Trade</td>
<td>-73.4</td>
</tr>
<tr>
<td>Finance and Insurance</td>
<td>-93.3</td>
</tr>
<tr>
<td>Hotels and Restaurants</td>
<td>-85.3</td>
</tr>
<tr>
<td>Other</td>
<td>-53.8</td>
</tr>
<tr>
<td><strong>Total Economy</strong></td>
<td>-58.3</td>
</tr>
</tbody>
</table>

1/ A number of manufacturing sectors have been excluded from the disaggregation. They are included in the category "other". Other sectors that are not included in the terms of reference, but for which it is possible to calculate gaps, have been included.
APPENDIX 1

A General Equilibrium Specification

In this section we develop the formal structure of a dynamic general equilibrium model that endogenously generates an underground economy. Much of the structure of our model is designed to permit numerical implementation for Pakistan. Our model has $n$ discrete time periods. All agents optimize in each period over a 2 period time horizon. That is, in period $t$ they optimize given prices for periods $t$ and $t+1$ and expectations for prices for the future after $t+1$. When period $t+2$ arrives, agents reoptimize for period $t+2$ and $t+3$, based on new information about period $t+2$.

Production

There are 8 factors of production and 3 types of financial assets:

1-5 Capital types 9. Domestic currency
7. Rural labor 11. Foreign currency
8. Land

The five types of capital correspond to five aggregate nonagricultural productive sectors.\(^{16}\)

An input-output matrix, $A_n$, is used to determine intermediate and final production in period $t$. The matrix is $27 \times 27$, using the disaggregation of Ahmad, Barrett and Coady (1985). The first 26 rows and columns of the input-output matrix correspond to domestic production. The final row and column represents imports of intermediate goods, which are treated as a single aggregate commodity. Corresponding to each sector in the input-output matrix, sector-specific value added is produced using capital and urban labor for the nonagricultural sectors, and land and rural labor in agriculture.

\(^{16}\) We could have any number of capital types without affecting the structure of the model.
The specific formulation of the firm's problem is as follows. Let $y_{K_i}^i, y_{L_i}^i$ be the inputs of capital and urban labor to the $j$th nonagricultural sector in period $i$. Let $Y_{Gi}$ be the outstanding stock of government infrastructure in period $i$. The production of value added in sector $j$ in period $i$ is then given by:

$$\text{va}_{ji} = \text{va}_{ji}(y_{K_i}^i, y_{L_i}^i, Y_{Gi})$$

(1)

where we suppose that public infrastructure may act as a productivity increment to private production.

Sector $j$ pays income taxes on inputs of capital and labor, given by $t_{K_j}, t_{L_j}$ respectively, in period $i$. The interpretation of these taxes is that the capital tax is a tax on firm profits, while the labor tax is a personal income tax that is withheld at source. It should be noted that the capital tax is equivalent to the standard formulation of the corporate income tax in economics models. The tax is not levied upon the stock of corporate capital, but on the returns to that capital, which is sector specific. There are no pure profits here, since production functions are constant returns to scale, and hence the corporate income tax is treated as a tax on returns to capital.

We suppose that each type of sectoral capital is produced via a sector-specific investment technology that uses inputs of capital and labor to produce new capital. Investment is carried out by the private sector and is entirely financed by domestic borrowing.

Let us define the following notation:

- $C_{Hi} =$ The cost of producing the quantity $H$ of capital of a particular type in period $i$.
- $r_i =$ The interest rate in period $i$.
- $P_{Ki} =$ The return to capital in period $i$.  

27
\[ P_{Mi} \] = The price of money in period \( i \).

\[ \delta_i \] = The rate of depreciation of capital.

Suppose, then, that the rental price of capital in period 1 is \( P_1 \). If \( C_{H1} \) is the cost-minimizing cost of producing the quantity of capital, \( H_1 \), then the cost of borrowing must equal the present value of the return on new capital. Hence:

\[
C_{H1} = \sum_{i=2}^{n} \left[ \frac{P_{Ki} (1 - \delta)^{i-2} H_1}{\prod_{j=1}^{i-1} (1 + r_j)} \right]
\]  \hspace{1cm} (2)

where \( r_j \) is the interest rate in period \( j \), given by:

\[ r_j = 1 / P_{bj} \]

where \( P_{bj} \) is the price of a bond in period \( j \). The tax on capital is implicitly included in the investment problem, as capital taxes are paid on capital as an input to production.

The to invest depends not only on the variables in the above equation, but also upon the decision the firm makes as to whether it should pay taxes.\(^{17}\) This decision determines the firm’s entry into the underground economy. We assume that the firm’s decision is based upon a comparison of the tax rate on capital with the rate of return on new capital. Formally, suppose that we were in a two period world. Suppose that:

\[ \frac{P_{K2}}{1 + r_i} \geq t_{K1} \]

\(^{17}\) In reality, we can regard the tax rate on capital as the generalized tax rate, including taxation, regulation, and corruption (bribes).
that is, the present value of the return on one unit of new capital is greater than the current tax rate on capital. In this case we assume the investor pays the full tax rate on capital inputs.

Suppose, on the other hand, that:

\[
\frac{P_{K_2}}{1 + r_1} \leq t_{K_1}
\]

Here the discounted rate of return is less than the tax rate. The extent to which the firm goes into the underground economy is determined by the gap between the tax rate and the rate of return to investment. That is, the firm pays a tax rate of \( t_{K_1} \) where:

\[
\bar{t}_{K_1} = t_{K_1} \left[ 1 - \left( \frac{t_{K_1} - \frac{P_{K_2}}{1 + r_1}}{t_{K_1}} \right)^\alpha \right]^{(3)}
\]

Here \( 0 \leq \alpha \) and higher values of \( \alpha \) lead to lower values of taxes actually paid. That is, the ratio \( \frac{\bar{t}_{K_1}}{t_{K_1}} \) reflects the share of the sector that operates in the above ground economy. Hence \( \alpha \) represents a firm-specific behavioral variable. An “honest” firm would set \( \alpha = 0 \), while a firm that is prone to evasion would have a high value for \( \alpha \).

We should note that a firm’s decision to evade taxes is based upon a comparison of the marginal return to capital with the tax rate.\(^{18}\) Hence a firm may be quite capital intensive, i.e., have a large stock of existing capital, yet have a low marginal return to new capital and hence be predisposed to evading taxes. Thus, for example, an “old” industrial firm might be a tax evader, while being relatively capital intensive. A service sector firm, while being relatively labor

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\(^{18}\) In the presence of corruption in tax administration, the effective tax rate takes into account evasion and bribes. The firms will compare marginal return to capital with effective tax rate in the presence of corruption.
intensive, might have a high marginal rate of return to new capital and hence not wish to evade taxes. Thus simply looking at relative capital intensities will not allow us to predict the degrees of tax evasions different firms will choose.

In general, economies with formal and informal sectors will see corresponding movements of labor to firms in the two sectors. In our model’s simplified treatment, the distinction between the formal and informal sectors is given the extent to which a firm evades taxes. Thus a firm that evades, for example, 30 percent of its taxes is considered to be 30 percent in the underground economy. It would not be correct to say that 30 percent of the firm’s workers are in the informal sector, as all workers are employed by the same firm. However, we might claim that the extent to which a firm evades taxes reflects its under-reporting of wages, and hence withholding taxes. The model itself generates a full employment equilibrium, so no unemployment is caused by tax evasion.

**Banking**

We will suppose that there is one bank for each nonagricultural sector of the economy. There are 5 such sectors, and hence 5 banks, corresponding to each of the aggregate capital stocks. Each bank lends primarily to the sector with which it is associated. The banks are, however, not fully specialized in the sector they correspond to. We make the simplifying assumption that each bank holds a fixed share of the outstanding debt of its particular sector. It then holds additional fixed shares of the debt of each of the remaining sectors. We make this assumption of diversification of assets in order to allow for a situation in which a firm that evades taxes, and thereby enters the underground economy, might receive varying degrees of credit rationing from the different banks to which it applies for loans.
Our premise is that banks have no direct way of knowing whether specific firms operate in the underground economy. We assume that banks only care about the amount of capital that they estimate the firm may have. If the firm defaults on its loan, then this represents the best estimate of the amount that the bank could seize. The bank would, presumably, be willing to lend an amount equal to at least the estimated firm capital.

We assume the borrower is required to show the bank his tax returns in order to obtain a loan. There is a single, flat corporate tax rate that the borrowing firm faces. Hence, suppose that $T_{K1}$ represents taxes actually paid by the borrower in period 1. This is known to the bank, as the potential borrower is required to present his tax returns. Thus if the borrower fully complied with his tax obligation, and hence carried out no underground activity, the value of his capital, $\hat{K}_1$, would be given by:

$$\hat{K}_1 = \frac{T_{K1}}{I_{K1}}$$

In this case the bank lends an amount $L_1$, where $L_1 < C_{H1}$, as the bank would not be able to seize the full value of the loan in the case of a default. The situation we have described would, in the case of perfect certainty, have credit rationing when the estimated value of the firm’s capital is less than its loan request. If the firm’s capital is greater than its loan request, there would be no credit rationing.

In a more realistic case of uncertainty about both the true value of the firm, as well as about the bank’s own ability to seize the firm, one might expect the lending process to be somewhat different. Accordingly, we will suppose that a simple functional form determines bank lending as
a function of the amount requested as well as the estimated value of the firm’s capital. We define the amount the bank lends, $L_1$, as:

$$L_1 = C_{H1} \left[ \frac{K_1}{C_{H1}} \left( 1 + \frac{K_1}{C_{H1}} \right)^{-\gamma} \right] = C_{H1} \left[ \frac{K_1}{C_{H1} + K_1} \right]$$ \hspace{1cm} (4)

Here $\gamma$ represents a measure of risk aversion by the bank. If $\gamma = 0$, there are no credit restrictions, and the bank ignores estimates of the borrower’s estimated net worth. As $\gamma$ rises, the bank increasingly restricts lending if the term in brackets is less than 1. Thus if a firm operates entirely in the underground economy it will not be able to borrow to finance investment. If banks are highly risk averse, they will never lend more than a firm’s estimated net worth, which is based on its tax return. This tax return therefore represents all the information the bank needs in order to determine its response to a request for a loan.

**Consumption**

There are two types of consumers, representing rural and urban labor. We suppose that the two consumer classes have differing Cobb-Douglas demands and endowments. The consumers maximize intertemporal utility functions, which have as arguments the levels of consumption and leisure in each of the two periods.

Formally, the consumer’s problem is then given by equation 5.\textsuperscript{19} The definition of the notation follows.

$$\text{Max } U(x), \quad x = (x_1, x_{lu1}, x_{lr1}, x_2, x_{lu2}, x_{lr2})$$ \hspace{1cm} (5)

\textsuperscript{19} See Feltenstein and Shamloo (2012) for a discussion of this modeling approach.
such that:

\[(1 + t_i)P_{i}x_{i} + P_{Lui}x_{Lui} + P_{Lri}x_{Lri} + P_{M1}x_{Mi} + P_{B1}x_{Bi} + e_iP_{BF1}x_{BFi} = C_i \]  
\[(5a)\]

\[P_{K1}K_0 + P_{A1}A_0 + P_{Lui}L_{ai} + P_{Lri}L_{ri} + P_{M1}M_0 + r_0B_0 + P_{B1}B_0 + e_iP_{BF1}B_{F0} + TR_i = N_1 \]

\[P_{K2}(1 - \delta)K_0 + P_{A2}A_0 + P_{Lui}L_{au2} + P_{Lri}L_{ru2} + P_{M2}M_1 + r_1x_{Bi} + e_iP_{BF2}x_{BF1} + TR_i = N_2 \]

\[C_i = N_i \]

\[\log P_{Bi}x_{Bi} - \log e_iP_{BF1}x_{BFi} = \alpha + \beta(\log r_i - \log \frac{e_{i+1}r_{Fi}}{e_i}) \]  
\[(5b)\]

\[\log(L_{ai}/L_{ri}) = a_i + a_2\log\frac{P_{Lui} - P_{Lri}}{P_{Lui} + P_{Lri}} \]  
\[(5c)\]

\[\log P_{M1}x_{Mi} = a + b(1 + t_i)P_{i}x_{i} \]  
\[(5d)\]

\[P_{B2}x_{B2} = d_0 + d_1(1 + t_2)P_{2}x_{2} + d_2\left[\frac{r_2 - \pi_2}{1 + \pi_2}\right] \]  
\[(5e)\]

where:

\[P_i = \text{price vector of consumption goods in period } i.\]

\[x_i = \text{vector of consumption in period } i.\]

\[C_i = \text{value of aggregate consumption in period } i \text{ (including purchases of financial assets)}.\]

\[N_i = \text{aggregate income in period } i \text{ (including potential income from the sale of real and financial assets)}.\]

\[t_i = \text{vector of value added tax rates in period } i.\]

\[P_{Lui} = \text{price of urban labor in period } i.\]

\[L_{ai} = \text{allocation of total labor to urban labor in period } i.\]

\[x_{Lui} = \text{demand for urban leisure in period } i.\]

\[P_{Lri} = \text{price of rural labor in period } i.\]
\( L_{ri} \) = allocation of total labor to rural labor in period \( i \).

\( x_{Li} \) = demand for rural leisure in period \( i \).

\( a_2 \) = elasticity of rural/urban migration.

\( P_{Ki} \) = price of capital in period \( i \).

\( K_0 \) = initial holding of capital.

\( P_{Ai} \) = price of land in period \( i \).

\( A_0 \) = initial holding of land.

\( \delta \) = rate of depreciation of capital.

\( P_{Mi} \) = price of money in period \( i \). Money in period 1 is the numeraire and hence has a price of 1.

\( x_{Mi} \) = holdings of money in period \( i \).

\( P_{Bi} \) = discount price of a certificate of deposit in period \( i \).

\( \pi_i \) = domestic rate of inflation in period \( i \).

\( r_i, r_{fi} \) = the domestic and foreign interest rates in period \( i \).

\( x_{Bi} \) = quantity of bank deposits, that is, CD's in period \( i \).

\( e_i \) = the exchange rate in terms of units of domestic currency per unit of foreign currency in period \( i \).

\( x_{BFi} \) = quantity of foreign currency held in period \( i \).

\( TR_i \) = transfer payments from the government in period \( i \).

\( a, b, \alpha, \beta \) = estimated constants.

\( d_i \) = constants estimated from model simulations.

The Government
The government collects personal income, corporate profit, and value-added taxes, as well as import duties. It pays for the production of public goods, as well as for subsidies. In addition, the government must cover both domestic and foreign interest obligations on public debt. The resulting deficit is financed by a combination of monetary expansion, as well as domestic and foreign borrowing.

**The Foreign Sector**

The foreign sector is represented by a simple export equation in which aggregate demand for exports is determined by domestic and foreign price indices, as well as world income. The specific form of the export equation is:

\[
\Delta X_{t0} = \sigma_1 \left[ \frac{\pi_i}{\Delta e_i + \pi_{fi}} \right] + \sigma_2 \Delta y_{wi}
\]

where the left-hand side of the equation represents the change in the dollar value of exports in period \(i\), \(\pi_i\) is inflation in the domestic price index, \(\Delta e_i\) is the percentage change in the exchange rate, and \(\pi_{fi}\) is the foreign rate of inflation. Also, \(\Delta y_{wi}\) represents the percentage change in world income, denominated in dollars. Finally, \(\sigma_1\) and \(\sigma_2\) are corresponding elasticities.

Imports are treated as a single aggregate good produced by the foreign sector. Consumption demand for imports is determined as part of the consumer’s demand given by equation (5), in which consumers have an elasticity of demand for imported goods. Thus imports are both intermediate and final goods, with intermediate demand being generated by the input-output matrix and final demand coming from consumer optimization.
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SUMMARY OF PUBLIC FINANCE (CONSOLIDATED FEDERAL AND PROVINCIAL)