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**Education Vouchers: Means
Testing Versus Uniformity**

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Abstract

This paper compares a uniform education voucher system with a means-tested scheme in which the voucher is subject to a taper or withdrawal rate as parental gross income increases. Parents are assumed to maximise a utility function which includes their consumption, leisure and the human capital of children. The human capital production function has inputs consisting of parental human capital and expenditure on education. The government faces a budget constraint such that the voucher and a social dividend are financed from a proportional income tax. Alternative combinations of voucher and tax and transfer schemes are evaluated using a social welfare function defined in terms of the utility of parents. It is found that for all combinations of policy variables, a uniform voucher turns out to be optimal. However, if a binding constraint is placed on the maximum tax rate, means-testing, with a low taper, is found to be optimal.

1   Introduction

The aim of this paper is to compare a uniform education voucher system with a means-tested scheme in which the voucher is reduced as parental gross income increases. Means-testing is usually advocated on the grounds that transfer payments should be designed to help the poor, so that universal benefits are wasteful and involve an excessive level of gross government revenue; that is, they are said to have low 'target efficiency'.[1] However, such a measure reflects only one characteristic of transfers. Alternative voucher schemes are evaluated here in terms of a social welfare function, in the context of a model allowing for the incentive effects, on both labour supply and educational choices, of a tax and transfer system combined with vouchers. The incentive effects are complicated by the highly nonlinear nature of the budget constraints facing parents, and play an important role in the analysis.

The analysis presented here therefore differs from a number of earlier studies of alternative voucher schemes, which have largely been concerned with examining majority voting outcomes. Thus they have had to deal with the complexities arising from the existence of double-peaked preferences.[2] Emphasis has been placed on the analysis of education in the context of endogenous growth models, with an interest not only in the resulting growth implications but in the evolution of inequality over time. These analyses generally consider sequential voting over different parts of the tax and voucher system (and do not include other types of transfer payment).

The present analysis is not directly concerned with growth but fits more closely into the optimal tax tradition, where a social welfare function is max-

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[1] Studies of target effeciency typically follow the influential study by Beckerman (1979). For broad discussions of means-testing versus universal benefits, see Mitchell, Harding and Gruen (1994), Atkinson (1995) and Creedy (1995, 1996), although these studies do not examine education vouchers.

[2] Much of this literature was stimulated by Glomm and Ravikumar (1992) and includes, for example, Cardak (1999, 2004), Bearse *et al.* (2000, 2004), Caucutt (2004) and Fernandes and Rogerson (2001). More detailed comparisons are made below. There are of course many issues relating to the use and design of voucher systems, which do not involve the types of model discussed here. For reviews of issues, see West (1997) and Cardak and Hone (2003). A critical review is by Ladd (2002). There is also a literature on the geographical mobility effect of vouchers; see for example, Nechyba (1999).

imised, involving a multi-dimensional search over policy variables. These include the income tax rate, the voucher taper (or withdrawal) rate, the maximum voucher and the social dividend, where a degree of freedom in those choices is lost because of the government's budget constraint.

The framework of analysis is described in section 2. Parents are assumed to maximise a utility function which includes their consumption, leisure and the human capital of children, where the latter is generated from a human capital production function with inputs consisting of parental human capital and expenditure on education. The expenditure on education is constrained to be at least as large as the maximum value of the voucher available. Parents differ both in their own human capital and their preferences regarding the education of children. The voucher, along with a social dividend and any other non-transfer government expenditure required, is financed from a proportional income tax within a pay-as-you-go system involving the generation of parents. Means testing introduces considerable complexities in consumption, education expenditure and labour supply decisions of parents, because of the non-convexities of the resulting budget constraints facing parents.

Section 3 applies this framework to the case of a uniform, or unconditional, education voucher system. The complications arising from means-testing are examined in section 4. The complexities of individual consumption, education expenditure and labour supply behaviour, combined with the nonlinearity of the government's budget constraint, mean that numerical iterative methods must be used to solve the model. A range of numerical analyses are presented in section 5. Alternative combinations of voucher and tax and transfer schemes are evaluated using a standard social welfare function defined in terms of the utility of parents, and including an aversion to inequality. The approach is therefore an extension of the type of model used in the optimal tax literature. The alternatives examined include the special situation in which means-testing is preferred, arising from a constraint on the income tax rate. Brief conclusions are in section 6.

2 The Framework of Analysis

This section describes the basic model. The utility functions and human capital production function are set out in subsection 2.1. The tax and transfer system is then described in subsection 2.2.

2.1 Utility Functions

In the following analysis, in order to avoid problems associated with joint utility maximisation and labour supply decisions, parents are essentially treated as a single individual. The population is made up entirely of such parents, with each person having a single child. The subscript, t, refers to the generation of parents while the subscript, $t+1$, refers to the generation of children. Each parent has endowments of time, normalised to unity, and human capital, denoted by $h_{i,t}$ for parent i. In the present context human capital actually reflects the productivity, or wage rate, of an individual. Earnings are the only source of income, other than government transfer payments.

Each parent is assumed to derive utility from its own consumption and leisure, and from the human capital of the child. Let $C_{i,t}$ denote the consumption of the ith parent, where the price is normalised to unity. The leisure of the parent is denoted $L_{i,t}$, and the human capital of the child is denoted $h_{i,t+1}$. The utility function of the parent is assumed to be Cobb-Douglas, expressed for convenience in logarithmic form, so that:[3]

$$U_{i,t} = \theta \log C_{i,t} + \alpha \log h_{i,t+1} + (1 - \theta - \alpha) \log L_{i,t} \qquad (1)$$

For convenience, no subscripts have been placed on the parameters. However, in section 5, the specification is extended to allow for a joint distribution of α and h_t.

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[3]This is similar to the form used by Preston (2003), except that here there is a non-unit coefficient on C. In Glomm and Ravikumar (1992) it is 'schooling quality' which enters U, not the human capital of the child; furthermore, the child is assumed to make a decision regarding the choice of work and time spent in education, and the adult has fixed labour supply. Cardak (1999, 2004) uses a similar model to that of Glomm and Ravikumar. In Bearse *et al.* (2000) there are no labour supply choices. Fernandez and Rogerson (2001) have a utility function including consumption and the expected value of next period's income of the child, with no labour supply variations.

The child's human capital results from a production function involving the parent's human capital and expenditure, made entirely by the parent, devoted to the education of the child. Define $E_{i,t}$ as education expenditure, and $h_{i,t}$ is the human capital of the parent. The human capital production function takes the Cobb-Douglas form:[4]

$$h_{i,t+1} = \delta E_{i,t}^{\gamma} h_{i,t}^{1-\gamma} \tag{2}$$

with $\delta > 1$. Hence $h_{i,t+1}$ is proportional to a weighted geometric mean of expenditure on education and the parent's human capital.

Substituting (2) into (1) gives:

$$U_{i,t} = k + \theta \log C_{i,t} + \alpha\gamma \log E_{i,t} + (1 - \theta - \alpha) \log L_{i,t} \tag{3}$$

where $k = \alpha \left\{ \log \delta + (1 - \gamma) \log h_{i,t} \right\}$.

2.2 Taxes and Transfers

Suppose there is an unconditional transfer payment of b per parent.[5] In addition, a non-transferable education voucher, worth $V_{i,t}$, is available to the parent. Each parent may supplement the voucher by spending an additional amount, $S_{i,t}$, on the child's education. Hence expenditure is given by:

$$E_{i,t} = V_{i,t} + S_{i,t} \tag{4}$$

Clearly, $S_{i,t} \geqq 0$, so the restriction, $E_{i,t} \geqq V_{i,t}$ must hold – otherwise the voucher is equivalent to a transfer payment which can be used by the parent to finance its own consumption.

Suppose that the voucher and unconditional transfer system is financed by a proportional tax on the earnings of parents at the rate, τ. Gross earnings

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[4]This is the form used by Preston (2003), except for the addition of the efficiency term δ. Glomm and Ravikumar (1992) have an additional term to allow for the proportion of time spent in education of the child.

[5]The models in Preston (2003), Cardak (1999, 2004), Glomm and Ravikumar (1992), Bearse *et al.* (2000) and Bearse *et al.* (2003) have no transfer payments in addition to the voucher. This means that all individuals work (since there are no other sources of income), and in means-tested systems, those not eligible for a voucher have fixed labour supplies (with Cobb-Douglas preferences).

are denoted $Y_{i,t}^G$, so that:

$$Y_{i,t}^G = h_{i,t} \left(1 - L_{i,t}\right) \tag{5}$$

Net income, $Y_{i,t}^N$, from earnings and the transfer payment, is given by:

$$Y_{i,t}^N = h_{i,t} \left(1 - L_{i,t}\right) \left(1 - \tau\right) + b \tag{6}$$

The parent's budget constraint is thus:

$$h_{i,t} \left(1 - L_{i,t}\right) \left(1 - \tau\right) + b + V_{i,t} = C_{i,t} + E_{i,t} \tag{7}$$

It is assumed that transfers and vouchers are financed on a pay-as-you-go basis for each generation of parents. With n parents, total income tax revenue is equal to $\tau \sum_i^n Y_{i,t}^G$. The total cost of the voucher system is $\sum_i^n V_{i,t}$ and the total cost of the unconditional transfer is nb. Hence the government's budget constraint is expressed as:[6]

$$\tau \sum_i^n Y_{i,t}^G = \sum_i^n V_{i,t} + nb \tag{8}$$

and:

$$\tau \overline{Y}^G = \overline{V} + b \tag{9}$$

where $\overline{Y}^G$ and $\overline{V}$ are average earnings and average voucher received by parents, and the t subscripts are omitted for convenience. The simple property that $\tau = \left(\overline{V} + b\right) / \overline{Y}^G$, so that the tax rate is equal to the ratio of benefits per person to average earnings, is nevertheless deceptive. In fact, it is highly nonlinear in view of the dependence of earnings on the tax and transfer parameters. This is shown in the following sections, which consider alternative voucher arrangements.

3 A Uniform Voucher

Suppose there is an unconditional uniform voucher system, so that:

$$V_{i,t} = \overline{V} \tag{10}$$

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[6]There is thus a pay-as-you-go system, involving the generation of parents, for the finance of vouchers and benefits. The possibility of tax smoothing is not examined here.

The full income of the parent, $M_{i,t}$, is therefore given by:

$$M_{i,t} = h_{i,t} (1 - \tau) + b + \overline{V} \tag{11}$$

The parent's problem is to select $E_{i,t}$, $C_{i,t}$ and $L_{i,t}$ to maximise utility, given by (3) subject to the budget constraint. Since the effective price of leisure per unit is the net wage, $h_{i,t} (1 - \tau)$, the budget constraint in (7) above can be rewritten as:

$$C_{i,t} + E_{i,t} + h_{i,t} (1 - \tau) L_{i,t} = M_{i,t} \tag{12}$$

There are the additional constraints that $E_{i,t} \geqq \overline{V}$ and $L_{i,t} \leqq 1$. The most convenient way to consider this optimisation problem is first to examine interior, or tangency, solutions and then to examine whether the inequality constraints are satisfied.

For interior solutions, the standard Cobb-Douglas results give, for consumption:[7]

$$C_{i,t} = \frac{\theta}{1 + \alpha (\gamma - 1)} M_{i,t} \tag{13}$$

with $1 + \alpha (\gamma - 1)$ equal to the sum of the coefficients on (the logarithms of) C, E and L in (3). Education expenditure is given by:

$$E_{i,t} = \frac{\alpha \gamma}{1 + \alpha (\gamma - 1)} M_{i,t} \tag{14}$$

And the demand for leisure is equal to:

$$
\begin{aligned}
L_{i,t} &= \left(\frac{1 - \theta - \alpha}{1 + \alpha (\gamma - 1)} \right) \frac{M_{i,t}}{h_{i,t} (1 - \tau)} \\
&= \left(\frac{1 - \theta - \alpha}{1 + \alpha (\gamma - 1)} \right) \left\{ 1 + \frac{b + \overline{V}}{h_{i,t} (1 - \tau)} \right\}
\end{aligned} \tag{15}
$$

since $h_{i,t} (1 - \tau)$ is the price of leisure.

These results hold only when the inequality constraints, $E_{i,t} \geqq \overline{V}$ and $L_{i,t} \leqq 1$, are satisfied. Attention needs to be given to possible corner solutions. From (14), the parent spends more on education than the voucher

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[7]For Cobb-Douglas utility functions, the demand for any good is expressed as the product of two terms. The first is the ratio of the exponent on the good to the sum of all exponents, while the second term is the ratio of full income to the relevant price.
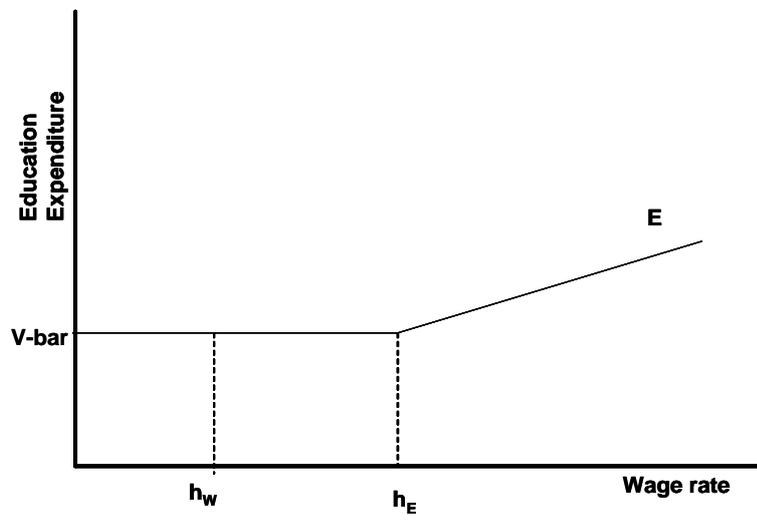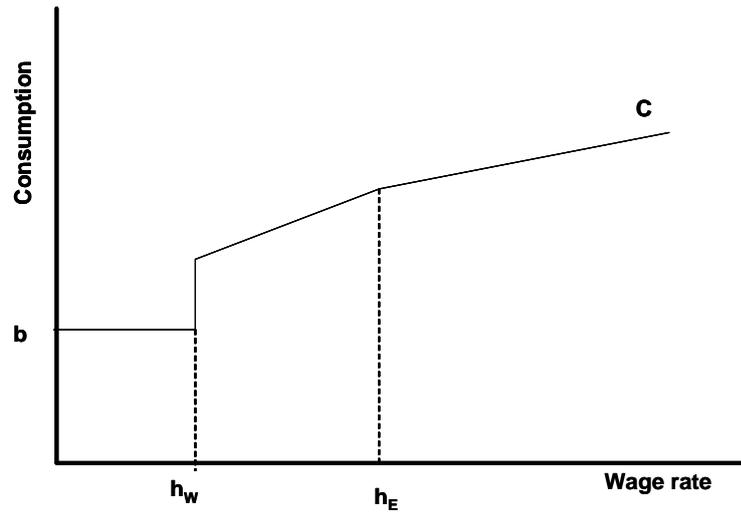
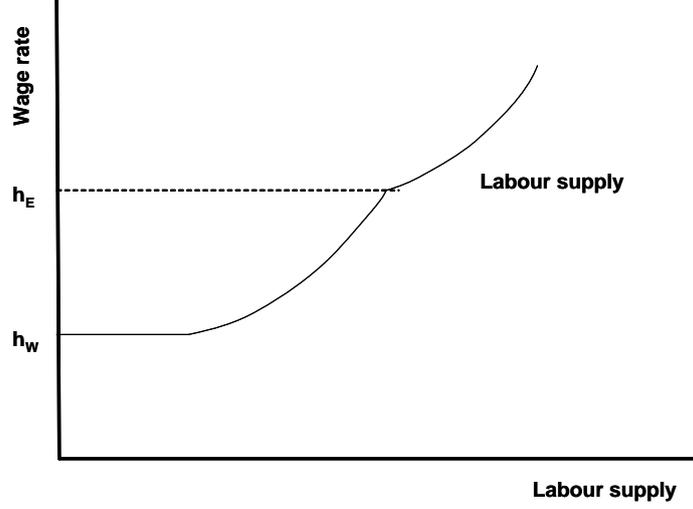Figure 1: Education Expenditure and Consumption

Figure 2: Labour Supply with Uniform Voucher

only when human capital exceeds a threshold, h_E, such that:

$$h_E = \left(\frac{b + \overline{V}}{1 - \tau}\right)\left(\frac{1 - \alpha}{\alpha\gamma}\right) \tag{16}$$

If $h_{i,t} < h_E$, it is necessary to set $E_{i,t} = \overline{V}$. As a consequence of this, consumption and leisure are determined by maximising:

$$U_{i,t} = k' + \theta \log C_{i,t} + (1 - \theta - \alpha)\log L_{i,t} \tag{17}$$

where $k' = \alpha\left\{\log\delta + (1 - \gamma)\log h_{i,t} + \gamma\log\overline{V}\right\}$. Applying the standard Cobb-Douglas properties gives consumption and leisure, instead of the above results, as:

$$C_{i,t} = \frac{\theta}{1 - \alpha}\left(h_{i,t}\left(1 - \tau\right) + b\right) \tag{18}$$

and:

$$L_{i,t} = \left(\frac{1 - \theta - \alpha}{1 - \alpha}\right)\left(1 + \frac{b}{h_{i,t}\left(1 - \tau\right)}\right) \tag{19}$$

Finally, from (19), the parent works if human capital (the wage rate) exceeds a threshold, h_W, where:

$$h_W = \left(\frac{b}{1 - \tau}\right)\left(\frac{1 - \theta - \alpha}{\theta}\right) \tag{20}$$

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For those with $h_{i,t} < h_W$, then $L_{i,t} = 1$, $E_{i,t} = \overline{V}$ and $C_{i,t} = b$.

These results show that the relationships between human capital and expenditure on education and consumption take the form of piecewise-linear schedules; Cobb-Douglas utility functions imply that both $E_{i,t}$ and $C_{i,t}$ are linear functions of $h_{i,t}$, between relevant ranges.[8] The typical forms are shown in Figure 1. Consumption and education expenditure are respectively b and $\overline{V}$, when $h_{i,t} < h_W$ and the parent does not work. Beyond h_W consumption jumps to a higher level, associated with the jump in labour supply shown in Figure 2. A feature of the model is that there is not a smooth transition into work, and this is caused by the fact that the non-transferable education voucher implies that individuals must, over a range of relatively low values of h, effectively spend more of their net income on education than they would otherwise spend. As shown in the lower segment of Figure 1, education expenditure does not exceed $\overline{V}$ until $h_{i,t} > h_E > h_W$. The discontinuity in the labour supply from non-work to work depends (in part) on the size of unconditional transfer payment relative to the education voucher, which prescribes the minimum education expenditure. A higher value of b means that the threshold value, h_W, is higher, but the initial work hours are lower.[9] Furthermore, the higher b also means that the threshold h_E is lower, so that education expenditure exceeds the fixed voucher level over a wider range of parent's human capital.

4 A Means-tested Voucher

Suppose the voucher is means-tested and subject to a taper rate of β. The maximum voucher, received by those who do not work, is V^* and the voucher

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[8]On Cobb-Douglas properties in basic optimal tax/transfer models, see Creedy (1996).

[9]Even where $b > \overline{V}$, there is a discrete jump on entry into work, so long as desired education expenditure is lower than $\overline{V}$.

received by the ith parent, $V_{i,t}$, is given by:[10]

$$V_{i,t} = \max\left(V^* - \beta Y_{i,t}^G, 0\right) \tag{21}$$

As above, $Y_{i,t}^G = h_{i,t}\left(1 - L_{i,t}\right)$ is the gross earnings of the ith parent. Those with $Y_{i,t}^G \geqq V^*/\beta$ receive no voucher and must fully fund their child's education from post-tax earnings. However, the constraint is imposed on all parents that they must spend at least V^* on education. The non-convexity in the budget set, introduced by means testing, complicates the labour supply and consumption behaviour of individuals in two ways. First, there can be multiple local optima, with one being at a corner solution involving no labour supply and the other being a tangency position where no voucher is obtained. There can also be simultaneous tangency positions on both segments of the budget constraint, along the same indifference curve. Secondly there can be discrete jumps in labour supply when a small change in the net wage causes an individual to move between segments of the budget constraint. The critical net wage is that giving rise to the two tangency position along a single indifference curve.

This section presents the analytics of means-tested vouchers. Subsections 4.1 and 4.2 examine consumption and labour supply for those eligible to receive the voucher and those who exhaust their entitlement, respectively. Subsection 4.3 considers the possible profiles which can arise from such a system.

4.1 Voucher Recipients

For those who are eligible for a means-tested voucher, the budget constraint is:

$$h_{i,t}\left(1 - L_{i,t}\right)\left(1 - \tau\right) + b + V^* - \beta h_{i,t}\left(1 - L_{i,t}\right) = C_{i,t} + E_{i,t} \tag{22}$$

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[10]This is the standard form of means-testing for transfer payments. Means testing is not considered by Preston (2003) or Cardak (1999, 2004). Fernandez and Rogerson (2001) consider two alternative types of means-testing. First, those below a threshold are given a fixed voucher, while those above the threshold receive no voucher. Second, those below the threshold receive a voucher depending on their income and the amount spent on education.

which can be rearranged as:

$$
\begin{aligned}
L_{i,t}h_{i,t}\left(1-\tau-\beta\right)+C_{i,t}+E_{i,t} &= h_{i,t}\left(1-\tau-\beta\right)+b+V^* \\
&= M_{i,t} \qquad (23)
\end{aligned}
$$

Hence for this group, the constraint looks exactly like the universal grant budget constraint except that the effective income tax rate is $\tau+\beta$ rather than simply τ. Grant recipients therefore face a higher effective tax rate than those who have $Y_{i,t}^{G} \geqq V^*/\beta$. This gross earnings threshold translates into a threshold in terms of labour supply, $1-L_{i,t}$, of $V^*/\beta h_{i,t}$. Hence this range of the budget set can be ruled out immediately if it results in $L_{i,t} < 1-V^*/\beta h_{i,t}$. As in the case of a uniform voucher, a check must be made to ensure that the resulting value of $E_{i,t}$ is at least V^*. If the constraint does not hold, the appropriate adjustment must be made, following exactly the same procedure as described in the previous section. As before, those who do not work receive the full voucher of V^*, and consume the universal transfer of b. However, if this corner solution applies to an individual parent, it may be only one local optimum. It is necessary to check the possibility that the parent may be better off by working relatively long hours and paying the lower marginal tax rate, while receiving no voucher.

4.2   Voucher Non-recipients

Those who have exhausted their benefit entitlement face a budget constraint of:

$$
h_{i,t}\left(1-L_{i,t}\right)\left(1-\tau\right)+b = C_{i,t}+E_{i,t} \qquad (24)
$$

In terms of full income, $M_{i,t}$, this translates to:

$$
\begin{aligned}
L_{i,t}h_{i,t}\left(1-\tau\right)+C_{i,t}+E_{i,t} &= h_{i,t}\left(1-\tau\right)+b \\
&= M_{i,t} \qquad (25)
\end{aligned}
$$

With this modification, the results for consumption, education expenditure and leisure in equations (13), (14) and (15) apply simply by setting $\overline{V} = 0$.[11] Any tangency solution giving rise to gross earnings, below the

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[11] If there were no transfer payment, the labour supply of non-recipients would be constant and independent of the income tax rate, τ.

threshold above which the voucher is exhausted, must of course be ruled out as infeasible.

It is necessary to check that the resulting value of $E_{i,t} \geqq V^*$, which holds when:

$$h_{i,t} \geqq \frac{1 + \alpha \left(\gamma - 1 \right)}{\alpha \gamma} \left(\frac{V^*}{1 - \tau} \right) - \frac{b}{1 - \tau} \qquad (26)$$

If individuals are constrained to set $E_{i,t} = V^*$, an adjustment must be made to the values of leisure and consumption, again following the approach discussed in the previous section. It is possible that this adjustment rules out this range of hours worked where no voucher is received, that is if the resulting $L_{i,t}$ is greater than $1 - V^*/\beta h_{i,t}$.

4.3 Possible Profiles

A possible relationship between the gross wage rate (human capital) and consumption is shown in Figure 3. This is similar to that shown in Figure 1 except that there is an additional range above h_E. For a wage rate of h_S, a small increase causes the individual to jump from working and receiving a means-tested voucher to a higher labour supply where no voucher is received. Figure 3 shows a situation in which the individual is voluntarily spending more on education than the maximum voucher, while continuing to receive a reduced (means-tested) voucher.

However, it is possible that the consumption profile could take alternative forms. Two possibilities are shown in Figures 4 and 5. In Figure 4, all those receiving a means-tested voucher are constrained by the need to spend more on education than they would otherwise wish, and some of those who exhaust their entitlement to the voucher face a similar binding constraint. Figure 5 illustrates the situation in which the taper rate is so high that it is never optimal to work and receive a voucher, but some of the workers face the binding constraint whereby $E_{i,t}$ is set equal to V^*. Yet another possibility, not illustrated here, is that $h_E < h_W$ in Figure 5.
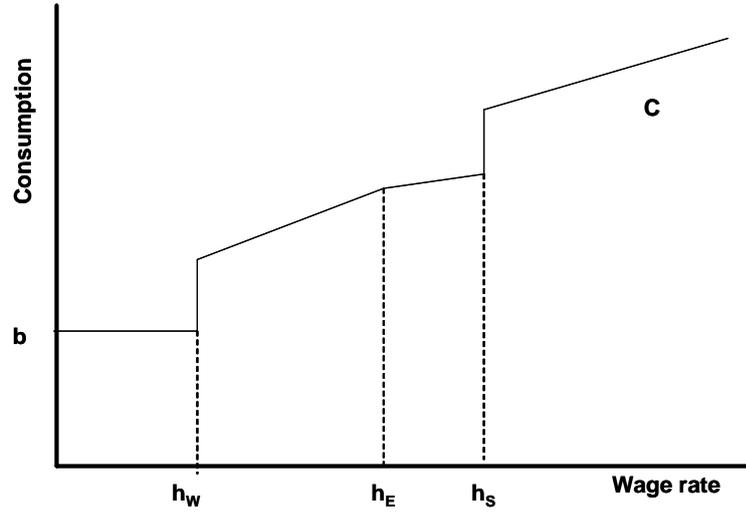
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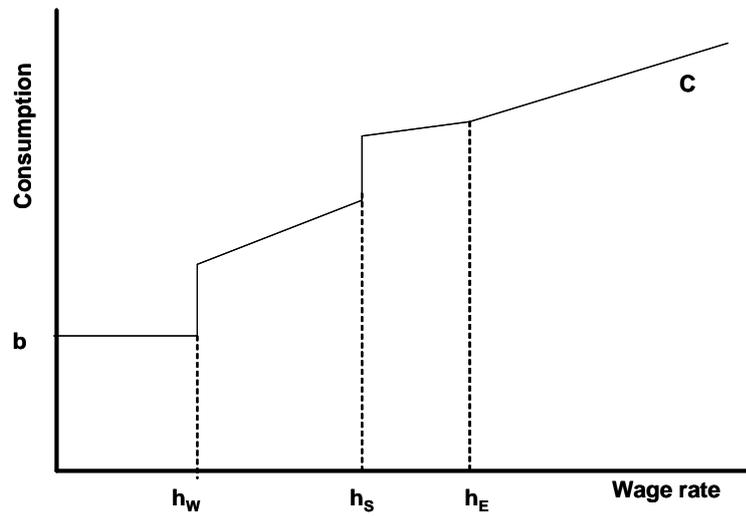Figure 3: Consumption with Means-tested Voucher

Figure 4: Consumption with Means-tested Voucher: Alternative Profile
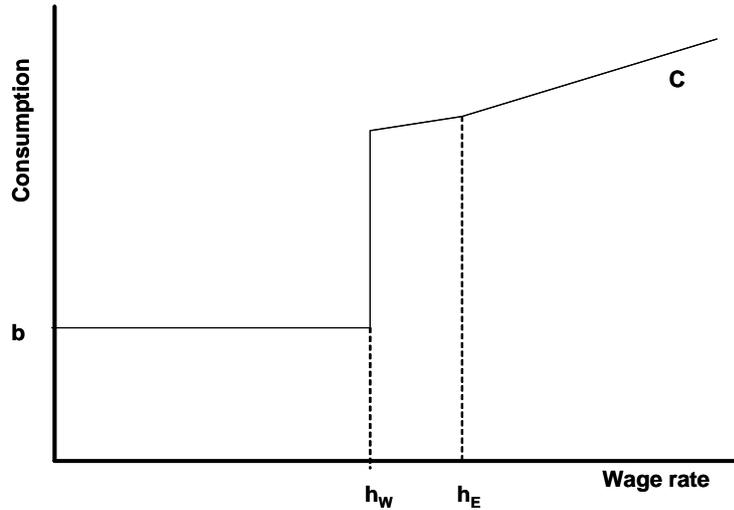
Figure 5: Consumption with Means-tested Voucher: Alternative Profile

5   Numerical Analyses

In view of the nonlinearity of the model, further results require the use of numerical simulation methods. This section reports a number of analyses designed to obtain the optimal values of the four policy variables τ, β, V^* and b. The simulation procedure is described in subsection 5.1. In order to interpret the results, it is useful to consider alternative profiles of consumption and other expenditure as human capital (the wage rate) varies; these are discussed in subsection 5.2. Variations in the income tax rate, for given values of the other parameters, are examined in subsection 5.3. Optimal rates are finally discussed in subsection 5.4.

5.1   Simulation Procedure

In simulating a population it is first necessary to specify the nature of population heterogeneity. In optimal tax models it is usual simply to assume that preferences are identical and only wage rates differ. However, suppose parents' preferences regarding the human capital of children, determined by the

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parameter α of the utility function, are correlated with human capital, $h_{i,t}$.[12] This can be modelled by specifying a joint distribution of h and α. Dropping subscripts for convenience, suppose that h and α are jointly lognormally distributed as:

$$\Lambda\left(h, \alpha \mid \mu_h, \mu_a, \sigma_h^2, \sigma_\alpha^2, \rho\right) \tag{27}$$

A simulated population may be obtained as follows. First, select a random observation from the marginal distribution of h. If v_i is a random variable drawn from an $N(0,1)$ distribution, the corresponding value of the ith parent's human capital, h_i, is obtained as:

$$h_i = \exp\left(\mu_h + v_i \sigma_h\right) \tag{28}$$

A corresponding value of α_i is obtained from the conditional distribution of α, given h. Let u_i denote another random draw from an $N(0,1)$ distribution. Then:

$$\alpha_i = \exp\left\{\mu_\alpha + \rho\frac{\sigma_\alpha}{\sigma_h}\left(\log h_i - \mu_h\right) + u_i \sigma_\alpha \left(1 - \rho^2\right)^{1/2}\right\} \tag{29}$$

It is assumed that individuals share a common value of θ, the coefficient on consumption in the utility function. Preferences for leisure also vary, since the coefficient is obtained as $1 - \theta - \alpha_i$.

In the following simulations, it is assumed that $\mu_h = 10$ and $\sigma_h^2 = 0.5$. In addition, a minimum value of h_i of 2000 is imposed. A simulated population of 5000 parents is used. In the majority of cases presented, it is assumed that $\sigma_\alpha = 0.02$, and $\rho = 0.5$, so that there is a tendency for those parents with relatively high human capital to have a higher preference for increasing that of their children. It is most convenient to specify the arithmetic mean value, $\overline{\alpha}$, and then to determine the appropriate value of μ_α using:

$$\mu_\alpha = \log\overline{\alpha} - \frac{1}{2}\sigma_\alpha^2 \tag{30}$$

Results are given for a range of values of the parameter, γ, in the human capital production function. The consumption, education expenditure

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[12]Heterogeneous preferences in education models were introduced by Cardak (1999). Preston (2003) assumed a uniform distribution of α.

and leisure choices of parents are independent of the parameter δ, so for convenience this is set at 2.6 in all cases presented below.[13]

Having produced a population of parents, the aim is to determine optimal values of the policy variables. These are the two tax rates, τ and β, and the two transfer levels, V^* and b.[14] But one degree of freedom is lost because of the government budget constraint in equation (9) above. Hence, a search over three dimensions is needed. The approach is as follows. For each combination of voucher parameters examined, β and V^*, a range of tax rates τ is considered, where of course the restriction $\beta + \tau < 1$ must limit the upper value of the income tax rate in the range. In addition, a minimum value of τ applies, given the need to finance the voucher system.

For each value of τ, the government budget constraint is solved iteratively to produce the corresponding value of the unconditional transfer payment, b. A trial value of b, say b_0, is used to examine each parent's choices, and hence the corresponding values of $\overline{Y}^G$ and $\overline{V}$ are computed. Then the resulting transfer, b_1, is obtained using:

$$b = \tau \overline{Y}^G - \overline{V} \tag{31}$$

If $b_1 > b_0$, the trial value is increased slightly, or if $b_1 < b_0$ the trial value is reduced, and another iteration is carried out until convergence is reached. It would not of course be appropriate to try to solve the budget constraint for τ, given a value of the transfer payment, b, because it is possible to have two different tax rates corresponding to any transfer level: holding other parameters constant, the feasible value of b first increases and then falls as τ is increased. However, the restriction on the range of τ, determined by the assumed value of β, means that b does not fall in all cases; examples are shown below.

The alternative tax structures can be evaluated using a social welfare function, which reflects the value judgements of an independent judge or hypothetical policy maker. Following the standard approach used in the optimal

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[13]This value is of course relevant in a growth context. In addition, given the form of the constant, k, in the utility function, it affects the absolute value of utility. This constant term differs among parents given the variability in α.

[14]If $\beta = 0$, then $\overline{V} = V^*$.

tax literature, the social welfare function is assume to take the individualistic Paretean form:

$$W = \sum_{i=1}^{n} \frac{U_i^{1-\varepsilon}}{1 - \varepsilon} \tag{32}$$

where ε is the constant relative inequality aversion parameter of the judge.

5.2 Individual Profiles

Examples of labour supply and expenditure profiles are shown in Figures 6 to 9. In each case $\theta = 0.45$, $\alpha = 0.35$ and $\gamma = 0.6$. The only difference between Figures 6 and 7 is that in the former the taper rate β is equal to 0.1 while in the latter it is increased to 0.3. The figures clearly show the extent to which the lower taper implies that a positive voucher is received over a wider range of the wage rate (the human capital of the parent). Furthermore, education expenditure exceeds the minimum required (determined by the maximum voucher V^*) over a slightly wider range, and parents work over a wider range of h. For the lower taper in Figure 6, the relevant wage thresholds are $h_W = 2300$, $h_E = 3800$ and $h_S = 27,800$. For the higher taper in Figure 8, the relevant wage thresholds are $h_W = 3300$, $h_E = 5300$ and $h_S = 10,800$; a larger number of parents receive no voucher at all.

Figure 8 illustrates profiles for higher levels of the unconditional transfer and the maximum voucher, and of the tax rates. In this case, all individuals below $h_W = 7000$ do not work, and human capital must reach $h_E = h_S = 17000$ until the level of education expenditure exceeds the minimum required amount: this arises after the discrete jump takes place onto the range where no voucher is received. Finally, Figure 9 has a relatively high unconditional transfer, financed by a higher income tax rate, but there is a uniform voucher ($\beta = 0$). Here education expenditure exceeds the universal voucher of 2800 when $h_{i,t} > h_E = 5700$.

5.3 Variations in the Income Tax Rate

An indication of the effect of varying the income tax rate, for given values of the other parameters, is given in Figures 10 and 11. In each case $\theta = 0.45$,
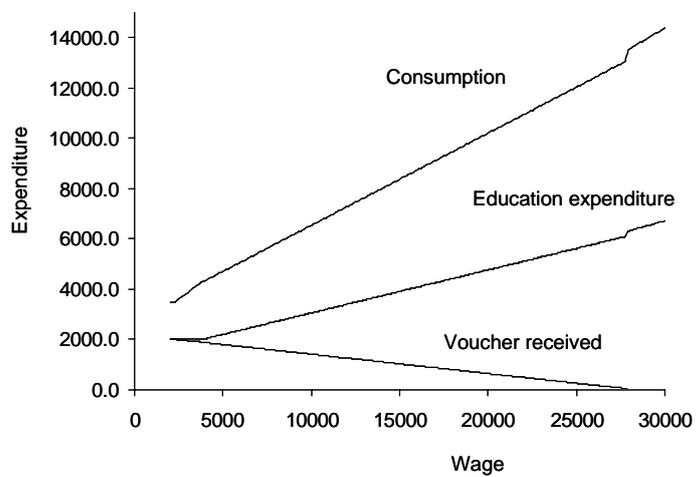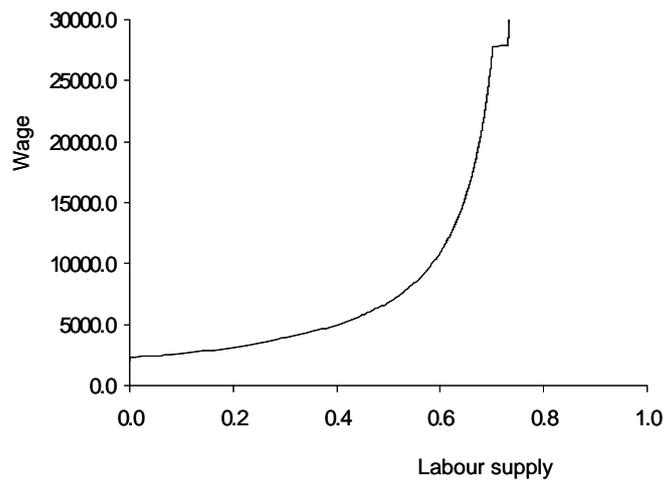
Figure 6: Labour Supply and Expenditure Profiles: $\overline{V} = 2000$; $b = 3500$; $\beta = 0.1$; $\tau = 0.2$
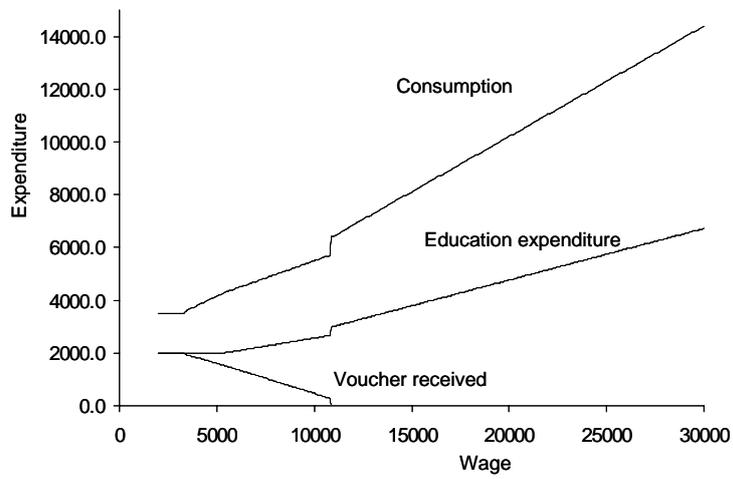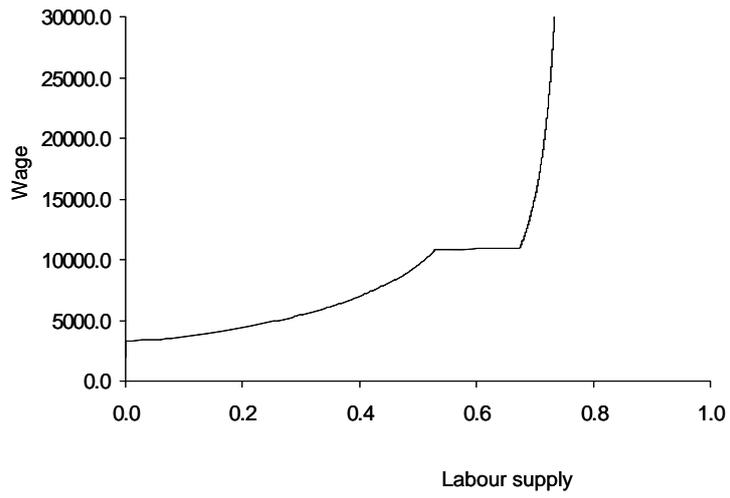
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Figure 7: Labour Supply and Expenditure Profiles: $\overline{V} = 2000$; $b = 3500$; $\beta = 0.3$; $\tau = 0.2$
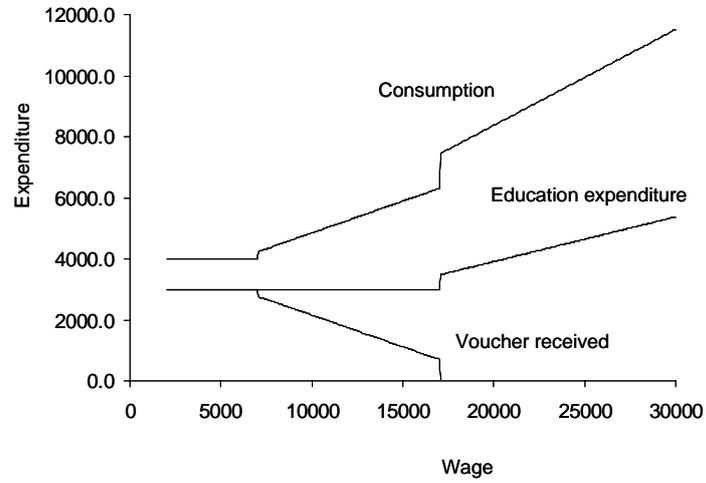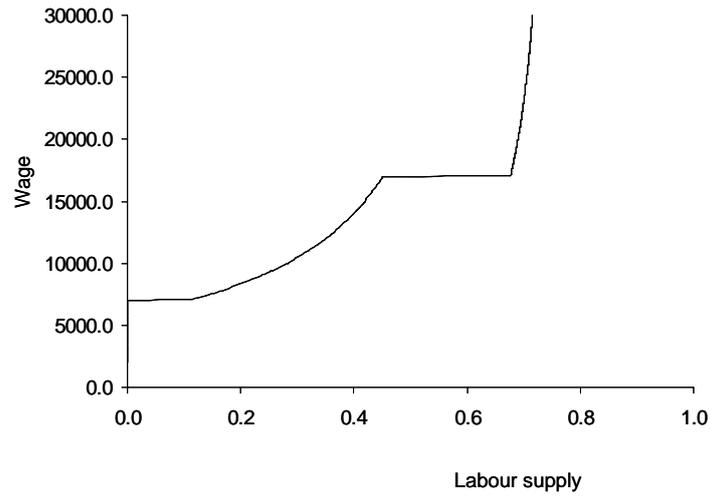
Figure 8: Labour Supply and Expenditure Profiles: $\overline{V} = 3000$; $b = 4000$; $\beta = 0.3$; $\tau = 0.4$
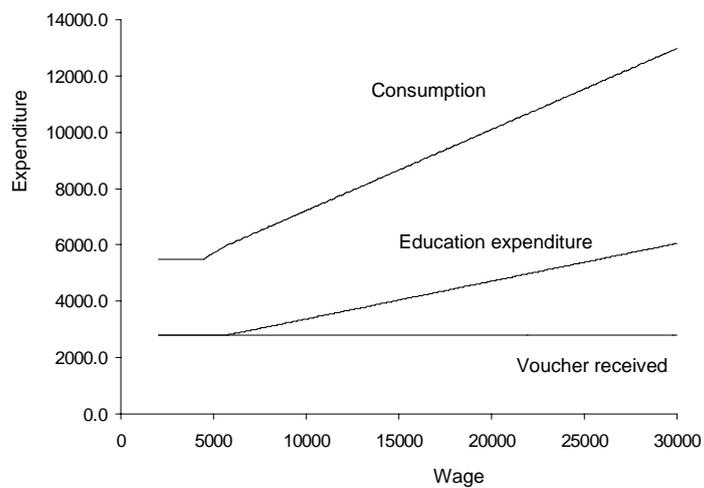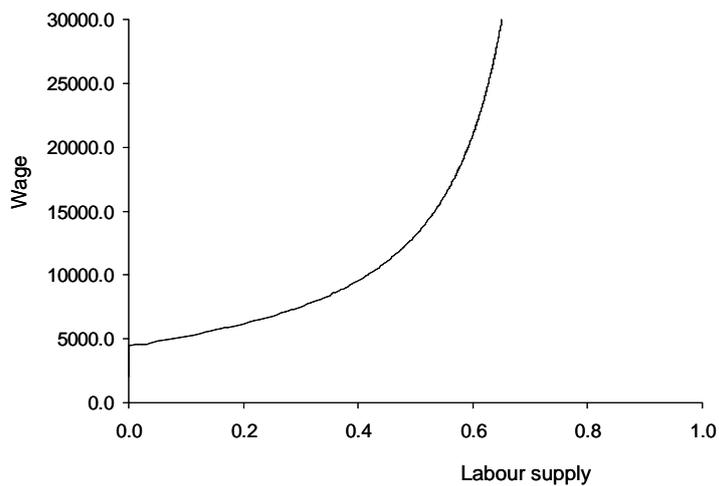
Figure 9: Labour Supply and Expenditure Profiles: $\overline{V} = 2800$; $b = 5500$; $\beta = 0$; $\tau = 0.45$

$\overline{\alpha} = 0.35$, $\sigma_\alpha^2 = 0.02$, $\gamma = 0.6$. Social welfare is evaluated using an inequality aversion of $\varepsilon = 0.5$. The difference in the two sets of diagrams is the maximum voucher level, set at 2000 and 5000 in Figures 10 and 11 respectively, and results are shown in each case for two taper rates.

For the lower value of $V^* = 2000$, the higher taper rate allows a slightly higher transfer payment to be financed, for a given value of τ, though the range of income tax rates is substantially restricted with the higher taper rate. The resulting social welfare is higher with the lower taper, for a given income tax rate. For the case where $V^* = 5000$, the profiles in Figure 11 are slightly different. The minimum value of τ must be higher with the lower taper rate, because the greater generosity of the voucher system means that insufficient revenue is obtained unless the tax rate is sufficiently high. The profiles of both b and W are somewhat different in this case, as they have a kink in them at a tax rate of around 0.34. In the case of the b profile, the transfer payment can actually be higher for a high taper rate compared with a low taper.

These results are explained by the behaviour of labour supply in the two ranges of the income tax rate. For example, with preferences of $\theta = 0.45$ and $\alpha = 0.35$, and tax parameters $V^* = 5000$, $b = 3500$, $\tau = 0.3$ and $\beta = 0.5$, the three wage (or human capital) thresholds are $h_W = 12,800$, $h_S = 17,400$ and $h_E = 24300$. Hence as the wage increases, individuals begin to work and receive a means tested benefit, then jump to the range of the budget constraint where no benefit is received, while continuing to spend only the minimum amount (of 5000) on education, until a wage of 24300 is reached. However, with the same parameters except that $b = 3800$ and $\tau = 0.38$, individuals jump directly from not working (and receiving the full voucher) to the segment of the budget constraint where no voucher is received; that is, $h_W = h_S = 18700$.[15] The simulations have a distribution of the parameter α, as explained above, but over the higher income tax rates the small number of people receiving a reduced voucher allows the unconditional transfer to be higher with a high taper than with the low taper rate. However, Figure 11 shows that social welfare is unambiguously higher, for given income tax rate,
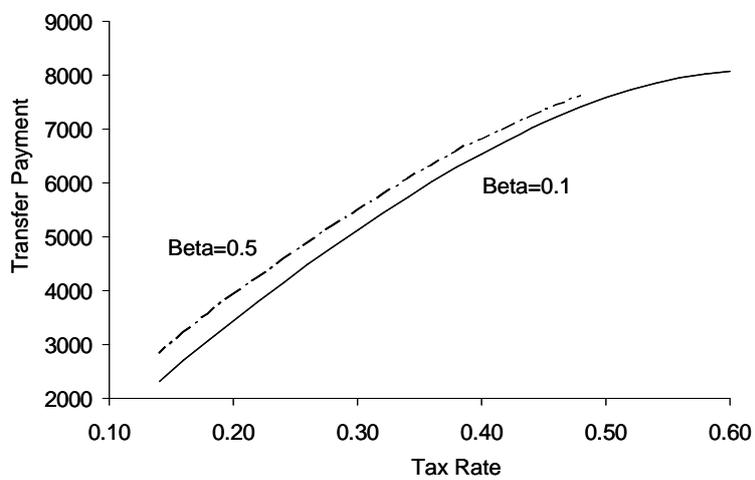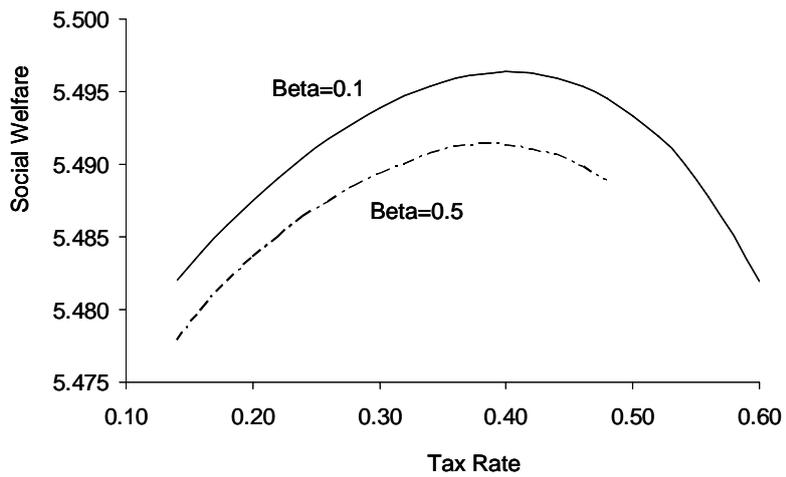
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[15]Furthermore, $h_E = 26900$.

Figure 10: Social Welfare and Transfer payment: $\overline{V} = 2000$
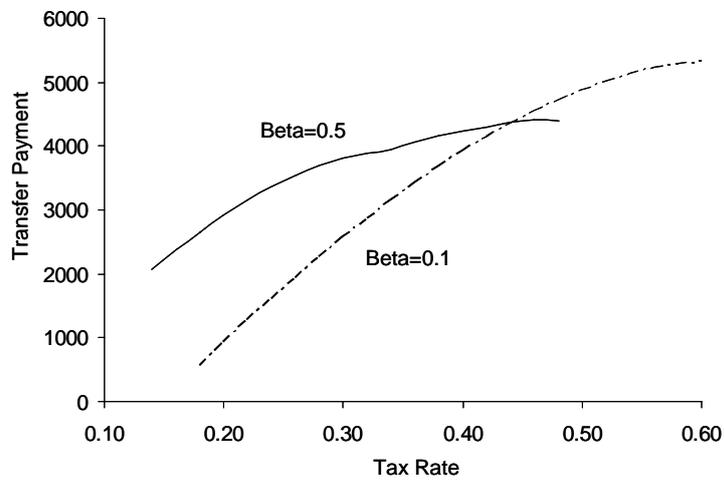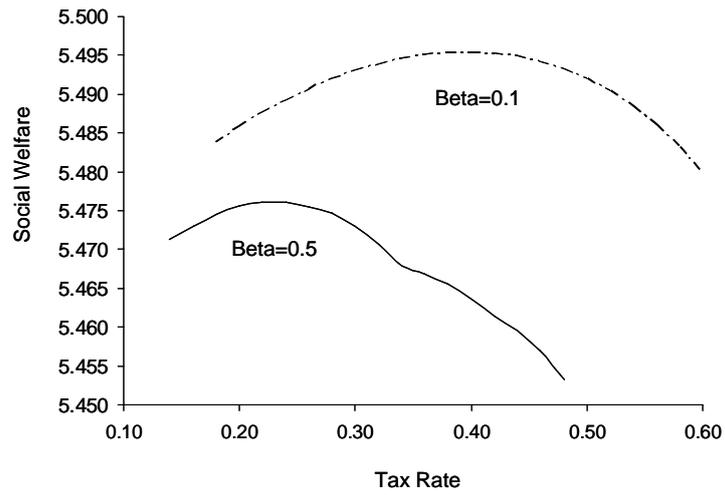
Figure 11: Social Welfare and Transfer Payment: $\overline{V} = 5000$

with the lower taper rate.

5.4   Optimal Tax Rates

The examples reported in the previous subsection have shown that, in these cases, social welfare is unambiguously higher with the lower taper rates, for given values of τ. But of course, depending on the choice of all the tax parameters, it is quite possible to achieve higher social welfare with a higher taper rate. For example, where $V^* = 2000$, social welfare is higher with the combination of $\beta = 0.5$ with $\tau = 0.2$, than when $\beta = 0.1$ and $\tau = 0.2$.

This subsection considers the optimal combination of the four tax parameters. This is achieved by carrying out a grid search over three dimensions, V^*, β and τ, where in each case the value of b is determined by solving (iteratively) the government budget constraint. At each iteration the optimal consumption, labour supply and education expenditure decisions of each of the 5000 simulated parents, as described above, are computed. For each of a range of inequality aversion parameters, ε, the value of the social welfare function, $W = \sum_{i=1}^{n} \frac{U_i^{1-\varepsilon}}{1-\varepsilon}$, are evaluated. The major result is that for all preference and human capital production function parameters examined, the optimal value of β was found to be zero; that is, the optimal voucher system is one involving an unconditional voucher.

The sensitivity of results to parameter values is shown in Table 1. Case B, in the first substantive row of the table, refers to the 'base' case used, and variations in the parameters are shown in the following rows: a space indicates that the previous value (in the row above the relevant row) applies. The other parameters are $\mu_h = 10.0$, $\sigma_h^2 = 0.5$, $\rho = 0.5$ and $\sigma_\alpha = 0.02$.[16] Consider first the optimal values shown in the three columns headed 'Full Model'. The optimal tax rates reported here are higher than those generally found in the optimal tax literature involving tax-transfer systems. This clearly results from the productivity of education expenditure, when combined with the

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[16] A positive correlation, ρ, seems the most sensible assumption, although some sensitivity results are reported below. It was also found that there is little effect of increasing the value of σ_α.

Table 1: Optimal Policy Variables for Alternative Parameter Values

Case	θ	$\overline{\alpha}$	γ	ε	Full model			$b=0$	
					τ	V^*	b	τ	V^*
B	0.45	0.35	0.6	0.5	0.47	2800	5674	0.24	4914
1				0.1	0.46	2800	5550	0.24	4914
2				1.5	0.48	2700	5895	0.22	4252
3				3.0	0.50	2800	6026	0.22	4252
4			0.4	0.5	0.46	1900	6100	0.19	3835
5			0.5		0.46	2300	5886	0.22	4470
6			0.7		0.47	3500	5134	0.28	5749
7			0.8		0.47	3600	5178	0.30	6196
8		0.30	0.6		0.44	2100	5196	0.20	3821
9	0.40	0.35			0.45	2500	4812	0.25	4642
10		0.30			0.43	2000	4392	0.22	3795
11	0.35	0.45			0.46	3800	4373	0.29	5715

Notes: $\mu_h = 10.0$; $\sigma_h^2 = 0.5$; $\rho = 0.5$; $\sigma_\alpha = 0.02$

parental human capital.[17] The optimal unconditional transfer payment, or social dividend, is higher than the maximum voucher level, though obviously many parents voluntarily invest more than V^* in education.

The optimal tax rate and transfer levels show very little sensitivity to the degree of inequality aversion of the social welfare function. Furthermore, a higher aversion involves a higher optimal social dividend, with very little effect on the voucher. Similarly, the parameter γ of the human capital production function has very little effect on the optimal tax rate, but does lead to a higher optimal voucher.

The 'redistributive' element involved in having a voucher in a progressive tax-transfer structure is to some extent reduced by the assumed positive correlation between parents' human capital and their preference for childrens' human capital. For the 'base case' a correlation coefficient of $\rho = -0.5$ produces a slightly higher optimal voucher of $V^* = 3200$, with b reduced to 5096 and $\tau = 0.46$. The negative correlation means that, on average,

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[17]The investment obviously increases the tax base for the next generation, but this effect is not allowed for here because the welfare function is defined over parents' utilities in one generation only, and pay-as-you-go financing of transfers and vouchers applies.

α is higher for those with lower h, and given the positive skewness in the distribution of h, it is not surprising that the effect of moving from a positive to a negative correlation is similar to that of increasing $\overline{\alpha}$.

The last two columns of Table 1 showing optimal combinations of τ and V^* for the case where there is no social transfer, so that b is set equal to zero in all cases. In this situation, all individuals work and with means-testing those who are no longer eligible for the voucher have an optimal labour supply that is independent of their net wage, and hence income tax rate. Here, the search for an optimal combination was carried out over two dimensions, by varying τ and β, while for each combination solving iteratively for the value of V^* which satisfies the government budget constraint. Again, a non-means-tested voucher turned out to be optimal for all combinations. In this situation, a higher degree of inequality aversion involves a very small reduction in the optimal tax rate: the education voucher system is not acting as a redistributive device.[18]

The approach used, of maximising a social welfare function defined in terms of individuals' utilities, allows the value of V^* to be determined simultaneously with other tax parameters. An alternative approach may take the view that an independent decision maker or social planner would have particular views about the minimum level of education expenditure per person. The social planner may, for example, take into account externality effects of investment in education (and possibly also consider later generations). However, if the voucher is exogenously set at a level above the values that are found to be optimal in Table 1, and a search is carried out over values of β and τ, while solving the government budget constraint for b, a universal voucher continues to be optimal (with $\beta = 0$). Furthermore, higher imposed values of V^* have very little effect on the optimal tax rate, with the main effect being a reduction in the social dividend.[19]

The model described above involves a 'pure' tax and transfer system in

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[18]Using the 'base case' parameters, a change to $\rho = -0.5$ with $\varepsilon = 3.0$ produces only a small increase in the value of V^* to 3000, with $b = 5536$ and $\tau = 0.48$.

[19]For example, for the 'base case' parameters, if $V^* = 4000$ the optimal values of τ and β are respectively 0.47 and 4513. A higher value of V^* of 6000 results in optimal τ and β of 0.50 and 3100 respectively.

Table 2: Optimal Values with Non-transfer Expenditure

Non-transfer expenditure	τ	V^*	b
1000	0.48	2671	5100
3000	0.50	2286	4100
5000	0.53	1968	3200
7000	0.56	1594	2400

Table 3: Optimal Values with Restricted Income Tax Rate

Restricted τ	β	V^*	b
0.15	0.14	3600	1675
0.20	0.13	3600	2478
0.25	0.12	3700	3140
0.30	0.10	3600	3808
0.35	0.08	3300	4593

which there is no government revenue allocated to non-transfer (including voucher) expenditure which has no effect on individuals' utilities. The introduction of such non-transfer government revenue, which has been found to be significant in the standard optimal tax literature, is easily carried out since it involves only a modification to the government's budget constraint. For the 'base' parameters described in Table 1, examples of the influence of alternative levels of non-transfer expenditure are reported in Table 2. Again, $\beta = 0$ is optimal in all cases. Table 2 shows that the transfer payment, b, is much more sensitive to the introduction of non-transfer expenditure than is the voucher.

In obtaining the optimal combination of policy variables, no constraints have been placed on the range of possible values. It might be argued that if there are exogenous constraints on the income tax rate, and hence the gross revenue which can be raised, the case for targeting the voucher is stronger. Indeed, it is an emphasis on gross revenue in a linear, or flat-tax, system which usually lies behind arguments for means-testing (though of course this is to a large extent arbitrary, since gross revenue is simply reduced if the system is administered by paying only net transfers and collecting only net

income tax). Table 3 reports, again for the 'base case', optimal values of the other policy variables if the income tax rate is restricted to a value below the corresponding value shown in Table 1. The restriction in the income tax rate has very little effect on the optimal value of V^*, with the burden of the constraint falling on the reduction in b. The restriction in the income tax revenue which can be collected does indeed mean that the optimal voucher system involves means testing. However, even with a very low τ, the taper rate is low.[20]

6 Conclusions

This paper has compared a uniform education voucher system with a means-tested scheme in which the voucher is subject to a taper or withdrawal rate as parental gross income increases. The model is one in which parents maximise a utility function which includes their consumption, leisure and the human capital of children. There is a human capital production function with inputs consisting of parental human capital and expenditure on education. The expenditure on education by each parent is constrained to be at least as large as the maximum value of the voucher available. Parents differ both in their own human capital and their preferences regarding the education of children.

In examining the choice of consumption, education expenditure and leisure, it was necessary to pay close attention to the complexities introduced by inequality constraints and non-convexities in the budget set of parents. The government faces a budget constraint such that the voucher and a social dividend are financed from a proportional income tax, within a pay-as-you-go system involving the generation of parents. Alternative combinations of voucher and tax and transfer schemes were evaluated using a standard social welfare function defined in terms of the utility of parents, and including an aversion to inequality.

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[20]Indeed, restricting V^* to be above the unconstrained socially optimal value, *and* constraining the income tax rate, such that $V^* = 5500$ and $\tau = 0.35$, produces an optimal β of only 0.12. Here β is effectively the only policy instrument over which there is freedom to choose, since b is determined to satisfy the government budget constraint.

Numerical methods were used to compare social welfare in alternative schemes. A systematic search was carried out over three of the policy variables, the income tax rate, the voucher taper rate and either the maximum voucher or the social dividend, with the fourth policy variable being determined by the government's budget constraint (which was solved iteratively in view of the nonlinearity involved).

It was found that for all combinations, a uniform voucher turns out to be optimal; that is, the optimal taper rate applied to the voucher is zero. The optimal taper showed little sensitivity to the degree of inequality aversion of the social welfare function. However, if a binding constraint is placed on the maximum tax rate (and hence gross revenue) which can be imposed by the government, means-testing is found to be optimal. However, the resulting taper rate is relatively small, even if both the tax rate and the maximum voucher are restricted. Although the paper has concentrated on comparing alternative tax and transfer structures using an approach based on the optimal tax literature and considering one generation of parents, it would be of interest in future work to examine the implications for growth and inequality in a multi-generational framework.

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