THE UNIVERSITY OF MELBOURNE

DEPARTMENT OF ECONOMICS

RESEARCH PAPER NUMBER 909

AUGUST 2004

**DYNAMIC ECOLOGICAL CONSTRAINTS
TO ECONOMIC GROWTH**

by

Anke D. Wurzbacher *†

Department of Economics
The University of Melbourne
Melbourne   Victoria   3010
Australia.

Dynamic Ecological Constraints to Economic Growth

Anke D. Wurzbacher[*][†]

August 2, 2004

Abstract

An important characteristic defining the threat of environmental crises is the uncertainty about their consequences for future welfare. Random processes governing ecosystem dynamics and adaptation to anthropogenic change are the source of prevailing ecological uncertainty and contribute to the problem of how to balance economic development against natural resource conservation. The aim of this study is to describe the implications for steady-state economic growth subject to non-linear dynamic environmental constraints. In a two-sector exogenous growth framework we model a stochastic environmental good, exhibiting uncertain ecological responses to environmental change and describe the economic and environmental trade-offs that ensue for a risk-averse social planner. Allowing for ecological risk tends to slow long run economic growth if environmental impacts are assumed to increase exponentially as the level of disturbance increases. A combination of low risk aversion and linear or concave ecological response functions leads to accelerated economic growth and environmental impact. This result is reversed for social planners characterised by high levels of risk aversion as conservation with zero environmental impact becomes the economically optimal policy choice. The environmental option value of preserving a natural resource in the face of uncertainty is treated in this paper as a Cicchetti-Freeman type risk premium.

[*]Department of Land Economy, University of Cambridge, UK.
[†]The author should like to thank John Creedy for valuable input.

1

1 Introduction

Uncertainty about the consequences for the welfare of current and future generations of looming environmental crises such as climate change, loss of marine and terrestrial ecosystems and loss of biodiversity, is perhaps their most defining characteristic. It explains why the debate on environmental conservation versus natural resource development has stimulated research in numerous disciplines and why it enters policy agendas across all levels of governance.

The growing body of economic literature on this topic is testimony to the widespread belief that the economic system itself may be the root-cause of many environmental problems. As a result, these studies explore the interdependencies between the economic system with its goal of long term consumption growth on the one hand and natural resource constraints on the other [1][2][3]. By contrast, many studies of ecosystems focus on the problem of prevailing ecological uncertainty. The simulation of non-linear and stochastic ecosystem dynamics, partially responsible for our difficulties in predicting the ecological response to a given environmental change, in a model environment is a major contribution in this area [Carpenter and Cottingham, 1997], [Perrings, 1998].

The aim of this study is to describe the implications for economic growth in the steady state if the environmental constraint is represented by a non-linear and stochastic dynamic resource. In a two-sector exogenous growth model we model a stochastic environmental good, characterised by uncertainty about the ecological response to environmental change and describe the economic and environmental trade-offs that ensue for a risk-averse social planner. The concept of ecological risk is closely connected to the notion of environmental option value, that is the value attributed to the option of preserving a given environmental resource in the face of uncertainty. This paper treats environmental option value as a risk premium in the sense of

---

[1]Krautkraemer [1998] finds that allowing for environmental constraints in a model with exogenous technological progress and positive steady-state rates reduces economic growth.

[2]Barrett [1992] concludes that normative parameters such as society's concerns for preservation and the welfare of future generations determine to a great extent the extent of environmental sacrifice for economic growth.

[3]Gradus and Smulders [1993] contrast the effect of increased environmental concern on exogenous versus endogenous economic growth and identify a negative relationship for the latter case.

Cicchetti and Freeman [1971].

In section 2 we construct the two-sector exogenous growth model, which describes the interaction between private sector growth and a stochastically dynamic natural resource, modelled in the public good sector. Section 3 characterises the economic and ecological trade-offs under ecological certainty. These results are contrasted with the outcome under the assumption of ecological uncertainty and a risk averse social planner in section 4. The analysis is followed by tests conducted to determine the model's sensitivity to assumptions made with respect to the ecological response function, the degree of risk aversion of the social planner and other, predominantly normative parameter values. This is done by means of a numerical sensitivity analysis for two scenarios, a pristine environment case (section 5.1) and for a managed environment with some prior history of land conversion (section 5.2). In section 6, the message emerging from this analysis is set within the context of the broader literature on economic growth and environmental conservation.

2   The Model

Using a two-sector exogenous growth model, we introduce ecological risk and uncertainty and characterise the trade-offs between the consumption of a private and a public good along the optimal growth path. The essential input to both sectors is a non-renewable natural resource. Its scale of development drives consumption growth in the private good sector and affects the dynamics governing the output of the public good, a renewable natural resource.

The model consists of three principal components: a social planner who aims at maximising total social welfare from joint consumption of the private good, agricultural production, and the public good, biodiversity. Production in the private sector is neoclassical with technology, capital and land inputs. The public good sector stylises the stochastic dynamics of an ecosystem that is susceptible to human impact. For the purpose of this model, human impact is defined as the rate, v, at which biodiversity reserve land is converted for use in the private sector.

Initially, the economy is endowed with capital K_0 and land of size one. The sectors are interlinked via the common input land, with $L(t)$ denoting the share of land in private good production and $R(t)$ denoting the remaining

reserve land supporting the public good, biodiversity.

$$L(t) + R(t) = 1 \tag{1}$$

The task of the social planner consists of finding the optimal steady state rate of land conversion that maximises the present value of social welfare from joint consumption over time. The three principal components of this model economy are dealt with separately in sections 2.1-2.3.

2.1 The Private Good Sector

Production in the private sector is restricted to a single good, $X(t)$, which can be consumed or invested in the capital stock. The production of X at time t depends on exogenously determined inputs such as production technology, A, and endogenously determined inputs, namely capital $K(t)$, and the share of land $L(t)$ that has been converted for use in the private sector. To keep the model simple, it is assumed that population and labour supply remain fixed over time and that labour productivity is implicit in the efficiency parameter A. The production function is Cobb-Douglas.

$$X(t) = AK(t)^{1-\alpha}L(t)^{\alpha}; \quad 0 < \alpha < 1 \tag{2}$$

Equation 2 exhibits all properties of a neoclassical production function: its marginal products are positive and diminishing with respect to capital and land, characterised by downward sloping and strictly convex isoquants for all positive values of K and L. Furthermore, the production function is linearly homogenous, exhibiting constant returns to scale and the Inada conditions hold [Bovenberg and Smulders, 1995]. Holding capital and land input constant an exogenous technological or human capital improvement will increase private good production by the same proportion [Chiang, 1984]. The parameter α is the relative share of private good output accruing to land, commonly referred to as the partial production elasticity.

The output of the private good sector at time t is either consumed, $C(t)$, or invested, $I(t)$, in the capital stock.

$$X(t) = C(t) + I(t) \tag{3}$$

Capital accumulation is driven by investments in excess of the rate of capital depreciation, denoted by δ. Thus the equation of motion of the capital stock

is:

$$\dot{K} = I(t) - \delta K(t) \tag{4}$$

where $0 < \delta < 1$.

2.2   The Public Good Sector

Exploring the inherent trade-offs between long term economic growth and natural resource development is most practically relevant if one allows for renewable rather than exhaustible resources. This is especially true for resource stocks subject to stochastic dynamics or non-linear dynamics. The growth and environment literature, having progressively moved away from exhaustible resource modelling to renewable resources [Krautkraemer, 1998], reflects this conjecture, as well as the importance that is attributed to the role of renewable resources in building sustainable economies.

The ecological processes responsible for the provision and abundance of the environmental resource are modelled in the public good sector. Its output is an environmental good, which is valued by society for its existence and option value. The public good considered here is biodiversity, as it is believed to contain valuable answers to yet unknown problems that may be encountered in the future. Furthermore, biodiversity is known to be highly susceptible to anthropogenic alterations to the environment. In the context of this model, biodiversity is represented by a bundle of populations exhibiting identical responses to environmental change over time.

The production dynamics in the public good sector are set apart from those in the private sector. The stock of biodiversity at time t, $P(t)$, is a function of the density of the biological population, $D(t)$, as well as the size of biodiversity reserve land, $R(t)$. The rate of land conversion, v, is constant along the optimal path, which means that land allocated to the public sector asymptotically approaches zero over time. These conditions are formally expressed in equations (5) and (6).

$$\frac{\dot{L}(t)}{L(t)} = v; \quad 0 < v < 1 \tag{5}$$

It follows from (5) that

$$L(t) = 1 - e^{-vt} R_0 = 1 - e^{-vt}(1 - L_0) \tag{6}$$

where $L_0 = L(0)$ and $R_0 = R(0)$. The approach taken to model the output in the public good sector at time t has been adapted from population biology. The value of biodiversity at time t, is the product of current population density and public sector land share.

$$P(t) = D(t)R(t) \tag{7}$$

Where $D(t)$ can be thought of as a measure of production efficiency in the public sector and takes a value between zero and one. The behaviour of population density over time is a function of the rate of anthropogenic disturbance and is explained in more detail in the following section.

2.2.1   Modelling the population response

Population density determines the productivity in the public good sector and represents the means by which we model the ecological response to environmental disturbance. For consumption of the private good to grow over time, the share of land set aside for its production has to increase. The rate at which land is converted governs the extent of ecosystem disturbance, which in turn affects the level of biodiversity in the public good sector. This section sets out the ecological response to anthropogenic disturbance and is the key contribution to the field of modelling dynamic ecological constraints to economic growth.

The ecological response to anthropogenic disturbance, biodiversity change, is modelled as the outcome of two independent processes: demographic stochasticity and evolutionary rescue [Gomulkiewicz and Holt, 1995]. Higher rates of land conversion, v, represent more severe environmental disturbance, leaving species populations more susceptible to demographic stochasticity and increasing the risk of extinction. In contrast, a counteracting ecological response has been identified, whereby ecosystem disturbances stimulate phenotypic evolution. Phenotypic evolution is the process by which a species, otherwise destined for extinction may be rescued by its own phenotypic traits that allow it to cope with and ultimately thrive in its altered environment. Hence evolutionary rescue works in the opposite direction of demographic stochasticity.

Biologists have developed birth-death models capable of simulating such processes reasonably accurately [Grummet and Stirzaker, 1992]. Here the simple birth-death model serves as the basis for modelling population density in the public good sector over time. The probability of species extinction,

μ, increases with rising rates of land conversion for private good production. The probability of evolutionary rescue is given by η. Assuming that the demographic effect dominates that of evolutionary rescue, η is a fraction of μ. The probabilities are summarised in equations (8) and (9).

$$P(extinction) = \mu = f(v) \tag{8}$$

$$P(rescue) = \eta = \beta\mu \tag{9}$$

where $0 < \beta < 1$. Using a Markov chain it can be shown that for given probabilities of extinction and evolutionary rescue, the expected population density at time t, $E(D_t)$, is an exponential function of the initial level of density with the rate of change equal to the net impact of land conversion on the population dynamics (equation (10)). The corresponding density variance is given in equation (11).

$$E(D_t) = D_0 e^{(\beta-1)f(v)t} \tag{10}$$

$$var(D_t) = \begin{cases} 2D_0\eta t & if \quad \beta = 1 \\ D_0\frac{\eta+\mu}{\eta-\mu}e^{(\eta-\mu)t}(e^{(\eta-\mu)t} - 1) & if \quad \beta \neq 1 \end{cases} \tag{11}$$

Equations (8 - 11) specify the stochastic nature of the public good, modelled as the outcome of the race between two opposing random processes, demographic stochasticity and evolutionary rescue. Both probabilities are a function of the anthropogenic disturbance of the ecosystem, represented by the rate of land conversion. Accordingly, this variable provides the link between both sectors in the economy and is the means through which the value judgement of the social planner with respect to the inevitable trade-offs between the private and public good are expressed. The characteristics of the social planner are discussed in more detail in the next section.

2.3   The social planner

The social planner's objective is to maximise the expected present value of social welfare from both goods by setting the rate of land conversion at the socially optimal level. As mentioned previously, the rate of land conversion along the equilibrium path is fixed over time. Adopting a social planner framework allows for the consideration of inter-temporal spill-over effects and

social benefits arising from a public good. One would expect that the optimal rate of land conversion under these circumstances is below the optimal rate that would be chosen under a laissez-faire approach.

2.3.1 Additive Social Welfare

Social welfare in this model is additive in the consumption of the public good, $P(t)$, and the private good, $C(t)$, implying substitutability between both goods. The variables represent total, rather than per capita, consumption levels. Rivalry in the private good sector implies that there is a price for the marginal unit, giving a clear indication of the sacrifice consumers are willing to make to consume the good rather than miss out. Public good consumption is non-rival, so an increase in its supply at the margin benefits every consumer equally. Public good consumption necessitates a social sacrifice in the form of land supply available to the private sector. The social planner's judgement of the relative worth of a unit of the private versus the public good is expressed through the weight parameter γ. The larger γ, the higher the value the planner places on an extra unit of the private good compared to an additional unit of the public good. Total social welfare at time t equals the weighted sum of the private and public good.

$$W(t) = \gamma C(t) + P(t) \tag{12}$$

where $\gamma > 0$. Despite equation (12) representing a social welfare function, we proceed and simplify the exposition by referring to welfare implications of the planner's choices in terms of changes to public and private good utility. The social planner's goal is to maximise total present value of welfare, Z, over an infinite time horizon.

$$Z = \int_0^\infty e^{-rt} W(t) dt; \quad 0 < r < 1 \tag{13}$$

The use of an infinite time horizon implies that future generations' welfare matter, albeit discounted by the social rate of time preference, r. In section 3 we solve this dynamic problem assuming environmental certainty. Environmental uncertainty and aversion to ecological risk are allowed for in section 4.

3 Socially Optimal Impact under Certainty

How much of the private good should be produced and how much biodiversity sacrificed to this end? This is the trade-off the social planner aims to balance by choosing the rate of land conversion, v, optimally. High rates of v fuel private good consumption growth while hurting biodiversity. Land is being converted optimally at the rate that maximises present value social welfare from the joint consumption of the private and the public good.

The use of a linear and additive social welfare function enables the dynamics in each sector to be treated separately in sections 3.1 and 3.2, before combining these for the steady state analysis in section 3.3. Initially we explore the trade-offs, assuming that the ecological resource is deterministic. Section 4 deals with the steady state analysis under the assumptions of a stochastic public good and risk aversion of the social planner.

3.1 Public good utility over time

Equation (12) yields a clear division of the social planner's problem into the private and the public good problem.

$$Z = \int_0^\infty e^{-rt}\gamma C dt + \int_0^\infty e^{-rt}D_0 e^{(\beta-1)f(v)t}R(t)dt \tag{14}$$

The second integral on the RHS of equation (14) can be solved using equation (6), since its time dependent variables can be expressed in terms of their initial value.

$$\int_0^\infty e^{[(\beta-1)f(v)-v-r]t}D_0 R_0 dt = \frac{D_0 R_0}{(\beta-1)f(v)-v-r}\left[e^{(\beta-1)f(v)-v-r}\right]_0^\infty$$

$$= \frac{D_0 R_0}{v+r+(1-\beta)f(v)} \tag{15}$$

Public good utility over time is constant and declining as the rate of land conversion increases for all $(\beta-1)f(v)-v-r < 0$, which is true by definition. In Figure 1 this relationship is demonstrated by the convex and decreasing public good utility schedule over a range of rates of land conversion from 0 to 30 per cent. The greatest environmental cost in terms of reduced biodiversity value is felt initially, as environmental impact from land conversion

9

Units of Utility
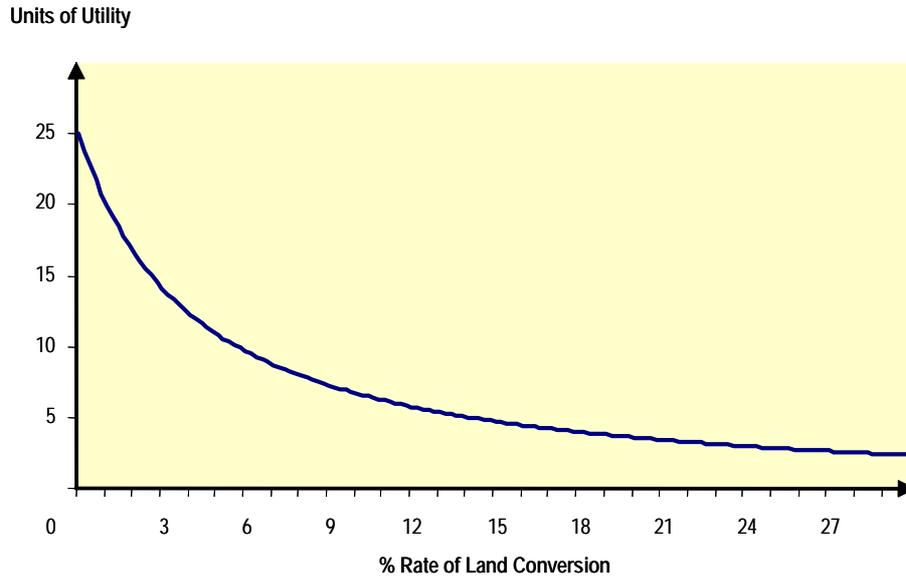
% Rate of Land Conversion

Figure 1: Rates of land conversion and public good utility

increases from 0 to 6 per cent, after which public good utility asymptotically approaches zero.

By replacing the second integral in equation (14) with the solution given in equation (15), the social planner's objective function has been simplified from a two-fold dynamic problem to the intertemporal optimisation of the private good sector. The value of the program is equal to the solution to the dynamic optimisation problem plus the constant representing present value public good welfare. The task of maximising the consumption of the private good, subject to the capital constraint is undertaken in the following section.

3.2   The optimal growth path in the private sector

Having solved for the present value of public good welfare the next step is to maximise the social welfare function (14) with respect to the control variable

$C(t)$, subject to the capital constraint and the state variable $K(t)$. That is,

$$\max_{C(t)} \int_0^\infty e^{-rt}\gamma C(t)dt + \frac{D_0 R_0}{r + v - (\beta - 1)f(v)} \qquad (16)$$

subject to

$$\dot{K} = AK(t)^{1-\alpha}L(t)^\alpha - C(t) - \delta K(t) \qquad (17)$$

The end conditions for the capital stock are the capital endowment of the economy, K_0, and the constraint that at infinity the capital stock has to equal at least some positive amount K_∞.

$$K(0) = K_0; \quad K(\infty) \geqslant K_\infty \qquad (18)$$

The Cobb-Douglas production function gives rise to concave objective (16) and growth functions (17) in $X(t)$ and $K(t)$. It satisfies the Arrow-Kurz conditions for the first-order Hamiltonians to be necessary and sufficient for maximising social welfare. To simplify notation, the time argument t is henceforth omitted from the control and state variables.

The current value Hamiltonian comprises of the integral in (16) and the capital constraint (17)

$$\tilde{H} = \gamma C + m(AK^{1-\alpha}L^\alpha - C - \delta K) \qquad (19)$$

where $m = e^{rt}\lambda(s)$. The Hamiltonian conditions are (20-22):

$$\frac{\partial \tilde{H}}{\partial C} = 0 = \gamma - m \qquad (20)$$

$$\frac{\partial \tilde{H}}{\partial K} = -\dot{m} + rm = (1 - \alpha)mAK^{-\alpha}L^\alpha - m\delta \qquad (21)$$

$$\frac{\partial \tilde{H}}{\partial m} = \dot{K} = AK(t)^{1-\alpha}L(t)^\alpha - C(t) - \delta K(t) \qquad (22)$$

The transversality condition arising from the endpoint constraint is:

$$m(\infty) \geqslant 0 \quad and \quad \left[\hat{K}(\infty) - K_\infty\right]m(\infty) = 0 \qquad (23)$$

It follows from equation (20) that the co-state variable, m, equals the positive constant γ for all t. Thus, at any point in time the current marginal value of

the maximum value of the program with respect to a small change in $K(t)$ is positive: the capital constraint is binding at all time. The time derivative of m, $\dot{m}$, is zero.

$$m(t) = \gamma \quad \forall t; \quad \dot{m} = 0 \tag{24}$$

Applying the result of (24) to (21) gives the capital-land ratio:

$$rm = (1 - \alpha)mAK^{-\alpha}L^\alpha - m\delta$$

$$\frac{K}{L} = \left(\frac{1 - \alpha}{r + \delta}\right)^{1/\alpha} A^{1/\alpha} \tag{25}$$

Thus along the optimal growth path the capital and land ratio remains constant. This result implies that increases in land allocated to the private sector have to be matched by equal increases in the capital stock through investment. The optimal level of the capital stock at time t, $\hat{K}$, is a function of the level of technology employed in the production of the private good, adjusted for the elasticity of production with respect to capital. The effective rate of discount for the capital stock is a composite of the social rate of time preference and the depreciation rate. Along the optimal growth path the growth rate of the capital-land ratio is zero, implying that, in the absence of further land conversions, only exogenous shifts in technology can yield sustained positive growth of capital .

$$\left(\frac{\dot{K}}{L}\right) = 0 \tag{26}$$

Consequently, the growth rate of capital over time is entirely governed by the rate at which land is converted for the production of the private good. Differentiating (25) with respect to time and dividing by K provides the formal proof. The rate of growth of the capital stock is v, the rate at which reserve land is converted for use in the private sector.

$$\frac{\dot{K}}{K} = \frac{\dot{L}}{L} = v \tag{27}$$

In view of the intermediate objective of maximising consumption of the private good the results of equations (25-27) are intuitively obvious: capital accumulation is limited only by the fixed supply of land and is non-decreasing over time. Land allocation to the private sector, $A(t)$, approaches the maximum, one, as t approaches infinity. The speed at which the capital stock

approaches its maximum increases proportionally to increases in the rate of land conversion. The capital stock along the optimal path at time infinity equals

$$\lim_{t \to \infty} \hat{K}(t) = \left(\tfrac{1-\alpha}{r+\delta}\right)^{1/\alpha} A^{1/\alpha} \tag{28}$$

Given the transversality condition (23) and given that $m = \gamma$ for all t, it follows that the capital stock at infinity is equal to the positive endpoint constraint K_∞.

Rearranging the third Hamiltonian condition (22) and using equations (25) and (27) it is now possible to express total consumption of the private good, $C(t)$, along the optimal growth path as a function of the capital stock and positive parameters v, α and δ (see Appendix A).

$$\hat{C} = \hat{K} \left(\frac{r+\delta}{1-\alpha} - \delta - v \right) \tag{29}$$

The term in brackets is the intertemporal opportunity cost of investment on consumption. As seen in equation (25) the capital-land ratio is constant along the optimal path. The rate of land conversion drives investment. Increases in v have to be met by increases in the capital stock through increased investment, crowding out current consumption. The capital depreciation rate, δ affects consumption in much the same way: increased investment is necessary to offset increases in capital depreciation, leading to lower consumption levels along the optimal path. The ratio in the parenthesis in (29) determines the extent of the trade-off between investment and consumption: the effective discount rate in the numerator favours consumption over investment, whereas the production elasticity of capital in the denominator represents the constraint on consumption through the investment requirement, which is higher the more elastic is production with respect to land.

While the effect of v mentioned above negatively impacts on the level of consumption along the optimal path, higher rates of v increase the capital stock, since $K_v > 0$, and thereby increase consumption. Thus, the rate of land conversion affects consumption in two opposing ways, resulting in an ambiguous relationship between the level of consumption and rates of land conversion. This relationship is shown graphically in Figure 2. For small rates of land conversion private good utility is positive and increasing. The private good utility schedule has a global maximum, which corresponds to the unique rate of land conversion for which the consumption of the private good is maximised. For conversion rates in excess of the optimal private
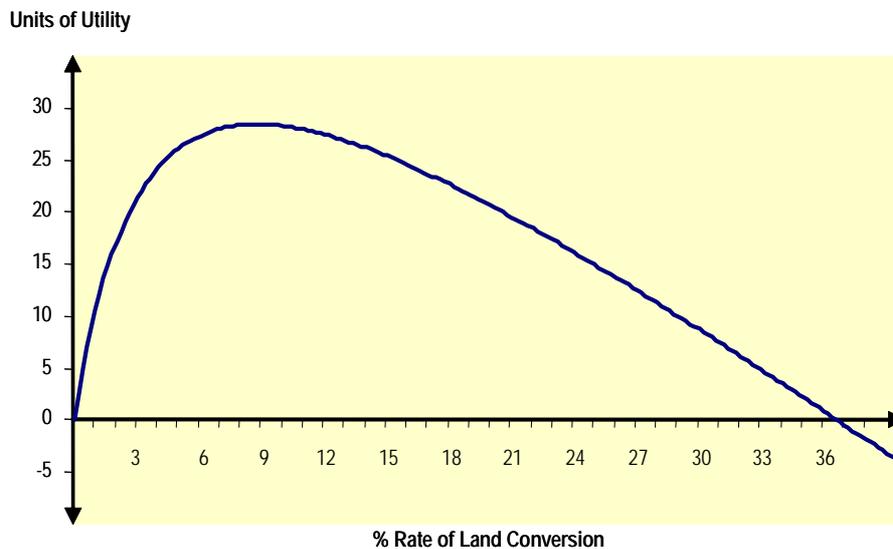
Figure 2: Rates of land conversion and private good utility

rate, investment pressure causes consumption to decline until it eventually becomes negative.

Differentiating equation (29) with respect to v further clarifies the relationship between consumption and the rate of land conversion. The subscripts denote the first derivative of the function with respect to the rate of land conversion.

$$C_v = K_v \left(\frac{r + \delta}{1 - \alpha} - \delta - v \right) - K \tag{30}$$

The first term on the RHS is negative for all $v > \frac{r + \alpha \delta}{1 - \alpha}$. This situation arises for small values of α, implying that production in the private sector is inelastic with respect to land. In this case $C_v < 0$ and it follows that increases in the rate of land conversion trigger investments large enough to cause private good consumption to decline. This case corresponds to the downward sloping part of the private good utility schedule in Figure 2. In the opposite case of private good production being highly elastic with respect to land, the first term on the RHS of equation (30) is positive so that the overall effect of a rise in v is dependent on the value of all other parameters chosen.

Hence all time dependent variables in this economy grow at the rate of

land conversion. The choice of v thus drives the dynamics in the private sector, controlling the growth rate of the economy in the steady state and determining the extent of human intervention in the biodiversity or public good sector. By setting v, the social planner can therefore influence the growth path of all variables in the private sector.

$$\frac{\dot{C}}{C} = \frac{\dot{I}}{I} = \frac{\dot{K}}{K} = \frac{\dot{L}}{L} = v \tag{31}$$

The following section discusses how best to reconcile the trade-offs involved in setting the rate of land conversion which fuels private good consumption growth at a certain cost of declining levels of biodiversity.

3.3 Private and public good trade-offs under certainty

It was shown in sections 3.1 and 3.2, that social welfare from the public and the private good is a function of the rate of land conversion, leading to the emergence of inevitable trade-offs in the steady state. In this section we explore the implications of these trade-offs between private good consumption and the public good for the optimal rate of land conversion, ignoring ecological risk for the moment. The first step is to find the present value of the program by inserting (29) into equation (13) and using (25) to solve the integral (see Appendix B for more detailed derivation).

$$
\begin{aligned}
Z &= \int_0^\infty e^{-rt}\gamma C\, dt + \frac{D_0 R_0}{r + v - (\beta - 1)f(v)} \\
&= \gamma\frac{K}{L}\left(\frac{r+\delta}{1-\alpha} - \delta - v\right)\left(\frac{v + rL_0}{r(r+v)}\right) + \frac{D_0 R_0}{r + v + (1 - \beta)f(v)}
\end{aligned}
\tag{32}
$$

Present value social welfare is the sum of social welfare from the consumption of the public and private goods until infinity, given a set of positive and normative parameters. Amongst the positive parameters are the initial allocation of land in public good production, R_0, the initial level of ecosystem health and productivity, represented by D_0, the level of technology, A, in the private sector, the elasticity of production, α with respect to its inputs and the capital depreciation rate, δ. Normative parameters are those that depend on the planner's judgement of social preferences, such as the weight parameter, γ, and the rate of time preference, r. All of these parameters ultimately determine $\hat{v}$, the optimal steady state rate of land conversion and

15

economic growth. The share of total social welfare from the consumption of the private good is captured in the first product of the RHS of equation (32). The contribution of the public good to total social welfare is given by the second term on the RHS.

We first consider the present value of social welfare derived from the public good until infinity. It equals the maximum achievable value of biodiversity, given the initial level of population density and initial endowment of land in the public sector, adjusted by the effective discount rate. The latter equals sum of the social rate of time preference, of the rate of land conversion and of its ecological net effect in terms of species population dynamics. Thus, land conversion leads to reduced public good utility, by reducing the size of the reserve land put aside for biodiversity conservation as well as causing disturbance to the ecosystem. The net ecological effect is the increase in the risk of species extinction net of the possibility of rescue through phenotypic evolution, which is triggered by disturbance.

The social welfare implications from the consumption of the private good are captured in the remaining products on the RHS of equation (32). These consist of the consumption parameter γ, the constant capital-land ratio, the intertemporal opportunity cost of investment on consumption and the discounted contribution of land conversion to private good welfare at each period in time. As v appears both in the numerator and denominator of this last term, the effect of an increase in the rate of land conversion on social welfare is not immediately obvious and requires more detailed analysis.

To begin with we explore the implications for total social welfare of the initial conditions. From (32) it is easy to show that if all available land were allocated to biodiversity conservation, $R_0 = 1$, and if no land were ever converted into agricultural land, that is $v = 0$, then $f(v)$ is also zero and total social welfare equals the maximum achievable value of biodiversity, discounted by the social rate of time preference. Conversely, if all available land is instantaneously converted for the production of the public good, social welfare from the consumption of the public good is zero.

Equation (32) shows that the contribution of social welfare from either good is a function of the rate of land conversion. More specifically, the social planner faces the trade-off between potentially increasing the welfare from the consumption of the private good and definitely decreasing social welfare from the consumption of the public good. Given this condition, it is necessary to determine the optimal level of environmental impact, represented by v, the rate of land conversion. This rate incidentally also represents the growth rate

16

in the economy along the optimal path.

Proposition 1 *The optimal rate of land conversion, $\hat{v}$, is set at the level that equates the marginal private good utility from a change in v, with the negative marginal public good utility from a change in v.*

Proof. The complexity of (32) makes solving explicitly for v difficult. Using the implicit function theorem, we obtain a function F by moving Z in (32) across to the RHS.

$$
\begin{aligned}
F &= -Z + \gamma A^{1/\alpha} \left(\frac{1-\alpha}{r+\delta} \right)^{1/\alpha} \left[\frac{(r+\delta)}{(1-\alpha)} - \delta - v \right] \left[\frac{v + rL_0}{r(r+v)} \right] \\
&\quad + \left[\frac{D_0 R_0}{r + v + (1-\beta) f(v)} \right]
\end{aligned} \tag{33}
$$

The implicit function theorem states that in the neighbourhood for which the function F is defined the partial derivative of Z with respect to v equals the following expression (see Chiang [1984]).

$$
\frac{\partial Z}{\partial v} = -\frac{\partial F_v}{\partial F_z} \tag{34}
$$

for $r \neq 0$; and $\alpha < 1$. With $F_z = -1$, equation (34) equals

$$
\begin{aligned}
\frac{\partial Z}{\partial v} &= \gamma \frac{K}{L} \left[\left(\frac{r+\delta}{1-\alpha} - \delta - v \right) \left(\frac{R_0}{(r+v)^2} \right) - \left(\frac{v + rL_0}{r(r+v)} \right) \right] \\
&\quad - \frac{D_0 R_0 (1 + (1-\beta) f'(v))}{(v + r + (1-\beta) f(v))^2}
\end{aligned} \tag{35}
$$

The first term in square brackets in (35) is the marginal utility from private good consumption for a unit increase in the rate of land conversion. The second term represents the marginal public good utility from an increase in v. We maximise total social welfare with respect to v, by setting equation (35) equal to zero and obtain the condition at the margin

$$
MU_{private} = -MU_{public} \tag{36}
$$

where $MU_{private}$ and MU_{public} denote marginal private and marginal public good utility. The optimal rate of land conversion and the optimal rate of

economic growth, $\hat{v}$, is the value for which the marginal utility from the consumption of the private good equals the marginal disutility from increased disturbance in the public good sector. ∎

As long as $f'(v) > 0$ holds, marginal utility from a certain public good is negative for all defined values of v and β, implying that the public good utility schedule under ecological certainty is a downward sloping function of the rate of land conversion. Hence, any increase in the rate of land conversion will lead to a decline in the present value social welfare derived from the certain public good. As already discussed, the marginal utility from the private good is ambiguous and depends on the relationship between the size of the relative magnitudes of the parameters and the rate of land conversion. From (32) we know that marginal private good utility is negative for all $\alpha < 1 - \left(\frac{r+\delta}{\delta+v}\right)$. Hence below this threshold for the production elasticity of land any increases in the rate of land conversion yield diminishing social welfare from private good consumption and, as a result, total social welfare. From (35) it is easy to see that for all $\alpha > 1 - \frac{R_0 r (r+\delta)}{r(2v+\delta+L_0)+v^2}$ social welfare from private good consumption increases as v increases.

Having discussed the behaviour of the utility schedules of the private and the public good for changes in the rate of land conversion it is now time to look at the optimal rate, $\hat{v}$, that will balance public and private good utility in the steady state.

Proposition 2 *The optimal rate of land conversion, $\hat{v}$, lies in the range of the values of v for which marginal utility from private good consumption is positive.*

Proof. Proposition one states that the optimal rate of land conversion, $\hat{v}$, is the rate which equates marginal private good utility with the negative of marginal public good utility. Given, that marginal public good utility is negative for all values of v it follows that in order for proposition one to hold, marginal private good utility has to be in the range of vs for which marginal private good utility is positive. ∎

In sections 3.1 and 3.2 it was shown that along the optimal growth path the rate of land conversion determines the productivity in both, the private and the public good sector. Social welfare from a certain public good is negatively related to the rate of land conversion. The relationship between the rate of land conversion and private good utility is ambivalent and depends on the parameter values used. Despite this ambiguity it could be shown that the

18

optimal rate of land conversion and economic growth, $\hat{v}$, is the rate at which the marginal private good utility equals the marginal public good disutility from environmental impact and that this rate lies in the range for which marginal private good utility with respect to v is positive. The problem of how the introduction of environmental stochasticity and risk aversion affects the optimal choice of the rate of land conversion along the balanced growth path is analysed in the next section.

4   Socially Optimal Impact under Uncertainty

Having discussed in section 3, the implications for the rate of land conversion and economic growth in an economy, which faces trade-offs between private good consumption growth and a decline in the environmental good, the model is now extended to include environmental risk. Section 4.1 introduces the utility function for a social planner who is averse to taking ecological risks. The effect of environmental risk on the optimal rate of land conversion is discussed in section 4.2, taking account of the public good only. Section 4.3 combines both goods under the social planner's objective function and explores the ensuing implications for the optimal rate of land conversion if the objective is to maximise joint consumption of the private and public goods.

4.1   The risk averse social planner

In this section we specify a social welfare function, that describes the behaviour of a social planner who is concerned with environmental uncertainty and risk. As explained in section 2.2.1, anthropogenic disturbances of biodiverse ecological systems affect prevailing population dynamics in a random manner. They increase the probabilities of demographic stochasticity and evolutionary rescue, but the magnitudes of these effects are stochastic. As a result, the net human impact on the environment is uncertain. The assumption of risk aversion implies that the social planner prefers a certain biodiversity outcome to an uncertain outcome of the same expected value. While the use of a general utility function characterised by risk aversion with respect to both goods, represents a fuller treatment of the problem, the model is sufficiently rich without adding the complexity of a general form. The following analyses the choices made by a social planner who is risk averse with respect to the environmental outcome but exhibits the previously used linear utility function with respect to the private good.

Social utility from the public good, $P(t)$, is defined by the weighted utility function:

$$U(P(t) = \begin{cases} \frac{P(t)^{1-\varepsilon}}{1-\varepsilon} & \varepsilon > 0 \quad and \quad \varepsilon \neq 1 \\ \ln P(t) & \varepsilon = 1 \end{cases} \qquad (37)$$

It satisfies the economically sensible characteristic of more of the public good being preferred to less, i.e. $U_P(P(t)) = P(t)^{-\varepsilon} > 0$. The second derivative $U_{PP}(P(t)) = -\varepsilon P^{-\varepsilon-1} < 0$ proves concavity in P, a necessary condition

for risk aversion. The social planner experiences decreasing absolute risk aversion as the level of biodiversity increases, as is demonstrated by the inequality: $\partial \left(-\frac{U_{PP}}{U_P} \right) / \partial P = -\varepsilon P < 0$. Relative risk aversion, on the other hand, is constant, such that the social planner's attitude toward losing a given percentage of species remains unchanged as the level of biodiversity in the ecosystem changes [Elton and Gruber, 1995]. The condition for constant relative risk aversion is given formally in equation (38).

$$R(P) = -\frac{P(t)U_{PP}}{U_P} = \varepsilon; \quad R_P = 0 \tag{38}$$

In addition, it is assumed that the social planner has a well-defined preference function for any combination of goods and is free from any constraints other than those defined in (38), which could introduce bias and lead to a suboptimal choice of the rate of land conversion [Milgrom and Roberts, 1992].

The value of the public good for a given rate of land conversion cannot be determined with certainty, as probabilities of extinction and evolutionary rescue are no longer assumed to be deterministic but rather governed by stochastic processes. Facing ecological risk and uncertainty of this type and given the hypothetical option of paying a risk premium to regain ecological certainty, a risk averse social planner would choose to have less of the public good with certainty over having more with uncertainty. Milgrom and Roberts [1992] show that the difference is the certainty equivalent of the public good outcome, $\hat{P}(t)$, given by:

$$\hat{P}(t) = E[P(t)] - 0.5R(P(t))var P(t) \tag{39}$$

The second term on the RHS of equation (39) represents the risk premium that the social planner would be willing to pay if certainty with respect to the public good could be purchased. In this case it represents the reduction of value of the public good due to its stochastic nature and consists of the coefficient of relative risk aversion, multiplied by the variance of the uncertain outcome halved. Substituting (8)-(11) into (39) yields the certainty equivalent of the value of biodiversity at time t, $\hat{P}(t)$.

$$\begin{aligned}
\hat{P}(t) &= R_0 D_0 e^{[-v-(1-\beta)f(v)]t} \\
&\quad -0.5\varepsilon R_0 D_0 \frac{\beta+1}{\beta-1} e^{[-v-(1-\beta)f(v)]t} \left(e^{-(1-\beta)f(v)t} - 1 \right)
\end{aligned} \tag{40}$$

21

The variance used to arrive at (40) applies to all $\beta \neq 0$, encompassing the range for which the parameter β has been defined in (11). The following section explores the implications of including uncertainty and risk aversion for the optimal rate of land conversion with respect to the public good.

4.2   Public good utility under uncertainty

In section 3, social welfare from the public good is a function of the expected value of the population density (14). This term is now adjusted for the certainty equivalent of the public good as derived in equation (40). The resulting integral can again be solved separately to yield the constant:

$$
\int_0^\infty e^{-rt}\hat{P}(t)dt \;=\; R_0 D_0 \left(\frac{1}{v + r + (1-\beta)f(v)} \right) \tag{41}
$$
$$
-0.5\varepsilon R_0 D_0 \frac{1+\beta}{1-\beta}\lambda
$$

where $\lambda = (v + r + (1-\beta)f(v))^{-1} - (v + r + 2(1-\beta)f(v))^{-1}$. The first term is the same as under ecological certainty (see equation (15), section 3.1). The second term is the present value of the risk premium from now until infinity. Given that the probability of species extinction increases as the rate of land conversion increases, i.e. $f(v)' > 0$, the value of the risk premium is positive for all values v as long as β satisfies $0 < \beta < 1$. Introducing ecological stochasticity and risk aversion, therefore leads to lower public good utility along the full range of v, as can be seen by the downward shift of the public good utility schedule in Figure 3. The magnitude of the downward shift is determined by the degree of risk aversion of the social planner. The higher the degree of risk aversion, i.e. the larger ε, the larger the value of the risk premium in equation (41) and the more important the loss of public good utility due to ecological uncertainty. Similarly, the size of the downward shift of the public good utility schedule is affected by the degree of ecological uncertainty. The fraction $\frac{1+\beta}{1-\beta}$ increases as β approaches one, implying overall greater ecological uncertainty as the effect of evolutionary rescue becomes more important in determining the final biodiversity outcome.

As shown in proposition 1, the optimal rate of land conversion is determined at the margin. Hence, it is important to determine the effect of introducing ecological uncertainty and risk aversion on marginal public good
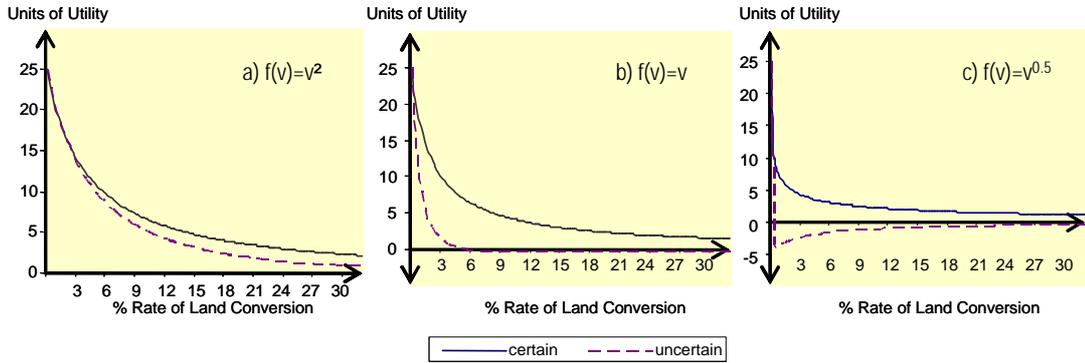
Figure 3: Effect of uncertainty on public good utility for various degrees of ecological resilience

utility. At this point, assumptions made with respect to the ecological response function become crucially important. The relationship between the probability of species extinction and the rate of land conversion, captured in the general functional form $f(v)$, governs the slope of the uncertain public good utility function. A range of curvatures of the ecological response function may be considered, all satisfying the requirement that ecological impact increases with disturbance so that $f'(v) > 0$. For any convex, linear or concave function $f(v) = v^x$, where $x > 0$, the risk premium is positive causing the public good utility schedule unequivocally to shift downwards.

However, the condition $f'(v) > 0$ alone does not provide sufficient information to determine the absolute effect of land conversion on public good utility under the assumption of ecological uncertainty. From equation (41) the derivative of the risk premium with respect to v decreases as x decreases. If x is sufficiently small, the risk premium itself may become negative causing marginal public good utility to increase and even turn positive. Hence the effect of environmental impact on the utility derived from an uncertain public good can take several forms and is summarised in Figure 3. Public good utility declines but may remain positive as shown in the first panel. Here the ecological response function is convex, implying rather high ecological resilience to environmental impact from land conversion. Therefore, even for high rates of land conversion the public good is able to yield positive utility to society. Case (b) depicts public good utility for a linear ecological response function. There is no tolerance to conversion, implying that

23

at every level of impact the ecological response is equally pronounced. The ecological risk associated with moderate and higher rates of land conversion is sufficient for a social planner with some degree of risk aversion to expect zero or even negative public good utility. Especially in ecosystems that are already under duress from economic development, additional stress might cause ecological collapse characterised by ecosystem function breakdowns. The cost to society of those ecological changes may manifest themselves in terms of loss of productive and aesthetic value of an area due to lake eutrophication, insect outbreaks or invasive plant and animal species. This case is simulated in part (c) of Figure 3 with the ecological response function assumed to be concave. In such circumstances even minimal environmental impact is expected to trigger ecosystem collapse, so that a policy shift from conservation to some positive value of land conversion carries with it a very large ecological sunk cost. Marginal public good utility increases as rates of land conversion and environmental impact increase, as it is assumed that the marginal environmental damage of an extra unit increase in the rate of land conversion decreases in ecosystems that have already undergone the move to a dysfunctional state.

The convex case in Figure 3(a) seems to be the most commonly applicable as it assumes that minor disturbances trigger only negligible ecological responses. The ecological response, however, increases exponentially as the disturbances become more important. The convex case also yields the most intuitive result, whereby maintaining a given present value of public good utility under the assumption of ecological uncertainty requires some reduction in the rate of land conversion as compared to the deterministic case. This restraint in economic growth is much higher if a linear response function is assumed, whereas zero consumption growth is the only option for a social planner who wants to ensure that a positive level of public good utility can be derived from an ecosystem that is considered unstable and highly susceptible to impact.

Determining the optimal steady state rate of land conversion, given ecological stochasticity, is not immediately obvious in view of the ambiguous relationships between the rate of land conversion and the marginal utilities from the private good and the stochastic public good. This problem is treated in depth in the following subsection.

4.3 Private and public good trade-offs under uncertainty and risk aversion

To fulfil the objective of maximising steady-state total social welfare from both goods the social planner's task is to maximise equation (42). As mentioned previously, the linear functional form of social welfare with respect to the private good has been retained. Hence equations (42) and (32) differ only with respect to the constant constituting the public good problem, which has been adjusted to allow for ecological stochasticity and risk aversion.

$$
\begin{aligned}
Z \;=\; & \int_0^\infty e^{-rt}\gamma C\,dt + \frac{D_0 R_0}{v + r + (1-\beta)f(v)} \\
& -0.5\varepsilon R_0 D_0 \frac{\beta+1}{\beta-1}\left[\frac{1}{v + r + 2(1-\beta)f(v)} - \frac{1}{v + r + (1-\beta)f(v)}\right]
\end{aligned}
\tag{42}
$$

The following describes the implications for the optimal rate of land conversion, $\hat{v}$, chosen by a social planner who is risk averse with respect to the uncertain ecological outcome to maximise total social welfare from the public and private good. The effect of allowing for ecological uncertainty and risk aversion on the optimal rate of land conversion in the steady state is ambiguous. The final outcome depends on the shape of the public good utility schedule, which is largely determined by three factors: the curvature of the extinction function, representing ecosystem resilience to disturbance, the degree of risk aversion of the social planner and, to a lesser extent, the probability of adverse ecological effects from anthropogenic change being offset by evolutionary rescue. Figure 4 sheds light on the interaction between the assumptions made with respect to the ecological response function $f(v)$, the degree of risk aversion ε and the optimal steady state rate of economic growth, $\hat{v}$.

The optimal rate of conversion at $\varepsilon = 0$ corresponds to the rate chosen by a risk neutral social planner or one who faces no ecological uncertainty. It is shown that, provided the curvature of the extinction function is sufficiently convex, i.e. $f(v) = v^2$ and $f(v) = v^{1.5}$, optimal anthropogenic disturbance under ecological risk and uncertainty never exceeds the optimal level under certainty. Since convexity in the ecological response function implies robustness of the ecosystem with noticeable ecological impact occurring only for larger rates of land conversion it is also true that small rates of land conversion might be optimal even for the most risk averse social planner. This
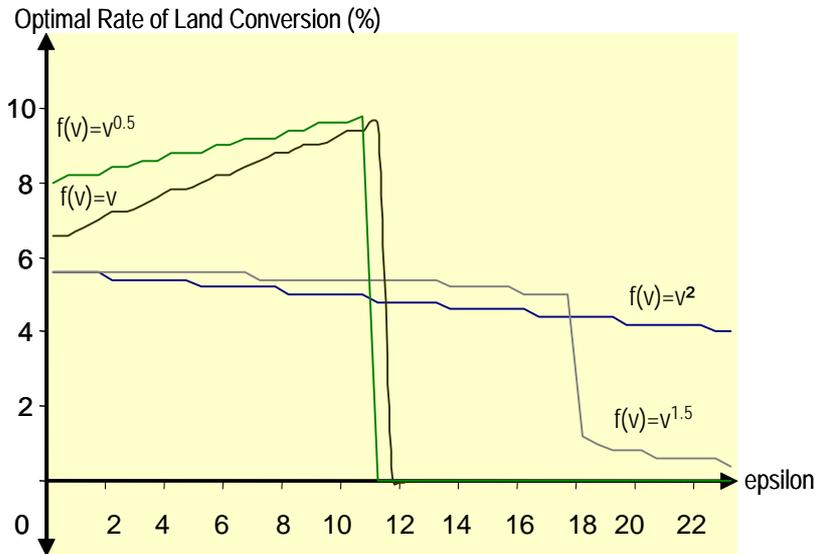
Figure 4: Optimal rates of land conversion for varying degrees of risk aversion

is the case for $f(v) = v^2$. For ecosystems that are characterised by little or no tolerance to even small disturbances, social planners with risk aversion parameters $0 < \varepsilon \leq 11$ will opt for higher rates of land conversion than they would under ecological certainty. The reason for higher optimal rates of impact under uncertainty can be seen clearly in the third panel of Figure 3. Ecosystems described by concave response functions are already on the brink collapse as is apparent by their low impact threshold. Once the threshold has been crossed the damage may be amortised and marginal public good utility increases as environmental impact increases. Social planners, characterised by higher degrees of risk aversion of $\varepsilon > 11$, on the other hand favour conservation over development and opt for $\hat{v} = 0$.

While the analysis allows for degrees of risk aversion that may seem extreme, a number of arguments, explained in more detail in the next section, can be put forward to support them. These point to the fact that most estimates of relative risk aversion are derived from experimental studies involving gambles or empirical studies of investment, saving or insurance data, with degrees of risk represented by some monetary value. Very little is known about the value that relative aversion to ecological risk should take and a number of characteristics of ecological risk suggest that higher values than

26

commonly adopted should be considered. These include the irreversible nature of the loss, unlike financial losses incurred on the stock market, which may be recoverable in the future. The nature of risky good itself differs from the nature of a financial asset, by being unique and without substitutes. Finally, whereas the meaning of a financial loss is tangible, it is difficult to grasp and evaluate the consequences of ecological losses, as we don't have the benefit of foresight to realise the true potential of biodiversity and ecosystem functionality to future generations. In light of these distinct characteristics we feel justified in considering levels of relative risk aversion of up to 20. At least, these estimates give an indication of the degree of risk aversion necessary for conservation to represent the economically optimal choice.

While the impact of ecological uncertainty on the optimal rate land conversion is ambiguous, it can be shown that allowing for ecological risk leads to a wider range of cases for which a policy of conservation and zero steady state consumption growth becomes optimal. This conjecture holds even for a convex ecological response function as demonstrated in Figure 5. It shows the range for the consumption preference parameter, γ, over given production elasticities for which conservation and zero economic growth in the steady state, i.e. $v = 0$, are optimal. For instance, given a partial production elasticity in the private sector with respect to land of 0.7, a social planner will choose to forego economic growth in favour of conservation as long as the certain public good is valued as much as the private good. Under ecological uncertainty this range is larger, as a slight preference for the private good of $\gamma = 1.1$ still yields conservation as the optimal policy choice for the social planner. This is true for all values of partial production elasticity α. The conservation area vanishes as α values tend to zero. This suggests that conservation is too costly for societies with private good production functions that are inelastic with respect to land. In the other extreme, the additional benefit in the private sector for additional land outweighs the public good benefit so that the range of conditions supporting conservation diminishes for high-end α values.

This section explored the effects of ecological uncertainty and risk aversion on the optimal rate of land conversion. Looking at the public good in isolation the social planner would opt for conservation or reduce environmental disturbance in order to obtain the same level of public good utility than under certainty. If social welfare implications from both goods are considered the result is ambiguous and depends on the ecological response function, the degree of risk aversion and the level of ecological uncertainty. It was shown
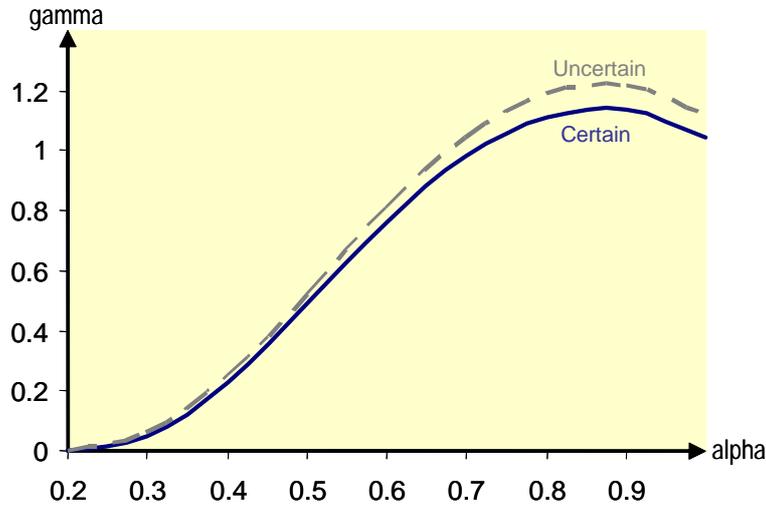
Figure 5: Range of consumption preferences for which conservation is optimal for given production elasticities with respect to land

that accounting for environmental risk leads to an increased number of zero environmental impact and zero consumption growth of the private good being optimal.

5   Testing the Robustness of Results

So far the analysis has been based on the assumption that the social planner is dealing with a pristine plot of land without history of land conversion. The following section describes and motivates the parameter values chosen for this pristine base case. It is followed by a section describing and motivating the introduction of a second scenario, that of an already managed ecosystem with a history of land conversion. Tests are conducted to show model robustness and identify the assumptions to which the model's outcomes are sensitive to.

5.1   Pristine Environment Case

The previous analyses were based on the case of an initially pristine environment, with no history of land conversion, supporting a healthy and func-

Table 1: Summary of Model Parameters

	Land	State of the environment	
		pristine	managed
R	Land in conservation	1	0.5
L	Land in agricultural production	0	0.5
	Private Good		
alpha	Production elasticity of land	0.7	0.7
delta	Rate of depreciation	0.1	0.1
A	Technological efficiency	1	1
	Public Good		
D	Population density	1	0.7
beta	Probability of evolutionary rescue	0.05	0.05
f(v)	Extinction function f(v)=v^x, x takes value of	2	2
	Society		
gamma	Relative consumption preference	2	2
epsilon	Degree of risk aversion	7	7
r	Rate of time preference	0.04	0.04
v	Rate of land conversion	0.05	0.03

tioning ecosystem. The following discussion motivates the values chosen for each parameter in the base case scenario. For ease of reference, these values are summarised and explained in Table 1 .

5.1.1 Private Sector Parameters

Production in the private sector is elastic with respect to the input land with $\alpha = 0.7$. The model emphasises the trade-off between biodiversity, representing a significant value of national wealth and agricultural production. So-called biodiversity hotspots, the most diverse and therefore most valuable natural areas, are predominantly situated in developing countries. In these countries, subsistence agriculture is still common, with a high proportion of income linked to the area of land under cultivation as opposed to modern agriculture where a larger proportion of agricultural output is due to capital intensive inputs such as machinery and fertilizers. Similarly, we assume that

the technological standard does not enhance nor hinder agricultural production in any way, hence implying that the value of the technology parameter should be unity, $A = 1$. Capital in this model is assumed to depreciate at 10 per cent, which is the standard rate applied when evaluating capital investment projects.

5.1.2 Ecosystem Parameters

For the pristine environment scenario we assume that the area of land under consideration has had no history of prior land conversion. Initially the entire area is therefore dedicated to biodiversity conservation, $R = 1$ and we are dealing with a fully functional and resilient ecosystem. In our model ecosystem resilience and health is represented by the population density parameter D, which we set equal to its maximum value, $D = 1$. Unless explicitly tested for the extinction function is assumed to be convex, $f(v) = v^2$, implying that the ecosystem is tolerant to small scale disturbance and significant ecological response is only triggered as the disturbance, represented by the rate of land conversion, v, increases to high levels. The ecological literature has a distinct lack of empirical tests of the phenomenon of evolutionary rescue and gives little guidance as to what value the probability of evolutionary rescue, β, should take. Gomulkiewicz and Holt [1995], suggest low likelihoods of such an event occurring. Given this conjecture we assume a five per cent probability of evolutionary rescue taking place. Fortunately, preliminary tests conducted on this parameter concluded that the outcome of the model is not very sensitive to the value of β, as long as it remains below ten per cent.

5.1.3 Normative Parameters

The social planner is characterised by a constant relative risk aversion of $\varepsilon = 7$, a four per cent rate of time preference and a relative consumption preference of $\gamma = 2$ for the private over the public good.

The magnitude of the parameter of relative risk aversion, ε, considered here may seem large in the light of the majority of estimates from empirical and experimental studies ranging from 0.3 to 2.5 ([Dubois and Vukina, 2003], [Johansson-Stenman et al., 2001]). However a number of arguments support this level. Empirical studies on aversion to environmental risk are rare and if studied they limit themselves to estimating risk premie using willingness to pay for risk reduction [Carson and Mitchell, 2000] or environmental damage

compensation, therefore giving little guidance in terms of choosing a parameter value for ε [Earnhart, 2000]. Most empirical work on risk aversion has been carried out in the area of insurance and investments, with only few exceptions such as studying livestock production contract data to reveal that hog growers' risk aversion lies in the range of 0.3 to 2.5 with a mean of 0.5 [Dubois and Vukina, 2003]. Despite these results seemingly contradicting the choice of ε, a particularly interesting phenomenon, often referred to as the equity premium puzzle, emerges from empirical studies on bond versus stock investments. It concerns the question of why people have been investing in bonds when the average annual real return on bonds is less than one per cent as compared to the equivalent return on stocks of seven per cent. Explaining the phenomenon purely on the grounds of risk aversion, places the estimated parameter value necessary to explain investor behaviour in excess of 30 [Johansson-Stenman et al., 2001]. Benartzi and Thaler [1995] investigate this phenomenon empirically and suggest myopic loss aversion as a possible explanation, a rationale that could similarly be applied to the loss of valuable ecological goods.

Many experimental studies are based on gambles with immediate positive wealth consequences, taken by individuals who are assumed to act according to expected utility theory. The degree of risk aversion derived from these studies may underestimate the true level of risk aversion in situations where losses, not gains, are at stake. It has been shown that individuals' attitude to risk is asymmetric with greater aversion to incurring losses than to the risk of not winning. This concept of loss aversion has been incorporated in prospect theory, which defines utility over relative gains and losses rather than absolute wealth. Benartzi and Thaler [1995] use prospect theory to explain why individuals act with more prudence if confronted with a one-off gamble involving relative gains and losses than if the same gamble was repeated several times. The link between loss aversion and one-off games is relevant to this study in that the social planner has only one chance of choosing the optimal rate of land conversion in the steady state all the while being confronted with the risk of irreversibly loosing biodiversity value. The combination of the risk of incurring irreversible ecological losses and playing the game only once justifies a higher value of the risk aversion parameter, ε.

Another criticism of many experimental studies of risk aversion concerns the lack of consideration given to the relationship between the individual's attitude to risk and the time horizon under which they are operating. Praag and Booij [2003] found that parameter values of relative risk aversion of

$0.5 < \varepsilon < 2.5$ are usually associated with extremely short time horizons of approximately two weeks, translating into a discount rate of 56 per cent. These figures emerge in studies where obtainable profits are negligible relative to the individual's wealth and are being earmarked for immediate consumption rather than long-term saving. In a lottery experiment with large prizes, inducing a longer time frame, they found a negative relationship between the degree of risk aversion and the rate of time preference used. Praag and Booij [2003] estimate the average value of relative risk aversion to be 1.54, with a standard deviation of 3.78 and a range from 0.01 to 39.99. In an experimental study of testing the subjects' preferences for the wellbeing of two generations ahead Johansson-Stenman et al. [2001] find that besides the expected cluttering of 32 per cent of individuals exhibiting relative risk aversion parameters of between 0.5 and 3, 19 per cent of individuals exhibited parameters of relative risk aversion of 8 and higher. These results make apparent the difficulty involved in estimating individuals' true attitudes to risk. In our model of ecological risk aversion, the social planner's rate of time preference of $r = 0.04$ is assumed to correspond to some long term savings rate such as a long term deposits. In addition, the four per cent rate of time preference is also within the range estimated by Praag and Booij [2003]. In their study the average subjective time discount rate of the respondents is 2.41 ranging from 0.01 to 6.99. In view of the large range of risk aversion found by Praag and Booij [2003] and Johansson-Stenman et al. [2001] and the negative relationship between the rate of time preference and the parameter of risk aversion we feel that our combination of $r = 0.04$ and for ε of 7 is justified.

The relative consumption preference of the private good over the public good, remains the last normative parameter to be chosen. We chose $\gamma = 2$, implying that the social planner prefers to consume twice as much of the private good than of the public good. While the choice of γ is arbitrary a number of tests, allowing for variations in the value or relative consumption preferences were conducted with the results explained in more detail in section 5.3.

5.2 Managed Environment Case

So far no consideration was given to the possibility that parts of the land in question may already have been converted in the past and that the state of the environment is no longer pristine. To allow for this possibility we intro-

duce a second scenario, which we refer to as the managed environment case. In this scenario, the size of the reserve land is reduced to half its original size, therefore $R(t)$ and $L(t)$ are 0.5 respectively. Considering the history of disturbance it is reasonable to assume that some degree of environmental deterioration has taken place during the course of disturbance. The consequences of disturbance are reflected in the reduction of ecosystem health to $D = 0.7$ as compared to $D = 1$ in the pristine environment scenario (see Table 1). The same argument could support the use of a revised, perhaps linear or concave, ecological response function to reflect the history of disturbance and increased sensitivity to potential disturbances in the future. We test this argument by running the same analysis as in Figure 4 of section 4 for the managed land case. Figure 6 shows that the the optimal $\hat{v}$ ranges from zero to five - about half the range of the pristine case. It shows that especially concern over much higher rates of land conversion becoming optimal as the ecosystem is run down and not worth preserving, are not warranted. The reason for this outcome may lay in the reduced potential value of the reserve land due to its reduced size. For these reasons and for ease of comparison the ecological response function remains the same, i.e. $f(v) = v^2$, for both scenarios. Similarly for comparison reasons all other parameter values remain unaltered and can be viewed in summary in Table 1. Under the managed environment case the optimal rate of land conversion is 3.4 per cent, compared to an optimal 5.2 per cent rate of economic growth and land conversion in the pristine case.

5.3   Model Sensitivity to Parameter Values

The analytical analysis conducted in section 3.3 revealed the importance of the relative value the partial production elasticity, α in determining whether marginal private good utility is positive or negative for a given rate of land conversion. Preliminary tests confirmed this theory, especially regarding the relative size of α and γ values and the rate of time preference, r, whereas tests on other parameter values such as β or δ showed the model's outcomes to be quite robust to variations in these parameter values. The aim of this section is to test the robustness of the results under both scenarios over a range of values of the parameters to which the model is known to be sensitive to.

Figure 7 demonstrates the robustness of the estimated optimal rates of land conversion to changes in partial production elasticity and relative con-
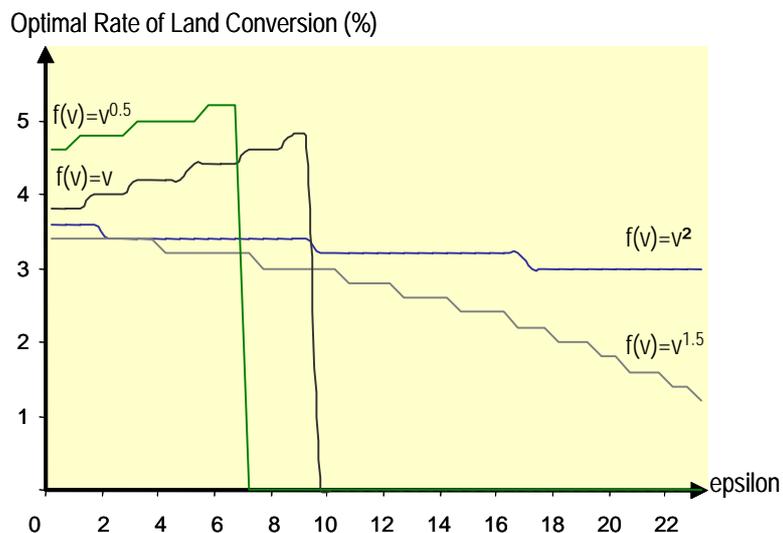
Optimal Rate of Land Conversion (%)

Figure 6: Optimal rates of land conversion for a managed environment for varying degrees of risk aversion and ecological resilience

sumption preferences under the pristine environment (panel a) and the managed environment scenario (panel b). For a partial production elasticity of 0.7 and a consumption preference of $\gamma = 2$ the model returns optimal rates of land conversion of 5.2 and 3.4 per cent for the two scenarios respectively (Table 1). The shaded area in Figure 7 shows the range of consumption preferences over partial production elasticities which yield optimal conversion rates in line with those estimated under the base case scenarios. Rounding errors limit the accuracy of the test outcome to plus/minus one percentage point. In the pristine case the five per cent result is robust for an α -range between 0.5 and 0.7. This means that the normative parameter of relative consumption preference of the private over the public good matter little in an economy characterised by such production elasticities for land and given a rate of social time preference of four per cent. As the production elasticity in the agricultural sector with respect to land increases, the model's results become more sensitive to the social planner's consumption preferences. The range of γ for which a five per cent rate of land conversion is still optimal narrows from $\gamma = [2.37; +\inf]$ for $\alpha = 0.6$, down to $\gamma = [1.69; 1.97]$ for $\alpha = 0.85$. It is interesting that undercutting the lower gamma threshold yields lower
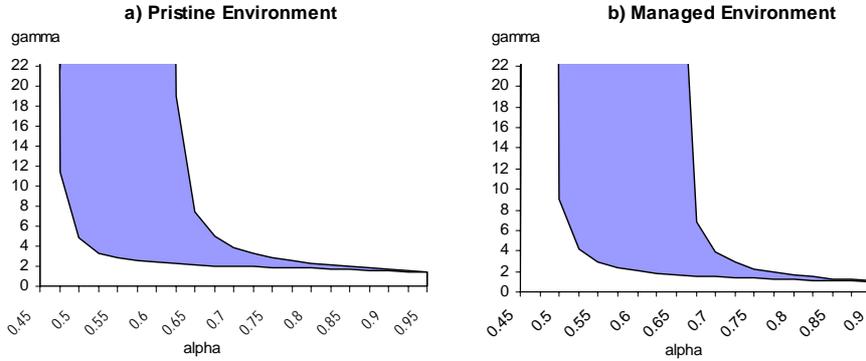
Figure 7: Robustness of base case outcomes to varying partial production elasticities and relative consumption preferences

optimal rates of land conversion, with the decrease proportional to the size of the undercut, as opposed to overshooting. For instance, doubling the value for γ in an economy whose private good sector is highly elastic with respect to the land input returns an optimal rate of land conversion of 24 per cent. The optimal rate of land conversion in the managed land scenario is three per cent. The range of γ for which this rate is optimal looks similar to the range in the pristine case, i.e. it narrows for higher values of α and a threshold for α exists below which a rate of land conversion as high as three per cent is not optimal (Figure 7b). While this threshold kicks in earlier than in the pristine case, the range of permissible γ is somewhat wider in the managed case, suggesting more room for heterogenous consumption preferences before the optimal conversion rate has to be adjusted.

Figures 8a and b show the model's sensitivity to the choice of interest rate for the pristine environment and managed environment scenarios. The γ-range for given values production elasticities in the private sector that warrants steady state conservation of a pristine area is larger for a ten per cent rate of time preference as opposed to a one per cent rate of time preference. This result is somewhat surprising given the widespread argument in favour of using low interest rates in support of environmental resource conservation. Figure 8b describes the results of the same analysis for the managed land case. Interestingly, for private good production that is inelastic with
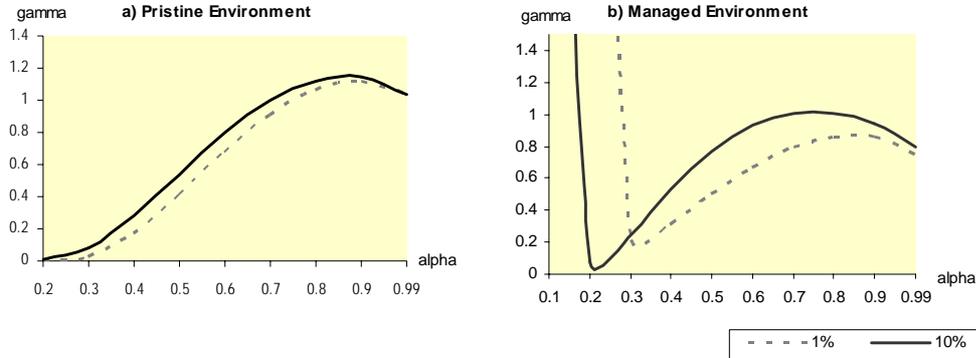
Figure 8: Sensitivity of conservation outcome to changes in production elasticity and consumption preferences for high and low rates of time preference

respect to land, further land conversion never becomes optimal even for infinitely high consumption preference for the private good over the public good. This can be explained by the fact that the existing allocation of land to the private sector already ensures some level of private good production and consumption. The small contribution of agricultural land to total private good production means that comparatively more benefit is gained by conserving the reserve land than by converting it for use in the private sector.

The aim of this section was to explore some attributes and robustness of the model with respect to the parameter values chosen, while relaxing some of the assumptions made. Where the model showed some initial sensitivity we defined the ranges of the parameter values for which the results of the model are credible and robust. The following section deals with the implications of introducing ecological risk and uncertainty as we have done here for the growth and environment debate.

6 Implications for Growth and Environment Debate

The aim of this paper was to allow for greater complexity when modelling the dynamics of the economic-ecological system. We introduce ecological risk

36

and uncertainty as characteristics of a dynamic environmental constraint to an exogenous growth framework. The environmental constraint is modelled as a stochastic variable, reflecting the considerable uncertainty surrounding the ecological response to anthropogenic disturbance. The model is a simple two-sector model and consists of a neoclassical private good sector and a public good sector, which provides the framework for modelling a stochastic and dynamic biodiverse ecosystem.

Using an additive social welfare function with linear private good utility and a concave public good utility function it was shown that the introduction of ecological stochasticity and risk aversion has ambiguous effects on the optimal steady state rate of economic growth and environmental impact, especially in cases where some positive level of growth is optimal in the long run. The analysis emphasised the importance of having information on the ecological response function to anthropogenic disturbance for determining magnitude and direction of the effect of allowing for ecological risk on optimal growth. If environmental impacts are assumed to increase exponentially as the level of disturbance increases, allowing for ecological risk tends to slow long run economic growth. A combination of low risk aversion and linear or concave ecological response functions leads to accelerated economic growth and environmental impact being optimal if confronted with ecological risk and uncertainty, whereas planners characterised by high levels of risk aversion should opt for conservation and a precautionary approach if confronted with such ecological circumstances.

Sensitivity analyses conducted to test the robustness of results showed that the model is sensitive to some positive parameters such as the prevailing production elasticities in the private sector with respect to land as well as the initial conditions of land allocation and history of disturbance. Normative parameters such as consumption preferences for the private over the public good and the degree of risk aversion are also important as is the choice of the social rate of time preference albeit not in the sense that is commonly advocated by conservationists.

The way the model is specified implies that the driving variable behind economic growth is the rate of land conversion. Specifically, all variables in the private sector, that is production, consumption and the capital stock, grow at the rate at which land supporting a biodiverse ecosystem is converted for the use in the private sector. Within an exogenous growth framework, these results yield ambiguous conclusions for the optimal growth rate along the balanced path if ecological stochasticity and risk aversion are allowed for.

37

Modelling the environment in such a way may accelerate or slow optimal economic growth, leading to increased or reduced rates of anthropogenic disturbance of the ecosystem. However, the results of the model show that the higher the degree of risk aversion the higher the level of cautiousness that should be applied to natural resource decisions.

The number of studies concerned with investigating the impact of environmental stochasticity on economic growth is limited. In addition, the present study assumes that the environmental quality appears only in the public good utility function and not in the production function of the private sector. Given this situation, it is difficult to draw explicit comparisons between the findings of this study and those from more than a handful of other pieces of work. Krautkraemer [1998] finds that introducing environmental constraints to economic growth leads to the emergence of a growth drag. This study finds that introducing an additional constraint, that of uncertainty, with respect to the environmental outcome, supports the growth drag argument in some circumstances, especially those of high risk aversion by the social planner and convex ecological response functions. Barrett [1992] (p.279) seminal finding that 'the fate of the natural environment depends as much on society's ethical views as on positive economic parameters' is confirmed here. Both the parameter γ, which can be interpreted to represent the relative preference between the two goods and the degree of risk aversion ε play important roles in determining the optimal rate of economic growth in the long run. Adding to Gradus and Smulders [1993] study of the impact of a shift in preferences for a cleaner environment on economic growth, the results of this model suggest that a shift in attitude from being neutral to adverse to environmental risk strengthens the case for environmental conservation and leads in most cases to results supporting a more cautious approach to economic growth where risks of causing ecological damage are prevalent. The finding that the introduction of environmental stochasticity increases the range of parameter values for which corner solutions are optimal confirms Hofkes [1996] result that increased complexity in the modelling of a natural resource leads to tighter ranges of parameter values for which a balanced growth path can be achieved.

Appendix A

Equations (2-4), yield the expression for consumption of the private good as a function of K:

$$C = X - \delta K - \dot{K}$$
$$= AK^{1-\alpha}L^\alpha - \delta K - \dot{K}$$

Replacing K with the solution for the optimal time path of capital, $\hat{K}$, (see equation (25)) yields the optimal time path for private good consumption $\hat{C}$:

$$\hat{C} = A\left[\left(\tfrac{1-\alpha}{r+\delta}\right)^{1/\alpha} A^{1/\alpha}L\right]^{1-\alpha} L^\alpha - \delta\left(\tfrac{1-\alpha}{r+\delta}\right)^{1/\alpha} A^{1/\alpha}L - \left(\tfrac{1-\alpha}{r+\delta}\right)^{1/\alpha} A^{1/\alpha}\dot{L}$$
$$= \left(\tfrac{1-\alpha}{r+\delta}\right)^{1/\alpha} A^{1/\alpha}\left[\left(\tfrac{r+\delta}{1-\alpha}\right) L - \delta L - \dot{L}\right]$$

From equation (26) it follows that the expression in front of the square brackets equals the constant capital-land ratio.

$$\hat{C} = \tfrac{\hat{K}}{L}\left[\left(\tfrac{r+\delta}{1-\alpha} - \delta\right) L - \dot{L}\right]$$
$$= \hat{K}\left[\tfrac{r+\delta}{1-\alpha} - \delta - \tfrac{\dot{L}}{L}\right]$$

We know from equation (26) that $\frac{\dot{L}}{L} = v$ and therefore, the optimal time path of private good consumption equals:

$$\hat{C}(t) = \hat{K}(t)\left[\frac{r+\delta}{1-\alpha} - \delta - v\right]$$

Appendix B

The present value of the program obtained by replacing private good consumption C in equation (13) with $\hat{C}$ (from (29)), private good consumption along the path that maximises the present value social welfare from private good consumption. P henceforth replaces the ratio denoting the share of the total present value welfare derived from the public good in the equation below.

$$
\begin{aligned}
Z &= \int_0^\infty e^{-rt}\gamma C dt + \frac{D_0 R_0}{r+v-(\beta-1)f(v)} \\
&= \int_0^\infty e^{-rt}\gamma \left(\frac{1-\alpha}{r+\delta}\right)^{1/\alpha} A^{1/\alpha} L \left(\frac{r+\delta}{1-\alpha} - \delta - v\right) dt + P \\
&= \gamma \left(\frac{1-\alpha}{r+\delta}\right)^{1/\alpha} A^{1/\alpha} \left(\frac{r+\delta}{1-\alpha} - \delta - v\right) \int_0^\infty e^{-rt}\left(1 - e^{-vt}R_0\right) dt + P \\
&= \gamma\frac{K}{L}\left(\frac{r+\delta}{1-\alpha} - \delta - v\right)\left[\int_0^\infty e^{-rt}dt - R_0\int_0^\infty e^{-(r+v)t}dt\right] + P \\
&= \gamma\frac{K}{L}\left(\frac{r+\delta}{1-\alpha} - \delta - v\right)\left(\frac{1}{r} - \frac{R_0}{r+v}\right) + P
\end{aligned}
$$

Simplification and replacing P with the public good constant yields the maximum present value of the program from the consumption of the private good along the optimal path and the present value welfare from the public good.

$$
Z = \gamma\frac{K}{L}\left(\frac{r+\delta}{1-\alpha} - \delta - v\right)\left(\frac{v + rL_0}{r(r+v)}\right) + \frac{D_0 R_0}{r+v+(1-\beta)f(v)}
$$

References

S. Barrett. Economic growth and environmental preservation. *Journal of Environmental Economics and Management*, 23:289–300, 1992.

S. Benartzi and R. H. Thaler. Myopic loss aversion and the equity premium puzzle. *The Quarterly Journal of Economics*, 110(1):73–92, 1995.

A. Bovenberg and S. Smulders. Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. *Journal of Public Economics*, 57:369–391, 1995.

S. Carpenter and K. Cottingham. Resilience and restoration of lakes. *Conservation Ecology*, 1:2, 1997.

R. T. Carson and R. C. Mitchell. Public preferences toward environmental risks. the case of trihalomethanes. Working Paper 2000-21, Department of Economics, University of California, 2000.

A. C. Chiang. *Fundamental Methods of Mathematical Economics*. Mc Graw-Hill Book Company, 3rd edition, 1984.

C. Cicchetti and A. Freeman. Option demand and consumer surplus: Comment. *Quarterly Journal of Economics*, 85:523–527, 1971.

P. Dubois and T. Vukina. Grower risk aversion and the cost of moral hazard in livestock production contracts. Working Paper 2003-08, Institut National de la Recherche Agronomique, 2003.

D. Earnhart. Liability for past environmental contamination and privatization. Working Paper 302, Department of Economics, University of Kansas, 2000.

J. Elton and M. Gruber. *Modern Portfolio Theory and Investment Analysis*. John Wiley and Sons, US, 5th edition, 1995.

R. Gomulkiewicz and R. Holt. When does evolution by natural seclection prevent extinction? *Evolution*, 49:201–207, 1995.

R. Gradus and S. Smulders. The trade-off between environmental care and long-term growth - pollution in three prototype growth models. *Journal of Economics*, 58:25–51, 1993.

G. Grummet and D. Stirzaker. *Probability and Random Processes.* Clarendon Press, Oxford, UK, 2nd edition, 1992.

M. Hofkes. Modelling sustainable development: An economy-ecology integrated model. *Economic Modelling*, 13:333–353, 1996.

O. Johansson-Stenman, F. Carlsson, and D. Daruvala. Measuring hypothetical grandparents' preferences for equality and relative standings. Working Papers in Economics 42, Department of Economics, Goeteborg University, 2001.

J. Krautkraemer. Nonrenewable resource scarcity. *Journal of Economic Literature*, 36:2065–2107, 1998.

P. Milgrom and J. Roberts. *Economics, Organization and Management.* Prentice Hall Inc., US, 1992.

C. Perrings. Resilience in the dynamics of economy-environment systems. *Environmental and Resource Economics*, 11(3-4):503–520, 1998.

B. M. v. Praag and A. S. Booij. Risk aversion and the subjective time discount rate: A joint approach. Discussion Paper TI 2003-018/3, Tinbergen Institute, 2003.